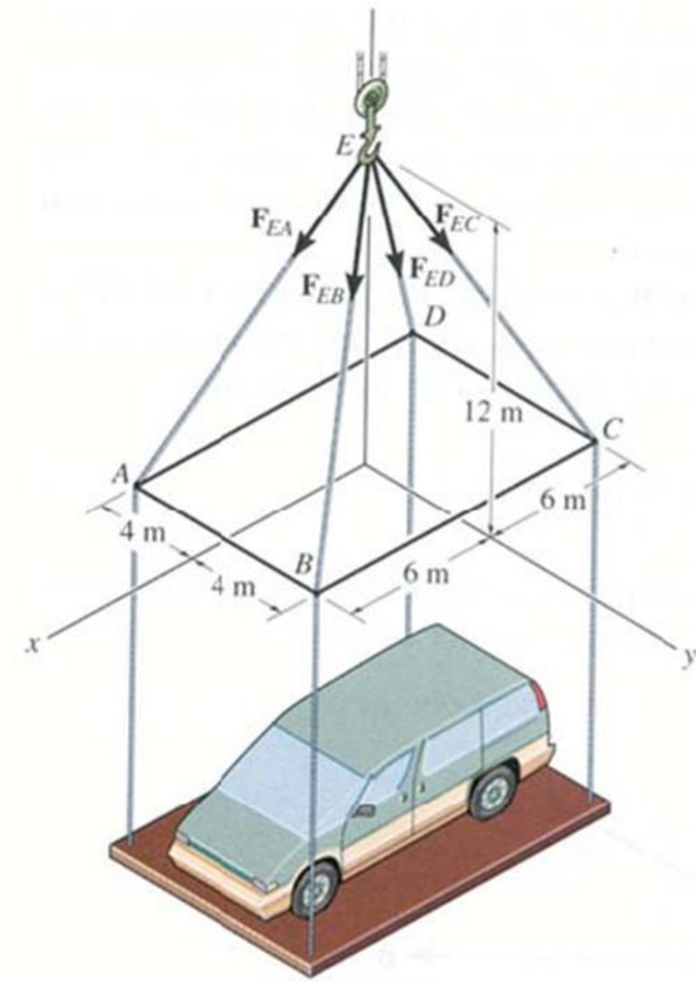
CHAPTER 2
SECTION B.

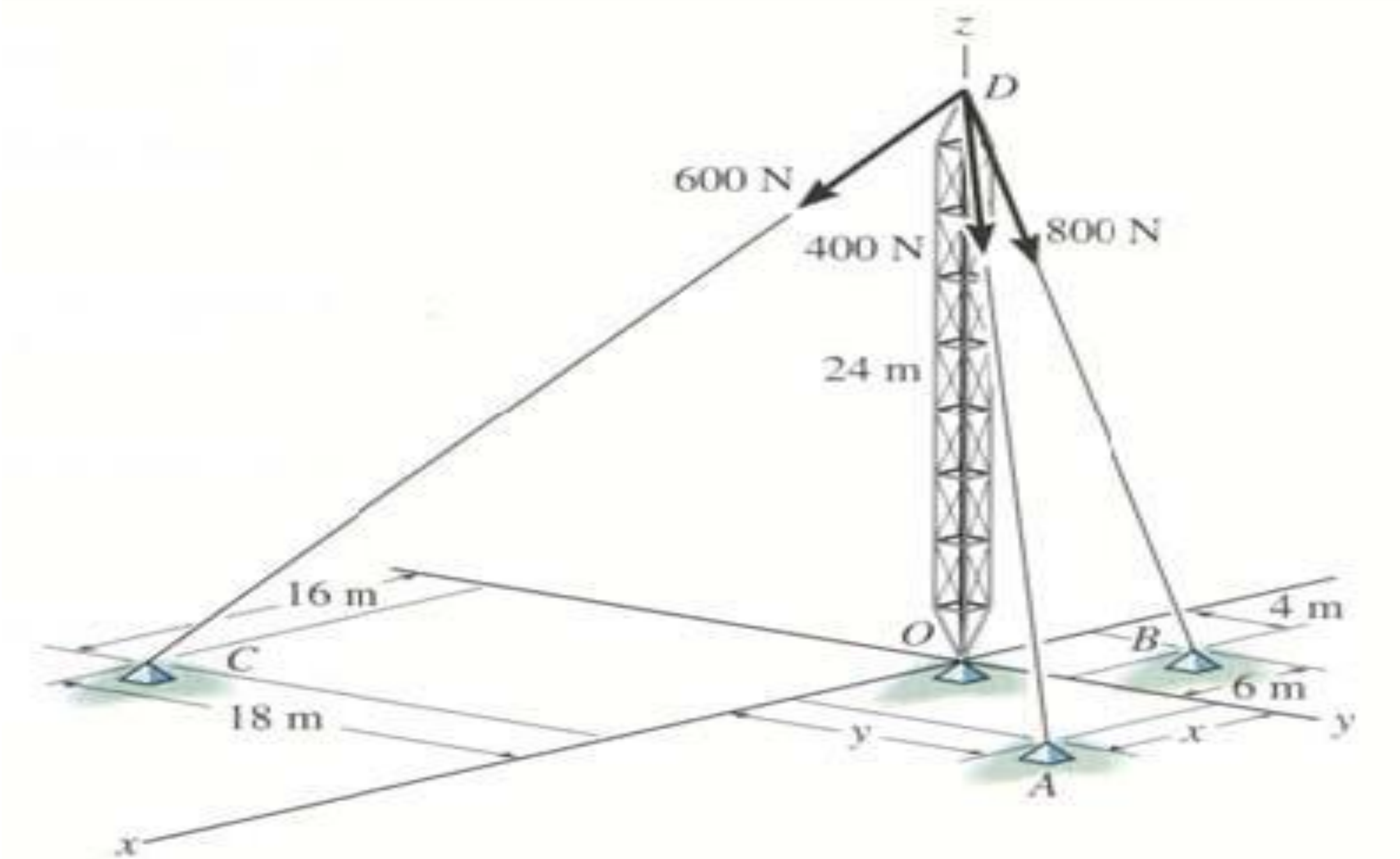
Three-Dimensional Force Systems

Three-dimensional Force Systems



Many problems in real-life involve 3-Dimensional Space.

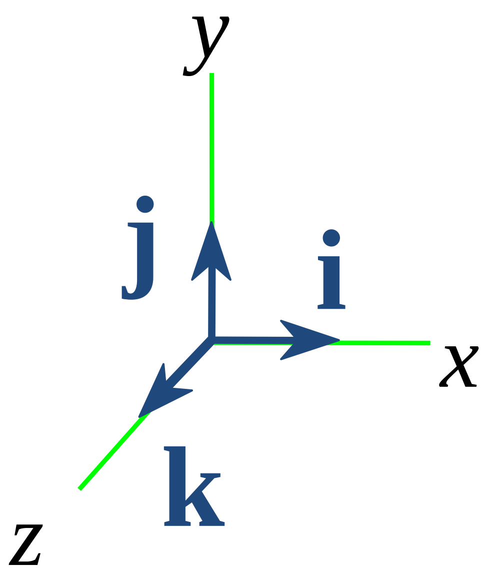
How will you represent each of the cable forces in Cartesian vector form?

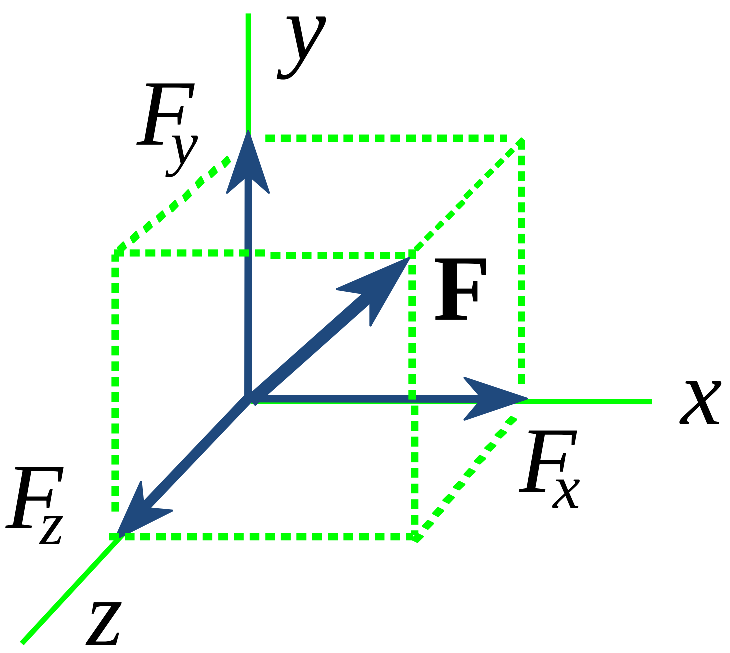


z
D
600 N
400 N
800 N
24 m
16 m
O
C
4 m
B
6 m
18 m
y
x
y
A
x

3D Rectangular Components

Important:
Use a *right-handed* coordinate system



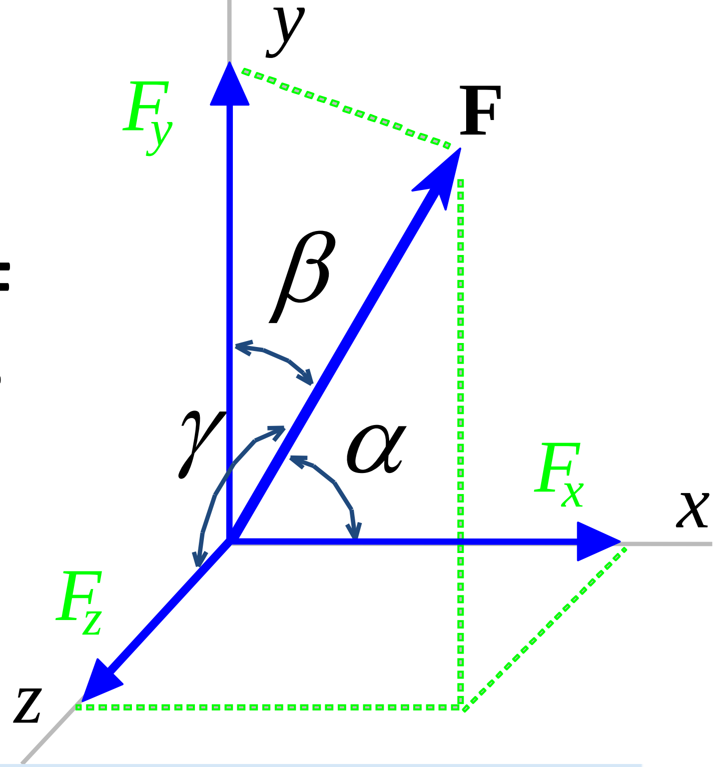


$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y + \mathbf{F}_z$$

$$= F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}$$

$$|\mathbf{F}| = F = \sqrt{F_x^{\,2} + F_y^{\,2} + F_z^{\,2}}$$

Direction angles & direction cosines



$\alpha(\theta_x)$ = angle between *x*-axis and **F**

$\beta(\theta_y)$= angle between *y*-axis and **F**

$\gamma(\theta_z)$ = angle between z-axis and **F**

Components

$$F_x = F\cos\theta_x \qquad F = \sqrt{F_x{}^2 + F_y{}^2 + F_z{}^2}$$

$$F_y = F\cos\theta_y \qquad \mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} \qquad \textbf{(2/11)}$$

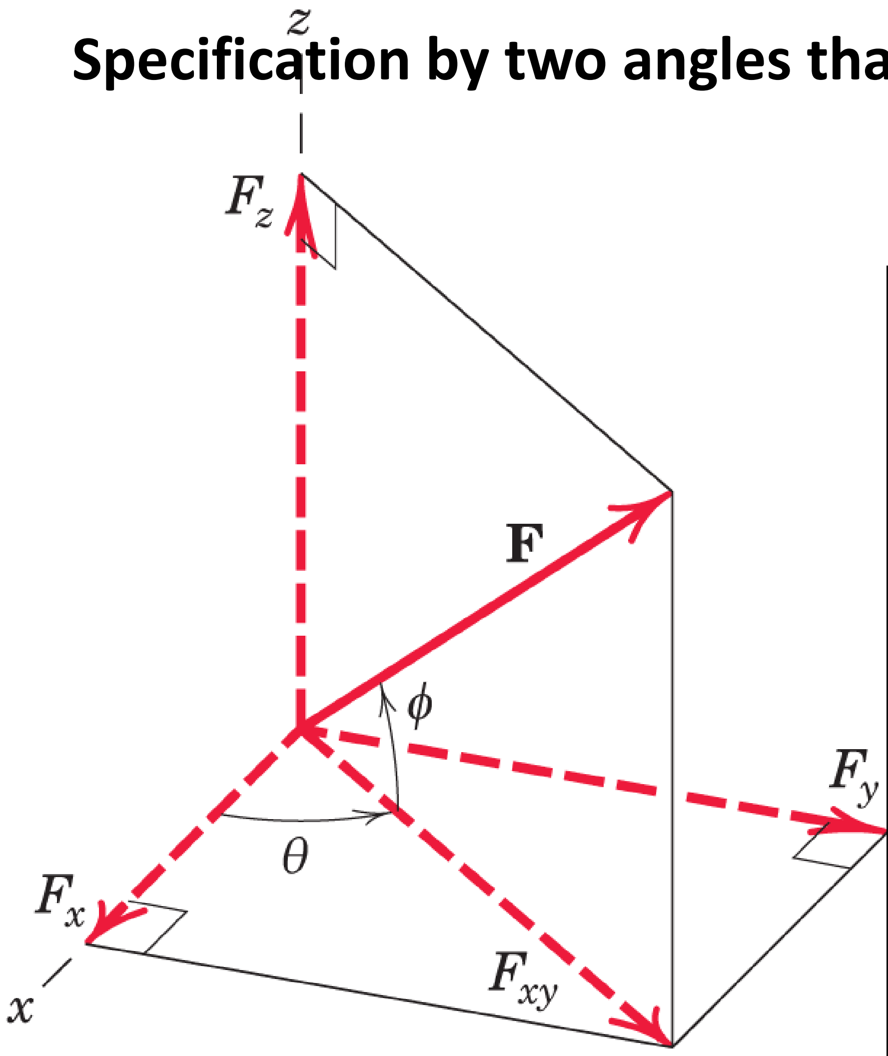
$$F_z = F\cos\theta_z \qquad \mathbf{F} = F(\mathbf{i}\cos\theta_x + \mathbf{j}\cos\theta_y + \mathbf{k}\cos\theta_z)$$

l=cosθ_x , m=cosθ_z , n=cosθ_z

$$\mathbf{F} = F(l\mathbf{i} + m\mathbf{j} + n\mathbf{k})$$

$$l^2 + m^2 + n^2 = 1$$

Specification by two angles that orient the line of action of the force



$F_{xy}=F\cos\phi$ $F_z=F\sin\phi$

$F_x=F_{xy}\cos\theta= F\cos\phi\cos\theta$

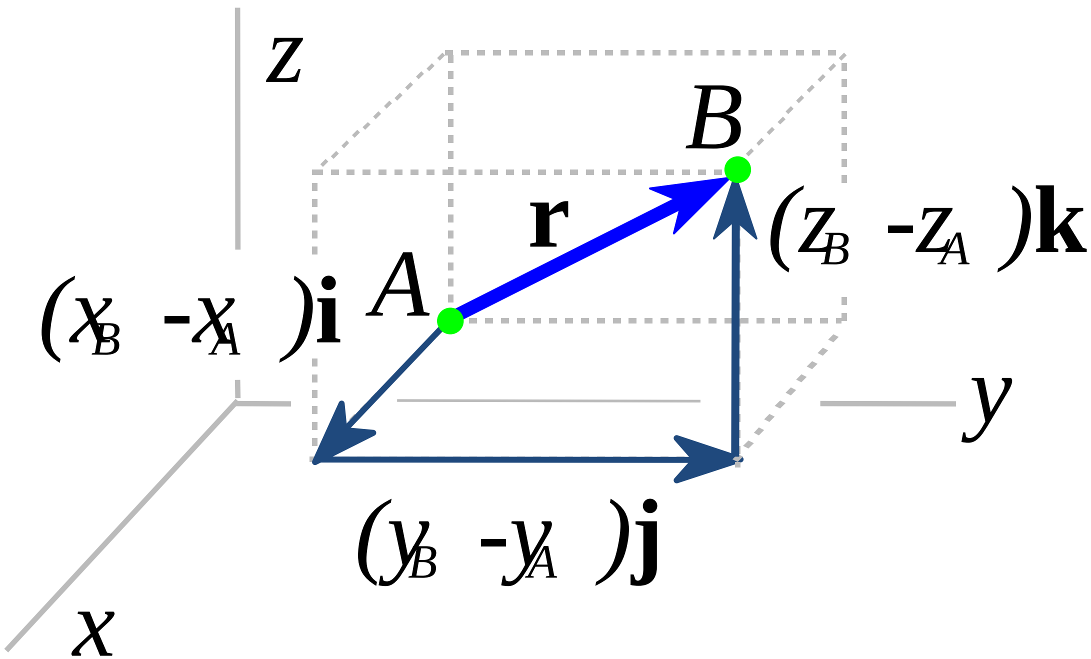
$F_y=F_{xy}\sin\theta= F\cos\phi\sin\theta$

Relative Position Vectors: The position of a point B relative to another point A:

$B(x_B, y_B, z_B)$ head

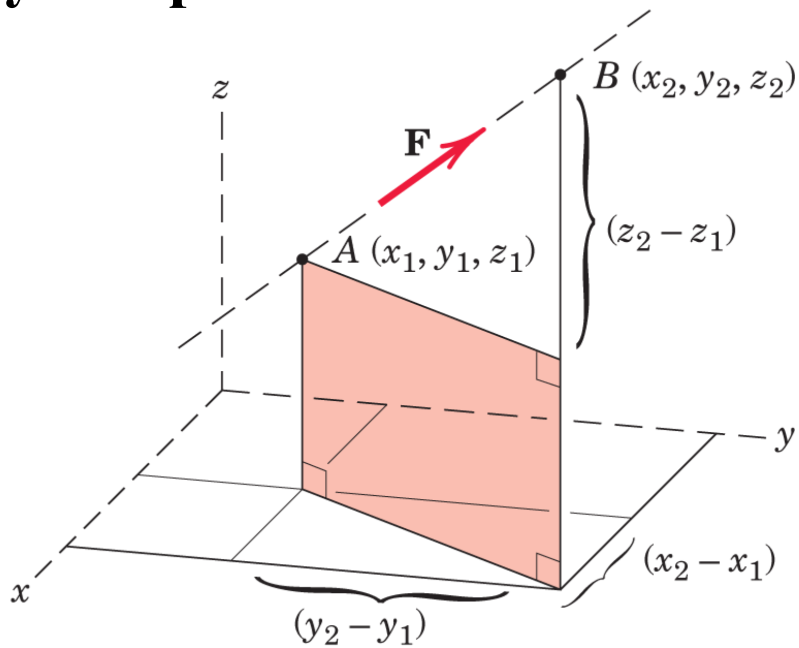
$A(x_A, y_A, z_A)$ tail

$$\mathbf{r} = (x_B - x_A)\,\mathbf{i} + (y_B - y_A)\,\mathbf{j} + (z_B - z_A)\,\mathbf{k}$$



Position vector components = coordinates of head - coordinates of tail.

Specification by two points on the line of action of the force



$$\mathbf{F} = F\mathbf{n}_F = F\frac{\overrightarrow{AB}}{\overline{AB}} = F\frac{(x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

- Unit vector $\mathbf{n} = \cos\alpha\ \mathbf{i} + \cos\beta\ \mathbf{j} + \cos\gamma\ \mathbf{k}$

$$\mathbf{F} = F(\cos\alpha\ \mathbf{i} + \cos\beta\ \mathbf{j} + \cos\gamma\ \mathbf{k})$$

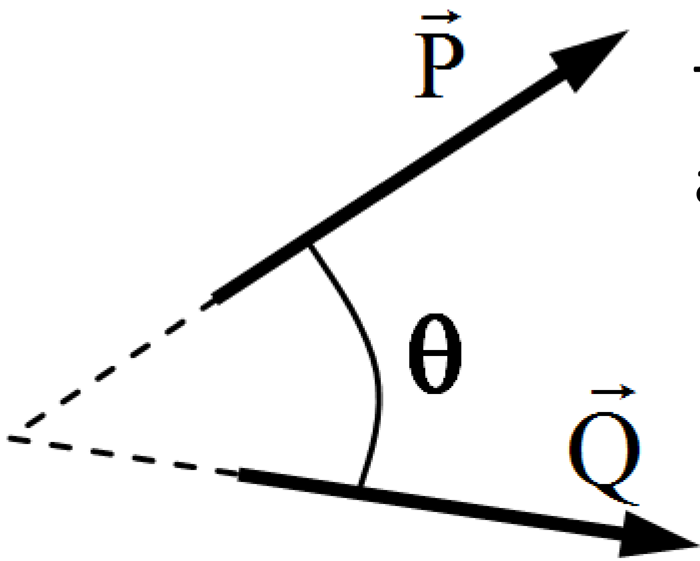
Force vectors directed along a line

Many of the structural members we will deal with will support forces that are directed along the axis of the member (e.g., cables, ropes, bars, often times columns, ...). Thus, by knowing the orientation of the member (position vector), we can conveniently express its force in vector form.

Procedure:

1) Determine the position vector $\mathbf{r}$ for the member.
2) Compute a unit vector along the axis of the member $\mathbf{n} = \mathbf{r}/r$.
3) The member's force vector is then $\mathbf{F} = F\mathbf{n}$.

Dot or scalar product



The dot product of vectors ***P*** and ***Q*** is defined as

$$\vec{P} \cdot \vec{Q} = PQ\cos\theta$$

Angle θ is the smallest angle between the two vectors and is always in a range of 0º to 180º.

Dot Product Characteristics:

1. The result of the dot product is a scalar (a positive or negative number).
2. The units of the dot product will be the product of the units of the ***A*** and ***B*** vectors***.***

Properties;

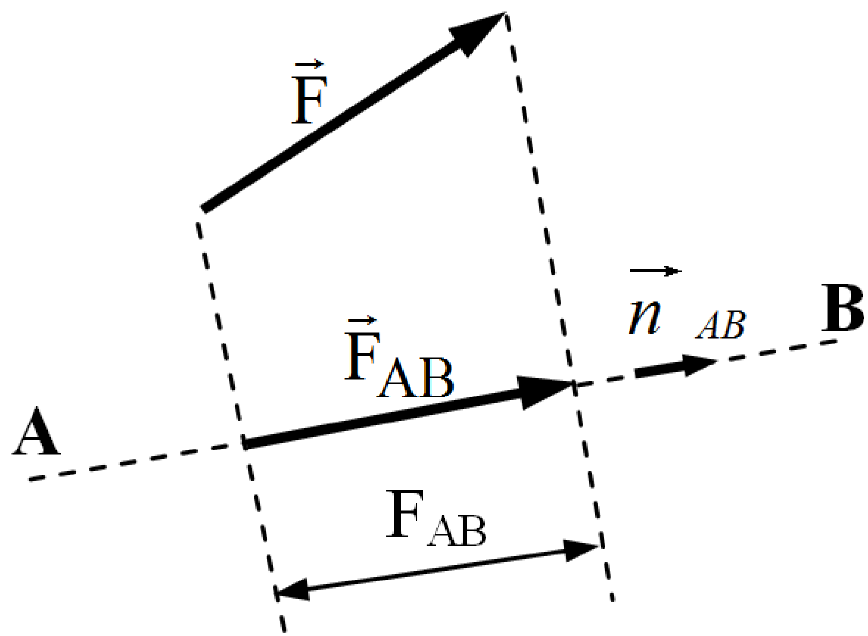
$\boldsymbol{i} \bullet \boldsymbol{j} = 0$; $\boldsymbol{j} \bullet \mathbf{k} = 0$; $\boldsymbol{i} \bullet \boldsymbol{k} = 0$

$\boldsymbol{i} \bullet \boldsymbol{i} = 1$; $\boldsymbol{j} \bullet \boldsymbol{j} = 1$; $\boldsymbol{k} \bullet \boldsymbol{k} = 1$

$$\boldsymbol{P} \bullet \boldsymbol{Q} = (Px\,\boldsymbol{i} + Py\,\boldsymbol{j} + Pz\,\boldsymbol{k}) \bullet (Qx\,\boldsymbol{i} + Qy\,\boldsymbol{j} + Qz\,\boldsymbol{k})$$

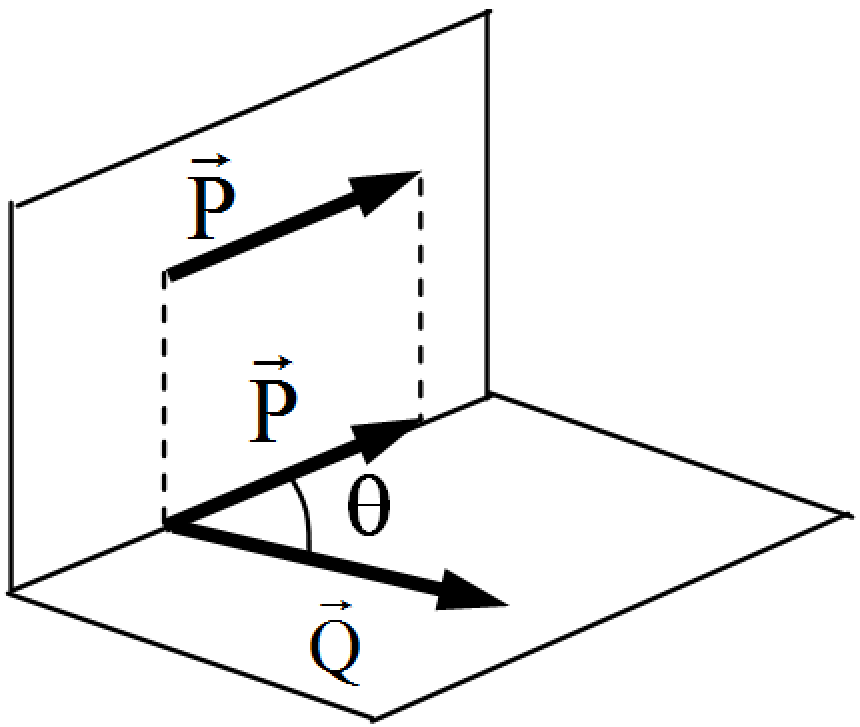
$$= Px\,Qx + PyQy + PzQz$$

The projection of the force along an axis



$$\vec{F}_{AB} = F_{AB}\ \vec{n}_{AB} \qquad F_{AB} = \vec{F} \cdot \vec{n}_{AB}$$

Angle between two vectors



$$\cos\theta = \frac{\vec{P}\cdot\vec{Q}}{PQ}$$

EXAMPLES

FORCE SYSTEMS

Three-Dimensional Force Systems-1

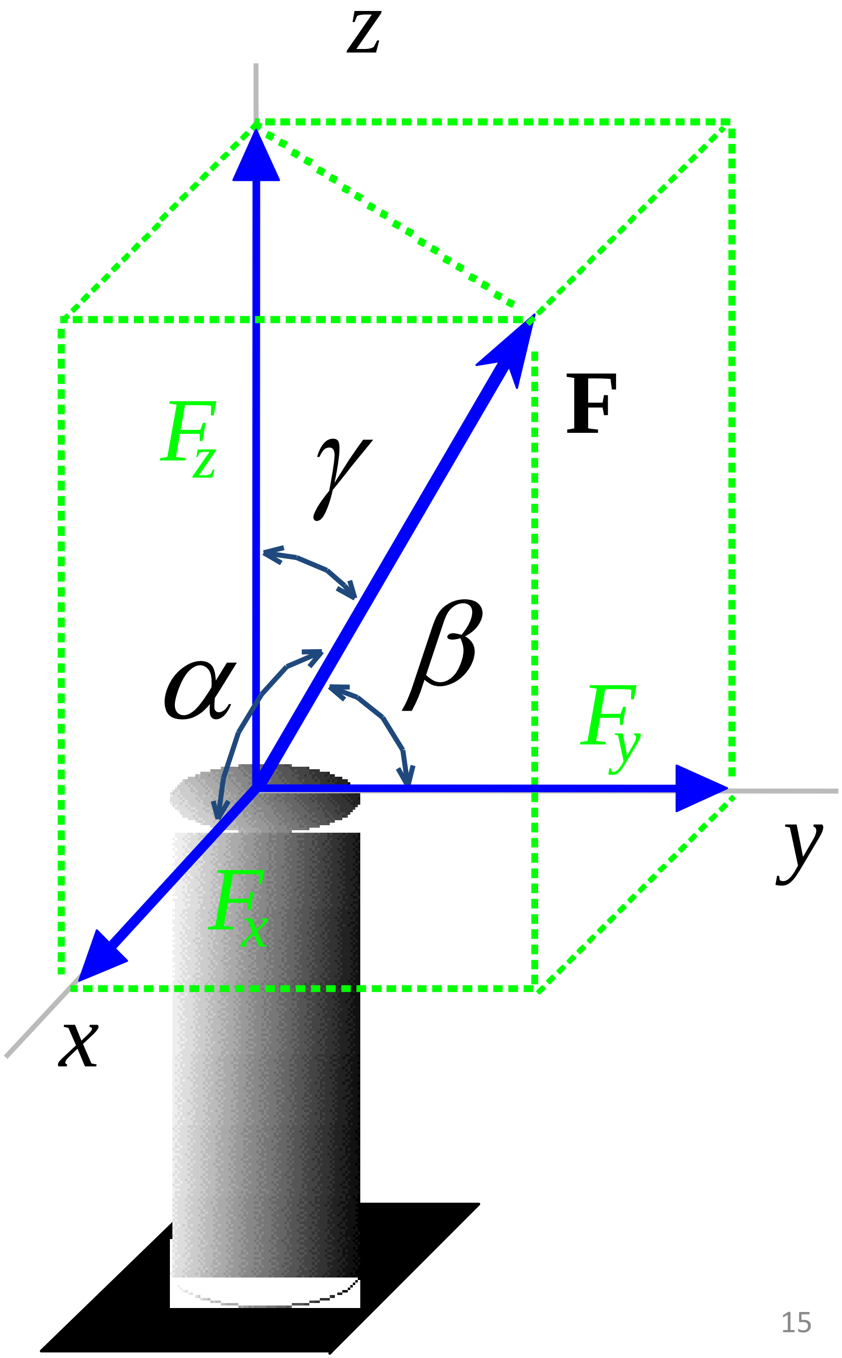
Example 1

A pole is subjected to a force **F** which has components F_x=1.5 kN and F_z=1.25 kN. If β = 75°, determine the magnitudes of **F** and **F**$_y$.

Answer:

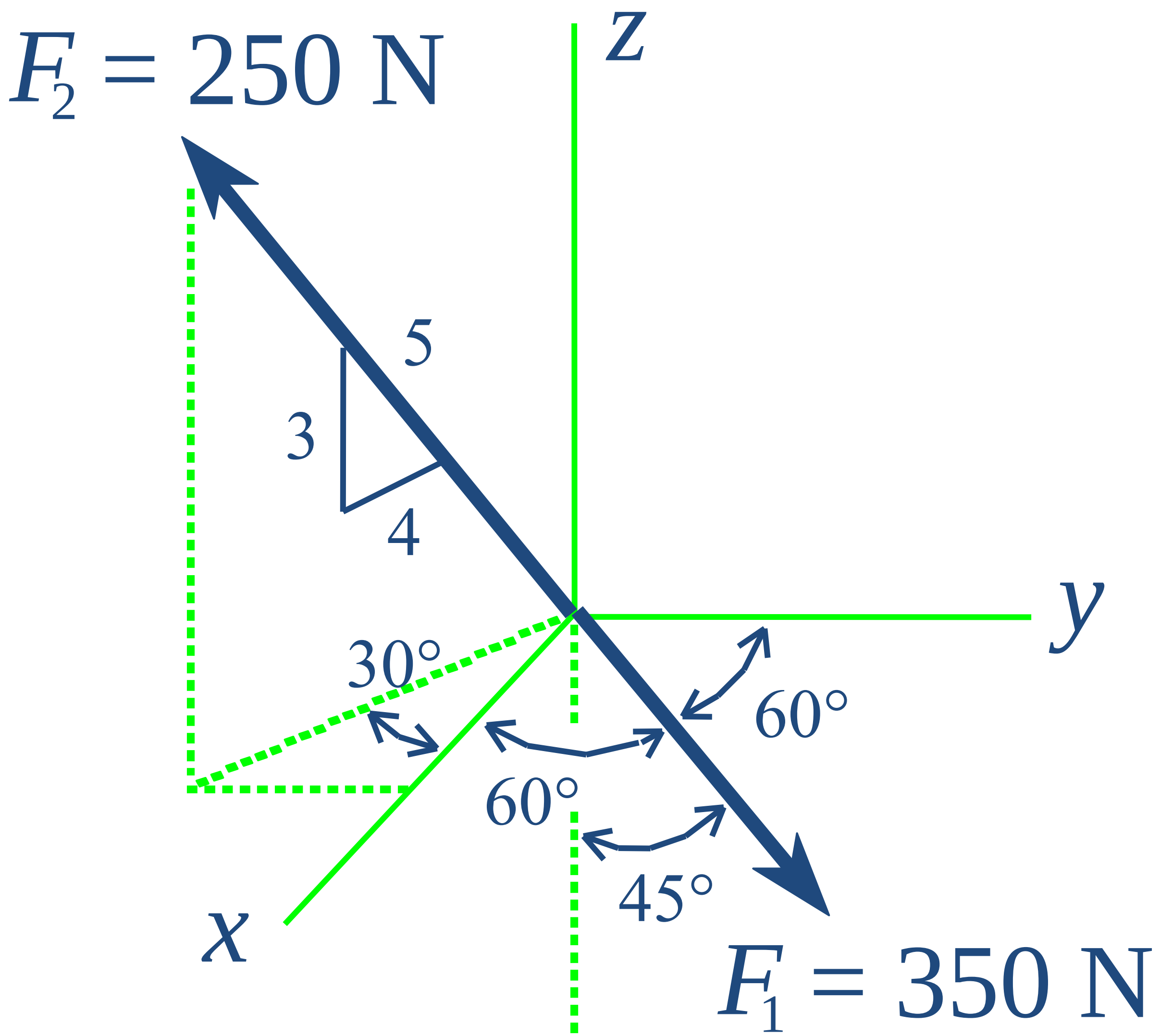
F = 2.02 kN

F_y = 0.523 kN

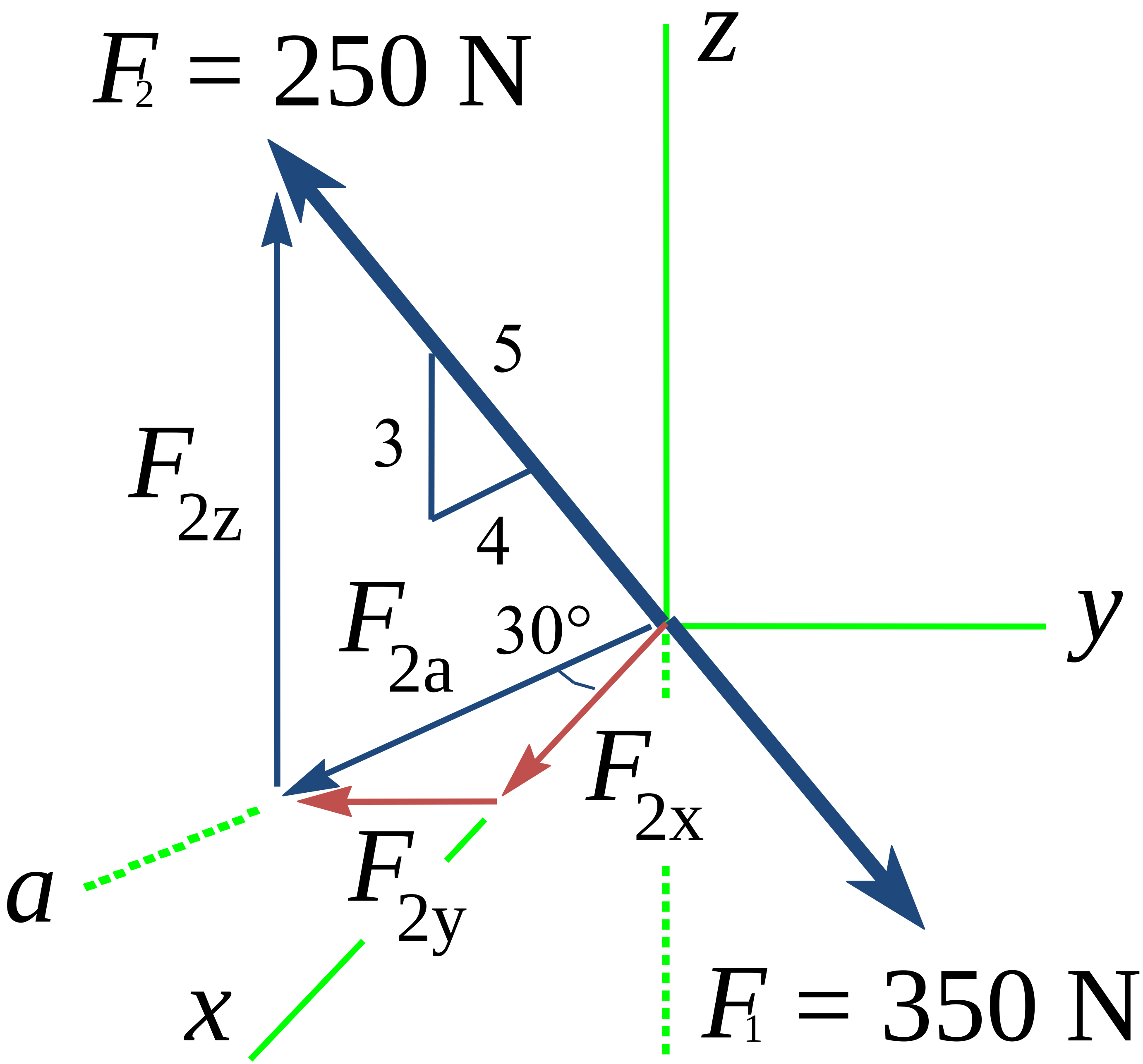


Example 2

Determine the magnitude and direction angles for the resultant force.



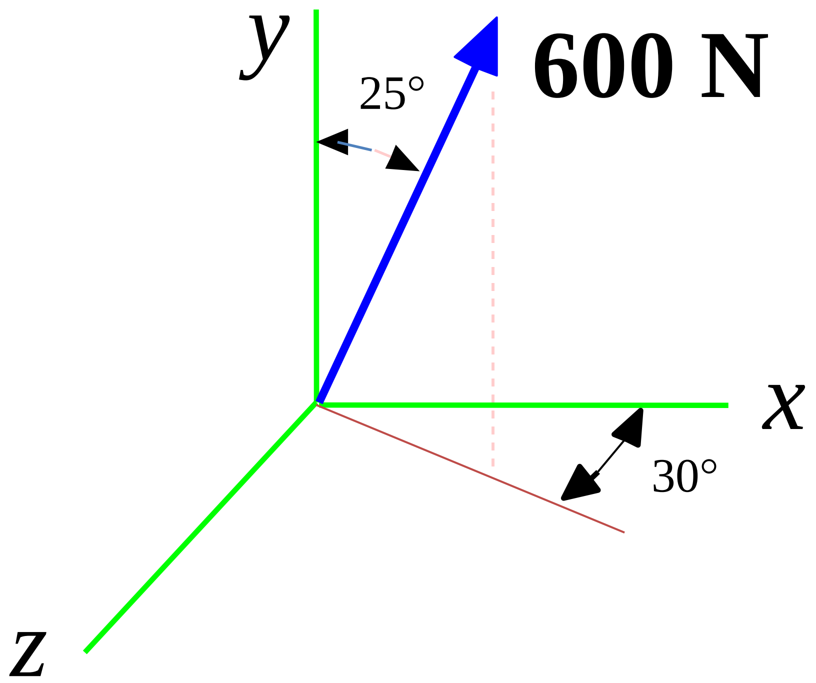
A:
R_x = 348 N
R_y = 75 N
R_z = -97 N
α = 19.4°
β = 78.3°
γ = 105°



$F_2 = 250$ N
z
5
3
4
F_{2z}
F_{2a}
30°
y
F_{2x}
F_{2y}
a
x
$F_1 = 350$ N

Example 4

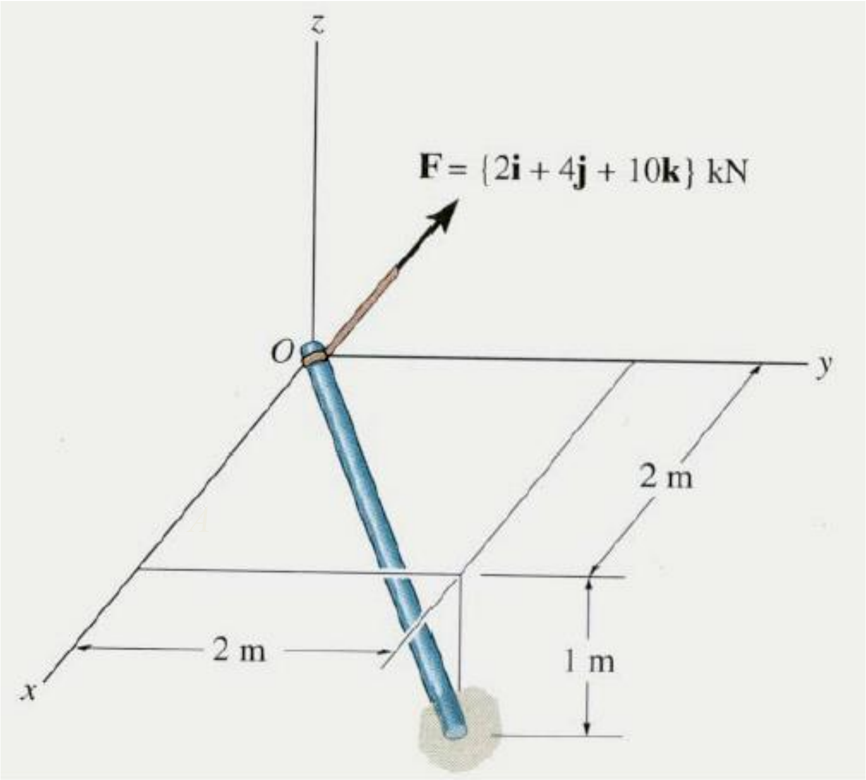
Determine the *x,y* and *z* components of the force vector shown. Also, determine the direction angles.



A: **F** = (220**i** + 544**j** + 127**k**) N, α = 68.5°, β = 25.0°, γ = 77.8°

Example 5

The magnitude of the projection of the force along the pole OA.



$\boldsymbol{r_{OA}} = \{2\ \mathbf{i} + 2\ \mathbf{j} - 1\ \mathbf{k}\}$ m

$r_{OA} = (2^2 + 2^2 + 1^2)^{1/2} = 3$ m

$\mathbf{n_{OA}} = 2/3\ \mathbf{i} + 2/3\ \mathbf{j} - 1/3\ \mathbf{k}$

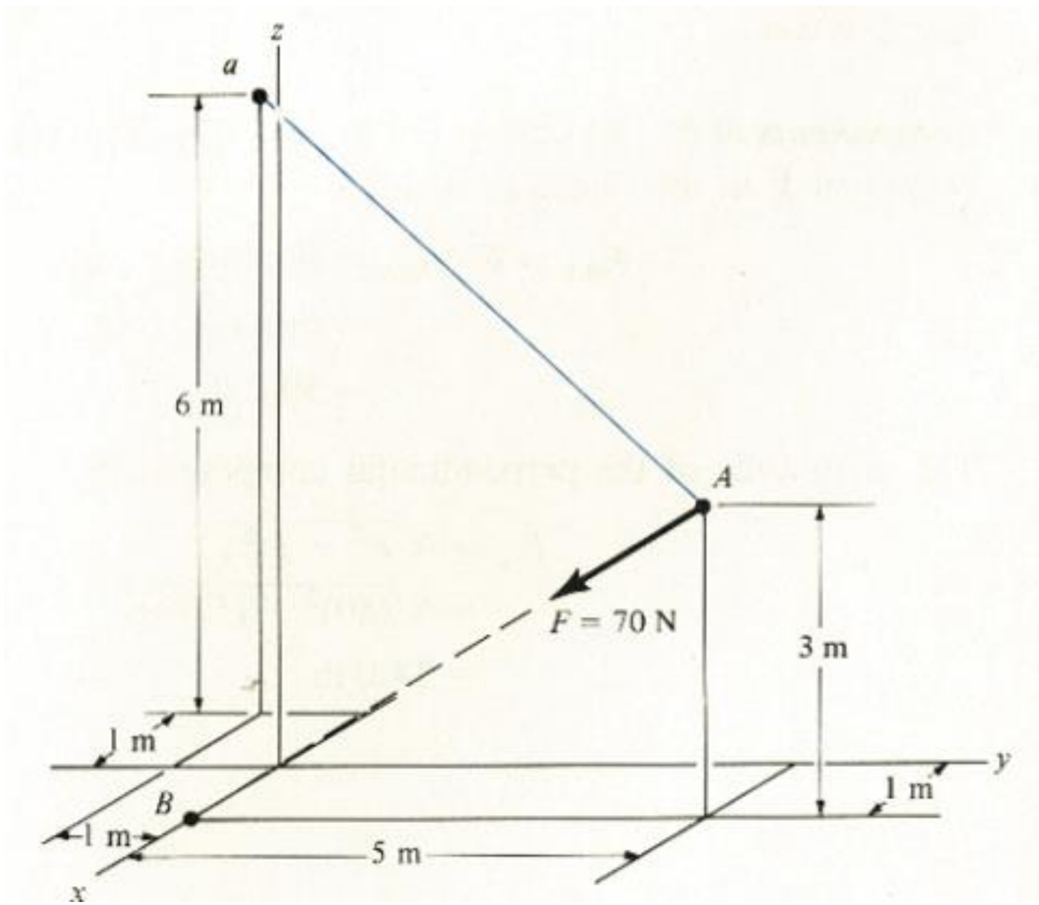
$\mathbf{F} = \{2\ \mathbf{i} + 4\ \mathbf{j} + 10\ \mathbf{k}\}$kN

$F_{OA} = \mathbf{F} \bullet \mathbf{n_{OA}}$

$= (2)(2/3) + (4)(2/3) + (10)(-1/3) = -2/3$ kN

Örnek 6

Determine the magnitude of the component of the force **F** along the *Aa* axis.



Answer

$F_{Aa} = 36.0\ \text{N}$