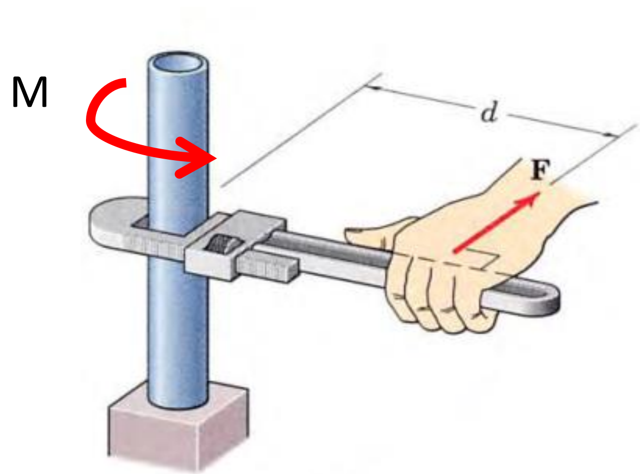
CHAPTER 2

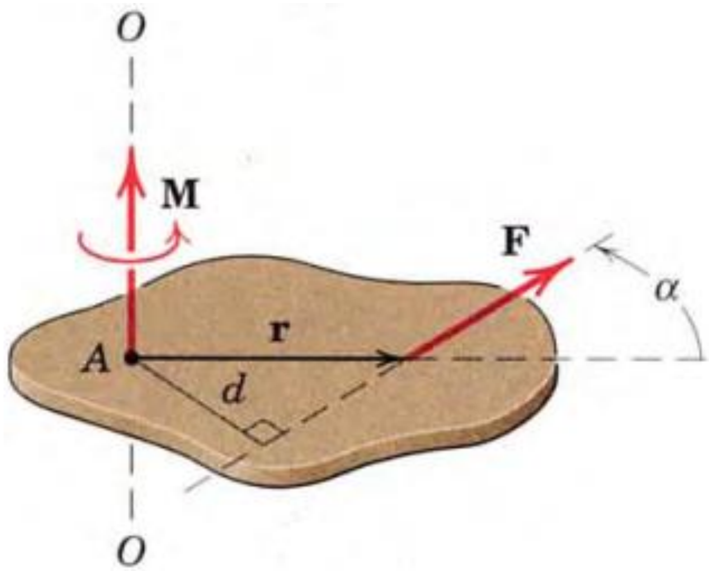
FORCE SYSTEMS

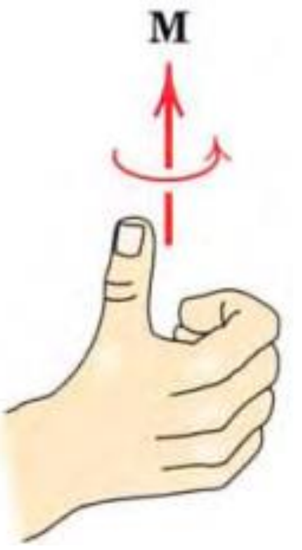
Moment

- *Moment of force* is the tendency of a force to twist or rotate an object.



$$M = Fd$$





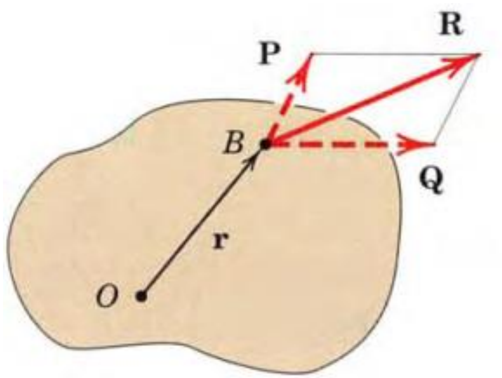
- The moment of F about point A may be represented by the vectorial form

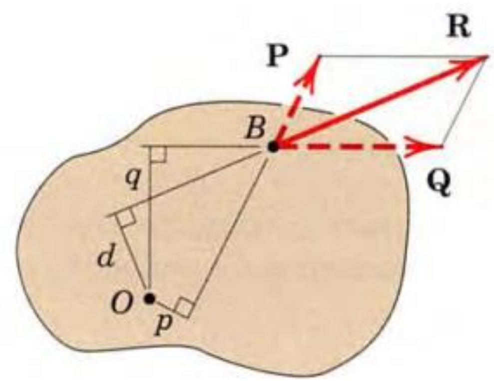
$$\vec{M} = \vec{r} \times \vec{F}$$

$$\vec{M} = F(r \sin \alpha) = Fd$$

$$\vec{r} \times \vec{F} \neq \vec{F} \times \vec{r}$$

- Varignon's theorem. Which states that the *moment of a force* about any point is *equal* to the *sum* of the *moments* of the *components* of the *force* about the same point.

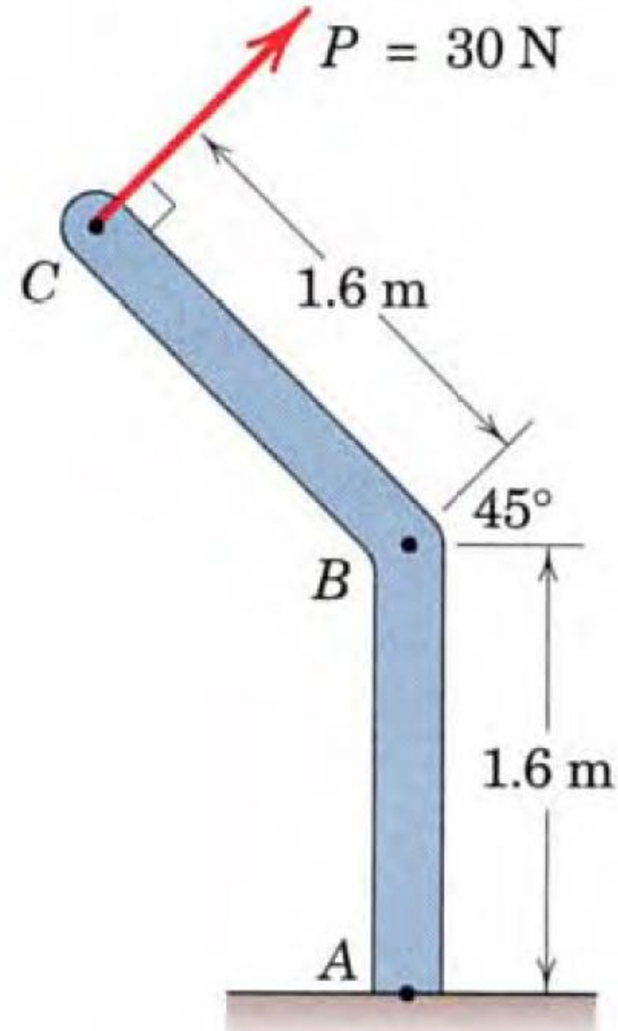




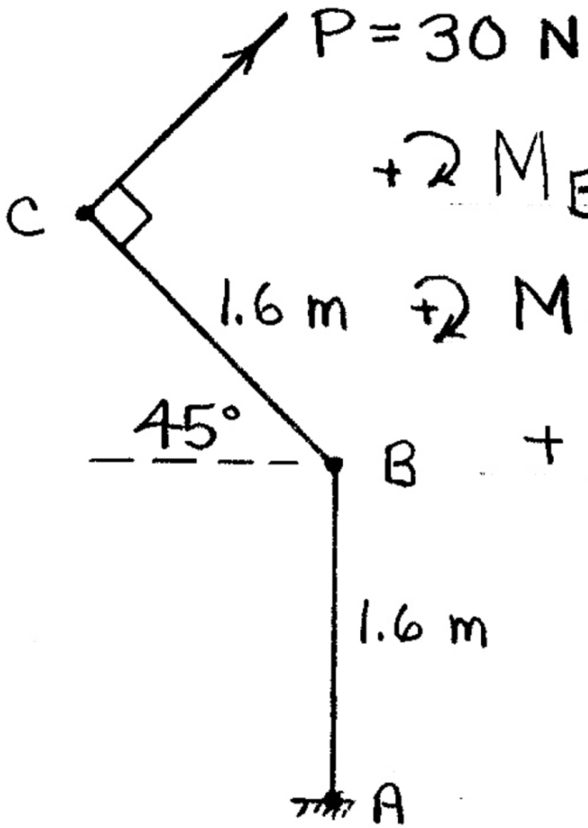
$$\vec{M}_o = \vec{r} \times \vec{R} \qquad \vec{R} = \vec{P} + \vec{Q} \qquad \vec{M}_o = \vec{r} \times \left(\vec{P} + \vec{Q}\right) = \vec{r} \times \vec{P} + \vec{r} \times \vec{Q}$$

Example

The 30-N force **P** is applied perpendicular to the portion BC of the bent bar. Determine the moment of **P** about point B and about point A.



SOL:

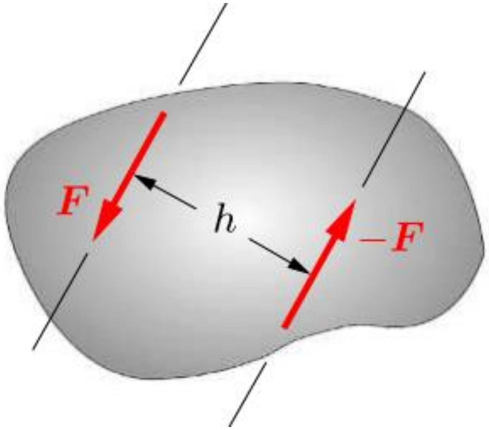


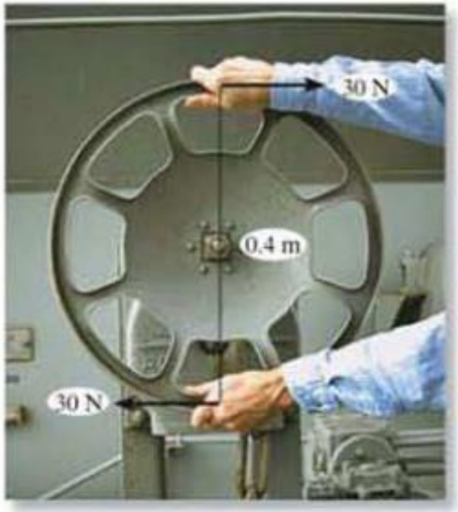
$$+\circlearrowleft M_B = 30(1.6) = 48 \text{ N}\cdot\text{m}$$

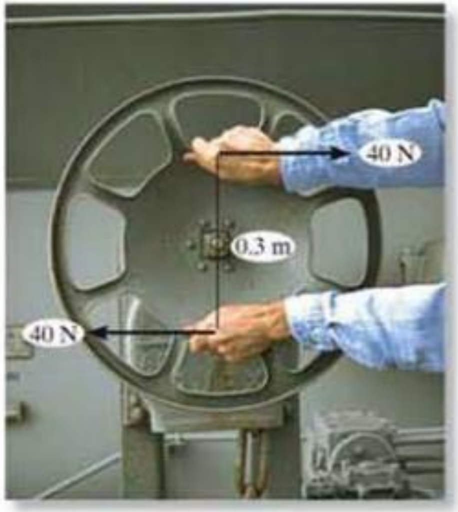
$$+\circlearrowleft M_A = 30\cos 45^\circ (1.6 + 1.6\sin 45^\circ) + 30\sin 45^\circ (1.6\cos 45^\circ) = \underline{81.9 \text{ N}\cdot\text{m}}$$

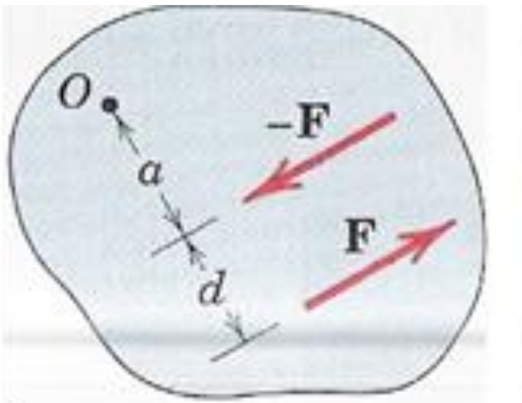
Couple

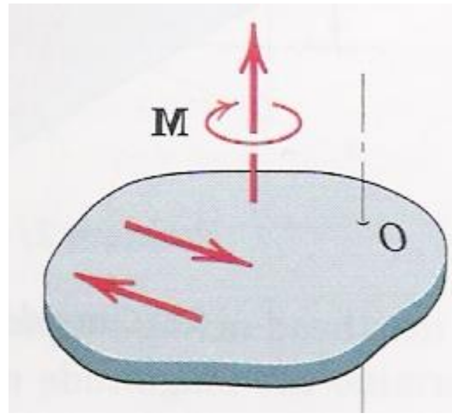
- Couple consists of two forces having equal magnitude, parallel action lines and opposite directions. The resultant force of a couple is zero the couple has an effect on the body on which it acts: it tends to rotate the body.

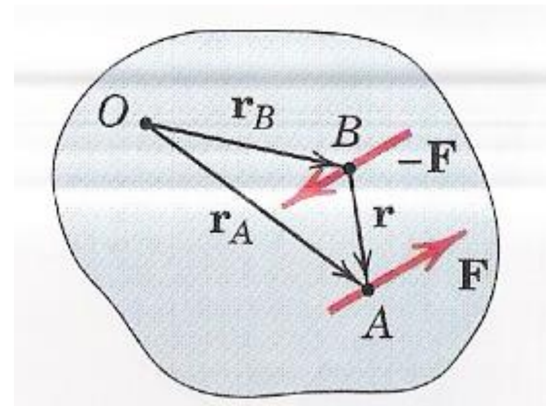








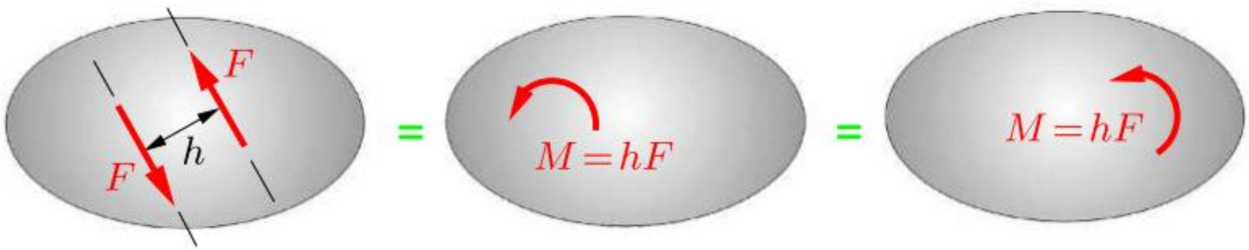


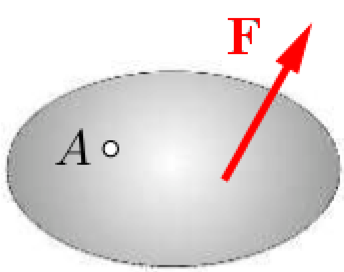


- Magnitude of Couple equal to the multiplication of perpendicular distance between the forces.

Difference Couple and Moment

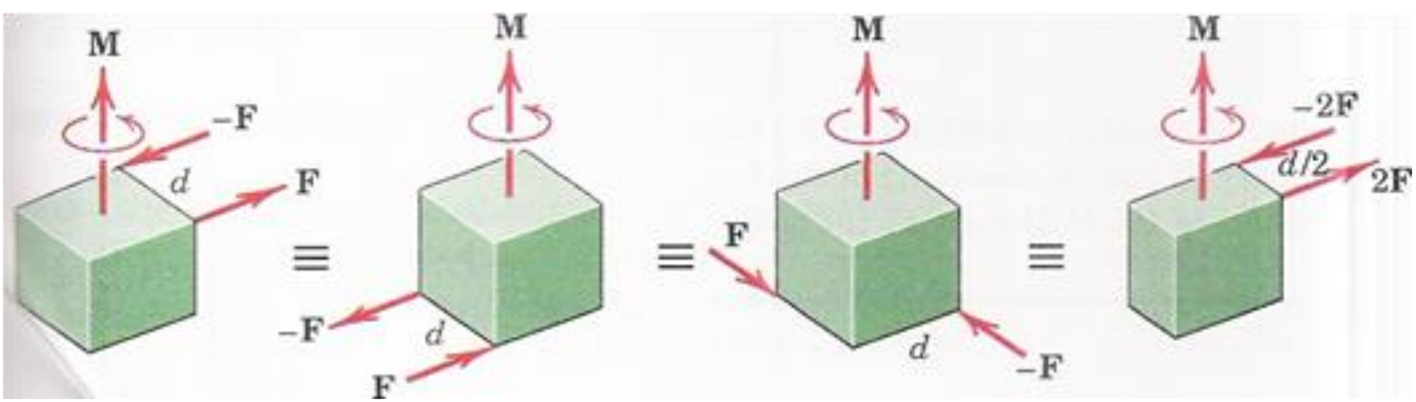
- Effect of the moment is about a point. But effect of the couple can be any point on a surface. The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e., it is a *free vector* that can be applied at any point with the same effect.



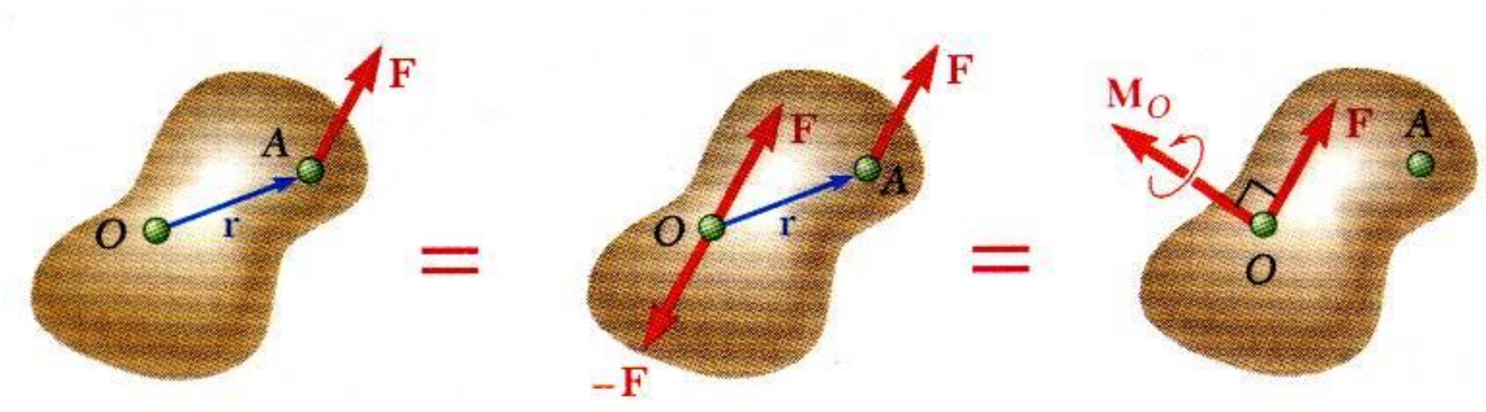


$M_A = d \times F$

- When magnitude and direction is constant, if force is increased and d decreased or vice versa, effect of couple does not change.



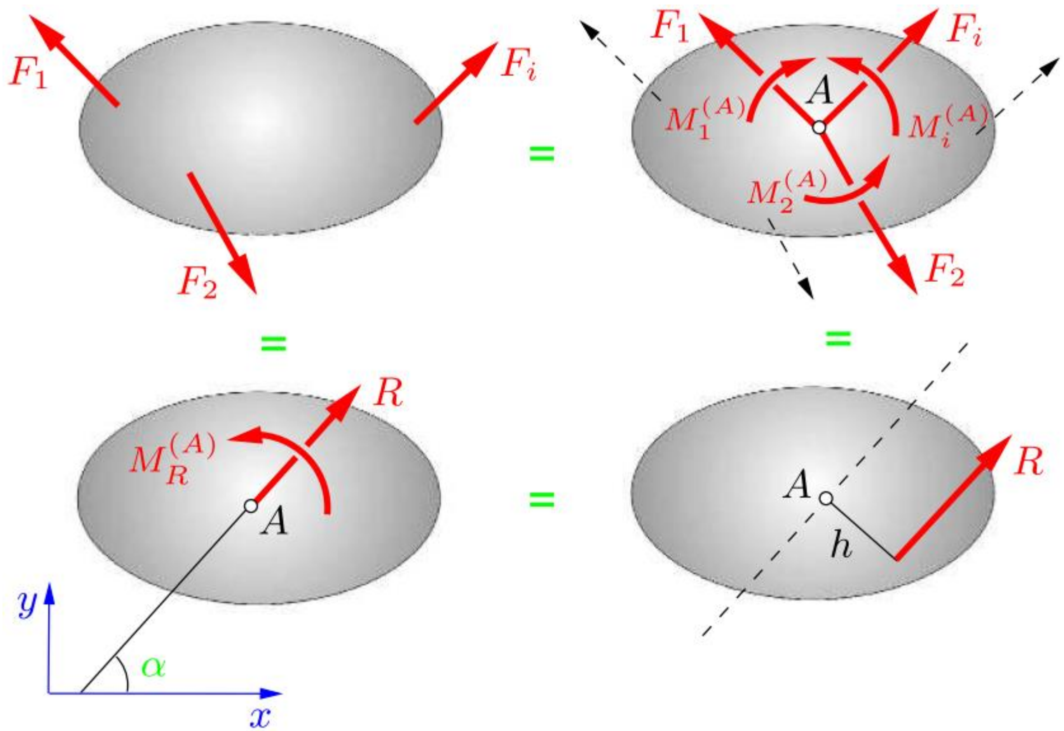
Resolution of a Force Into a Force at *O* and a Couple

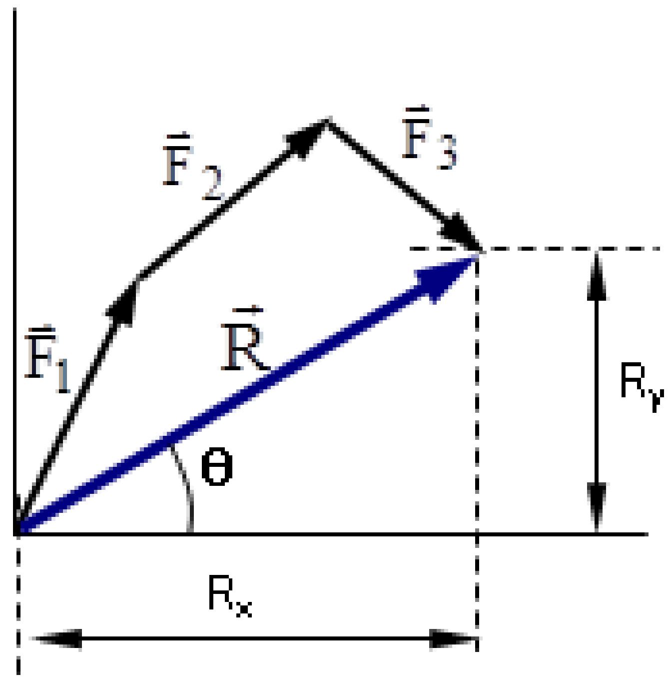


- Force vector ***F*** can not be simply moved to *O* without modifying its action on the body.
- Attaching equal and opposite force vectors at *O* produces no net effect on the body.
- The three forces may be replaced by an equivalent force vector and couple vector, i.e, a *force-couple system*.

Resultants

- Most problems in mechanics deal with a system of forces, and it is usually necessary to *reduce* the *system* to its *simplest form* to *describe* its *action*. The *resultant* of a *system* of forces is the *simplest force combination* which can be *replace* the *original forces*.





$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma \mathbf{F}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

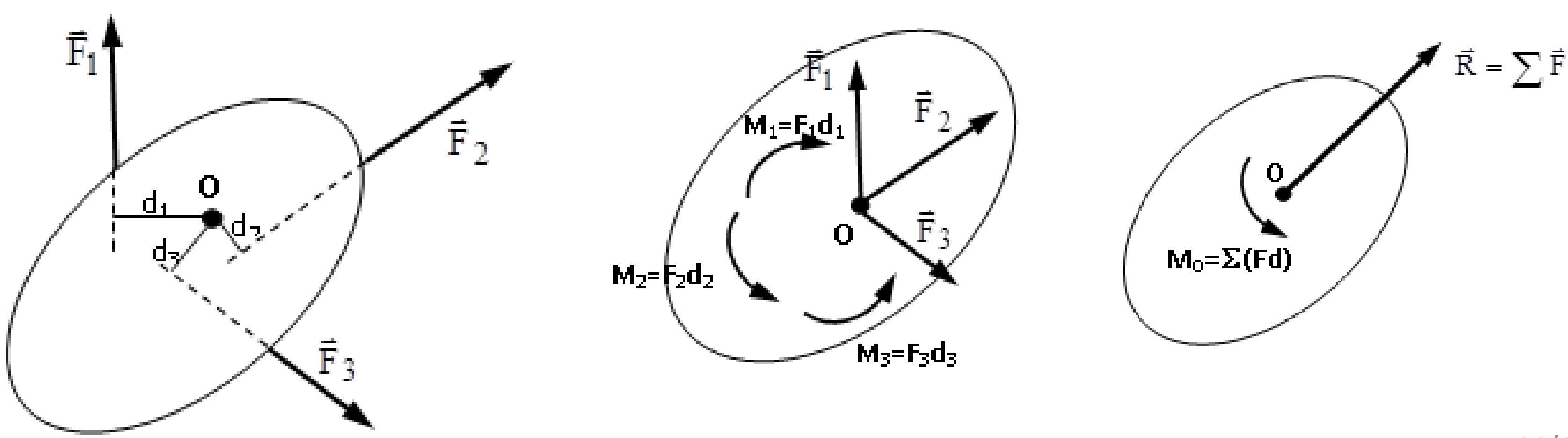
Algebraic Method

- We can use algebra to obtain the resultant force and its line of action as follows:

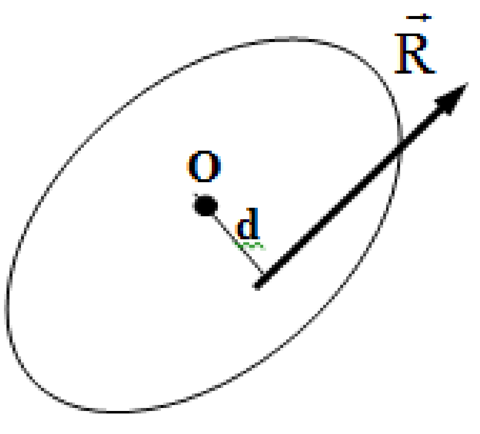
➢Choose a convenient reference point and move all forces to that point.

➢Add all forces at *O* to form the resultant force **R**, and add couples to form the resultant couple *Mo*.

➢Find the line of action of **R** by requiring **R** to have a moment of *Mo* about point *O*.



- The forces may be replaced by an equivalent force vector and couple vector system or a single force vector system.



$$Rd = M_o$$

$$d = \frac{M_o}{R}$$

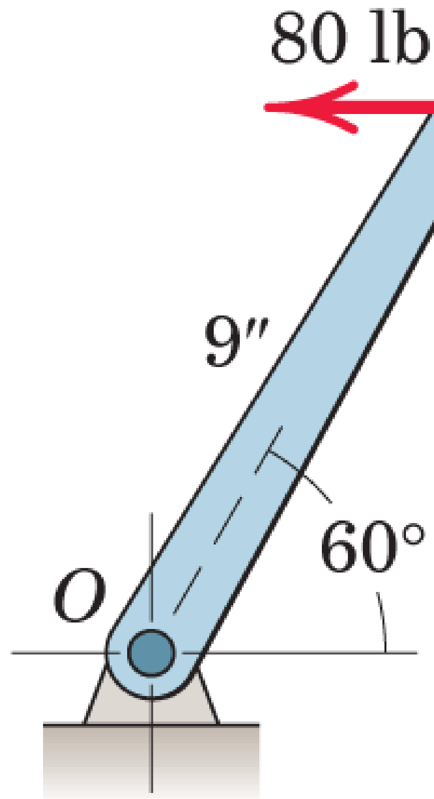
EXAMPLES

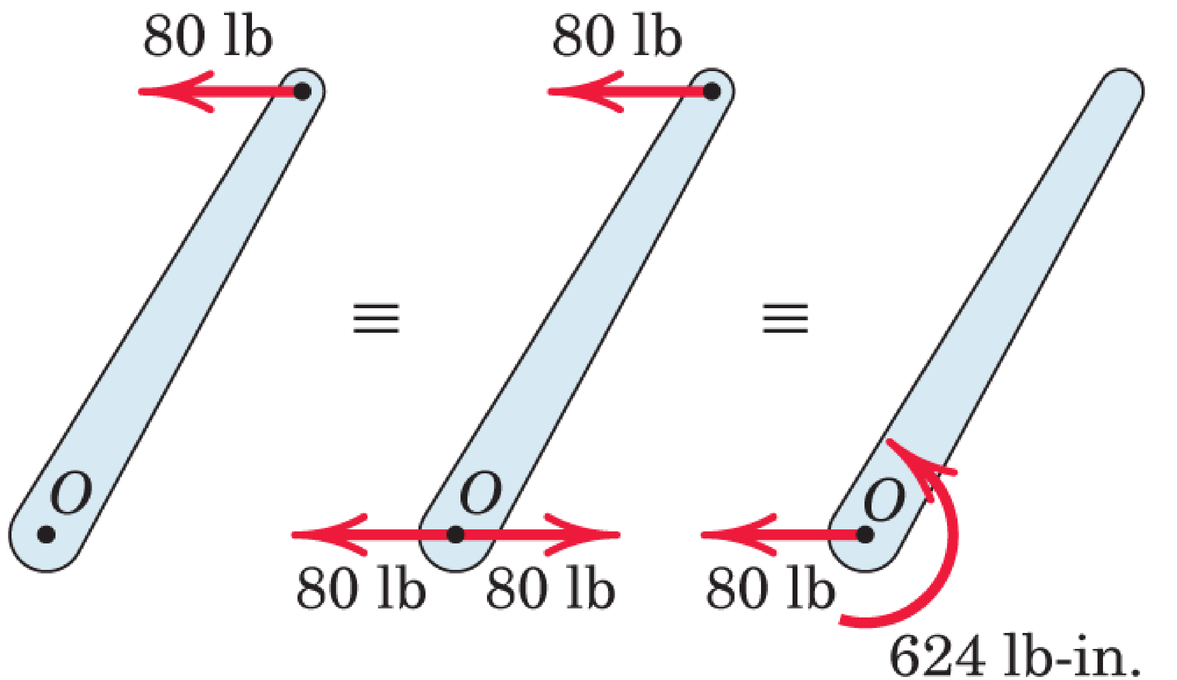
FORCE SYSTEMS

2D Moment and Couple

Example 1

- Replace the horizontal 80 lb force acting on the lever by an equivalent system consisting of a force at O and a couple.



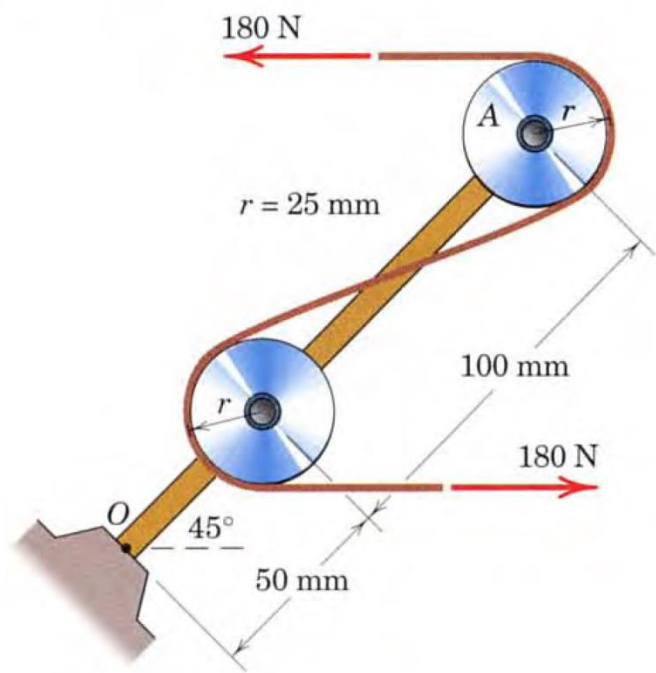


$[M = Fd]$ $M = 80(9 \sin 60°) = 624 \text{ lb-in.}$

Example 2

The system consisting of the bar OA, two identical pulleys, and a section of thin tape is subjected to the two 180-N tensile forces shown in the figure. Determine the equivalent force–couple system at point O.

- **M**=21,728 Nm



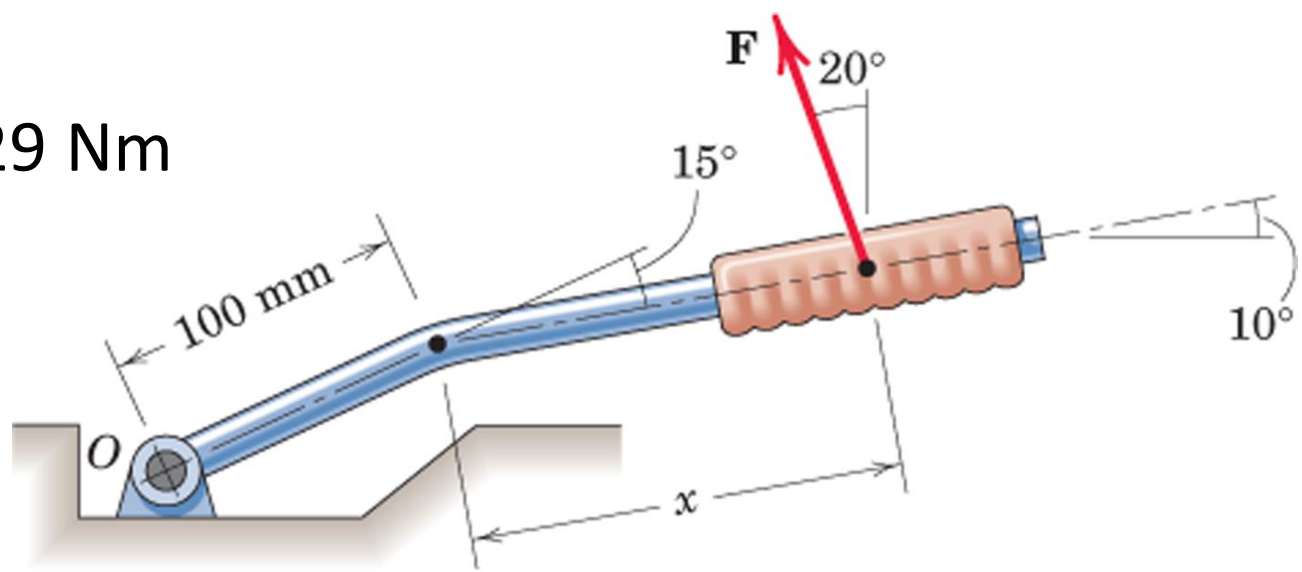
Example 3

A force **F** of magnitude 50 N is exerted on the automobile parking-brake lever at the position $x = 250$ mm. Replace the force by an equivalent force–couple system at the pivot point O.

Answer

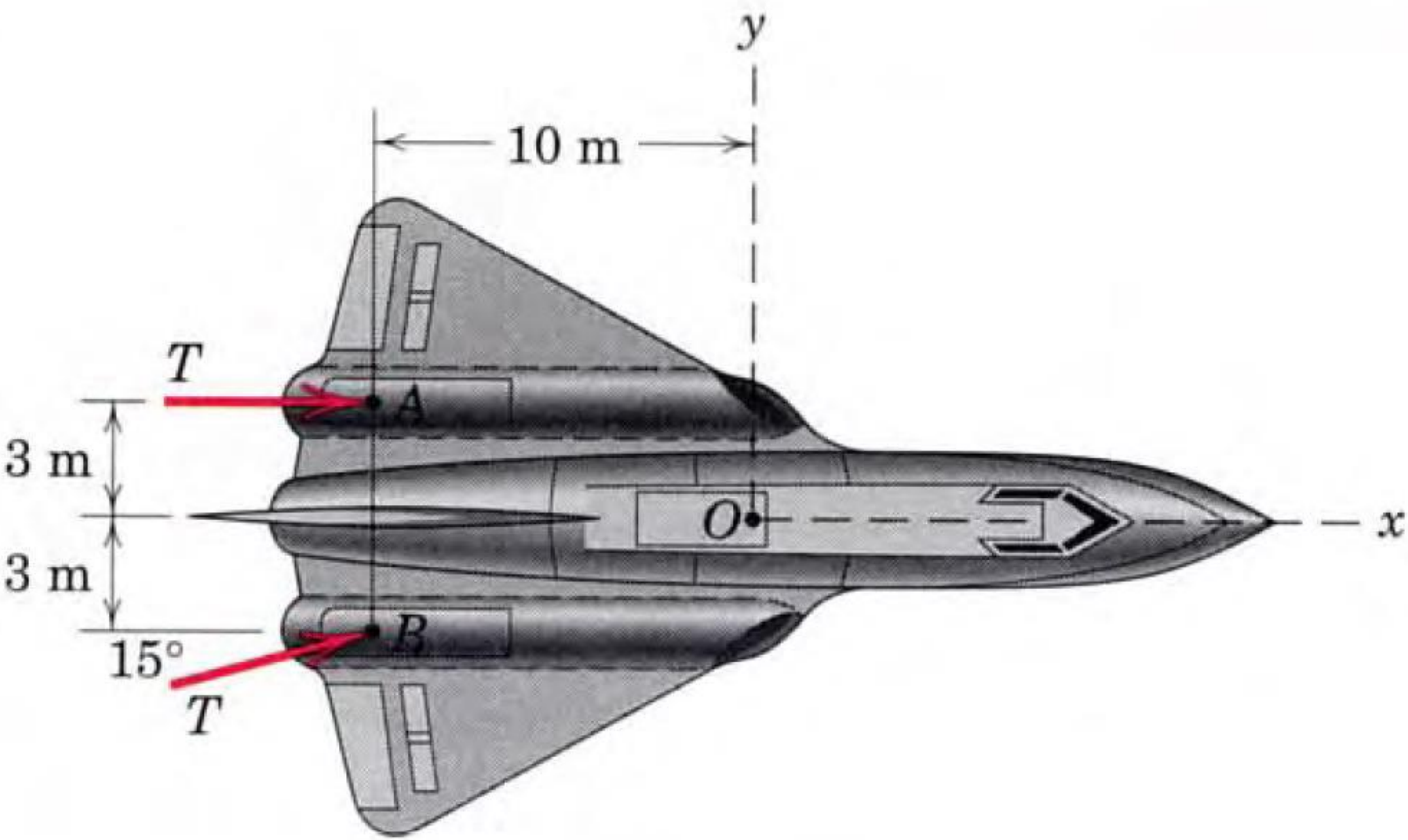
R=50 N;

M_o=17,29 Nm



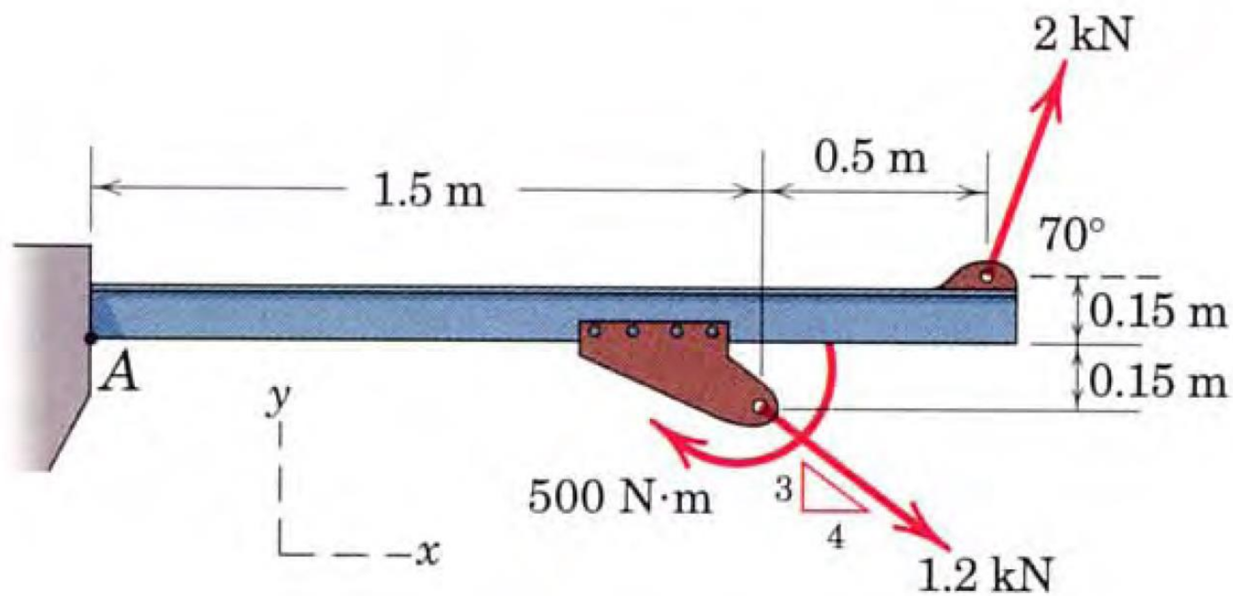
Example 4

The directions of the two thrust vectors of an experimental aircraft can be independently changed from the conventional forward direction within limits. For the thrust configuration shown, determine the equivalent force–couple system at point O. Then replace this force–couple system by a single force and specify the point on the x-axis through which the line of action of this resultant passes. These results are vital to assessing design performance.


y
10 m
T
A
3 m
O
x
3 m
B
15°
T

Example 5

The flanged steel cantilever beam with riveted bracket is subjected to the couple and two forces shown, and their effect on the design of the attachment at *A* must be determined. Replace the two forces and couple by an equivalent couple *M* and resultant force **R** at *A*.



Answer :

R= -1,644i+1,159j kN

M_A= 2,22 kNm