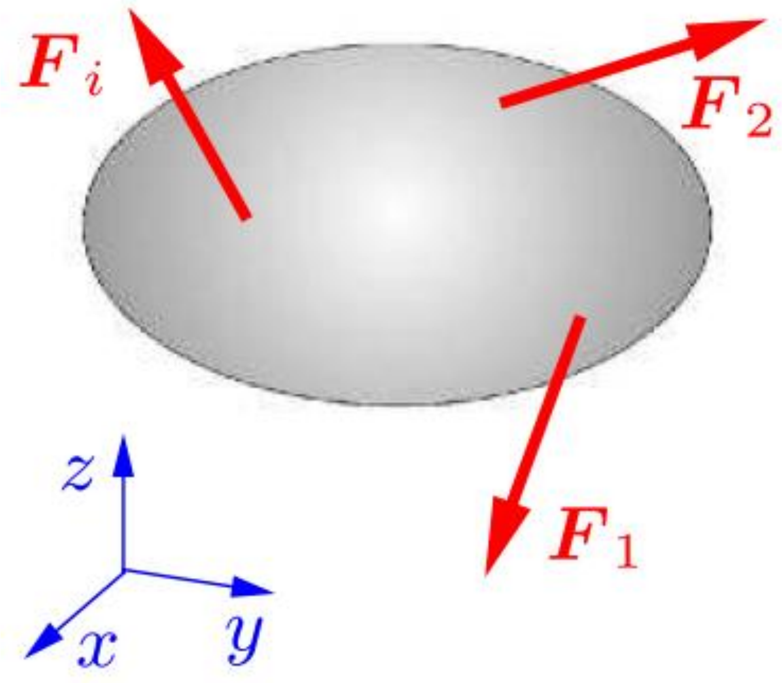
CHAPTER 3

EQUILIBRIUM in 3D

- We extend our principles and methods developed for *two-dimensional* equilibrium to the case of *three-dimensional* equilibrium.

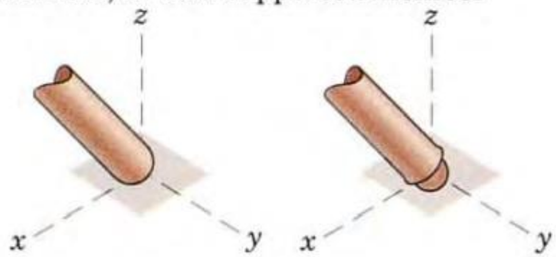
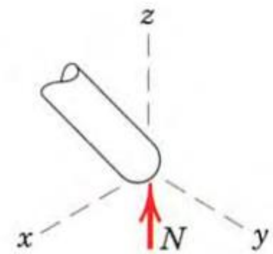
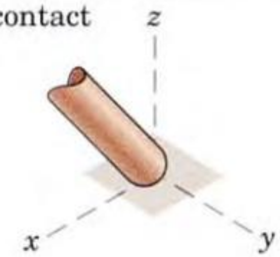
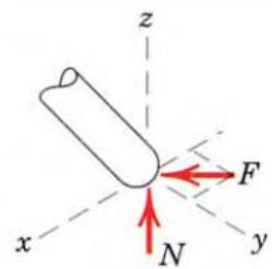
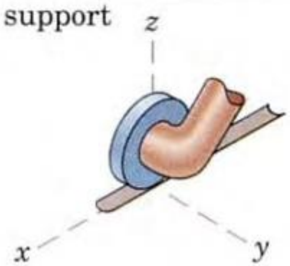
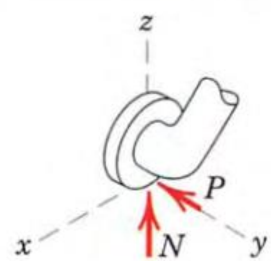
$$\sum \vec{F} = 0 \quad , \quad \sum \vec{M} = 0$$



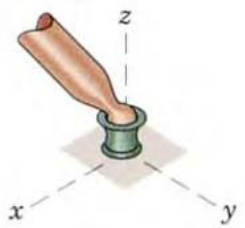
$$\sum \vec{F} = 0 \quad , \quad \sum \vec{M} = 0$$

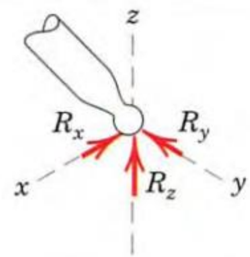
Free-Body Diagram

- For drawing free-body diagram of 3D problems, we use same steps with 2D problems.
- *Step 1*. *Decide* which *system* to isolate.
- *Step 2*. Next *isolate* the chosen *system* by drawing a *diagram* which represents its complete *external boundary*.
- *Step 3*. *Identify* all *forces* which act on the *isolated system* as applied by the removed contacting and attracting bodies.
- *Step 4*. *Show* the choice of *coordinate axes* directly on the diagram.

MODELING THE ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Member in contact with smooth surface, or ball-supported member 	 Force must be normal to the surface and directed toward the member.
2. Member in contact with rough surface 	 The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.
3. Roller or wheel support with lateral constraint 	 A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.

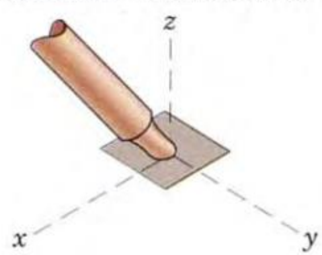
4. Ball-and-socket joint

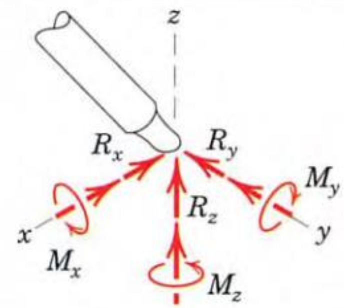




A ball-and-socket joint free to pivot about the center of the ball can support a force **R** with all three components.

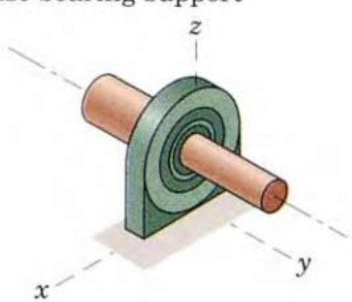
5. Fixed connection (embedded or welded)

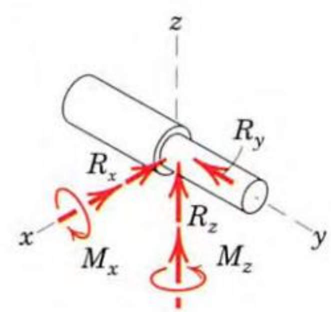




In addition to three components of force, a fixed connection can support a couple **M** represented by its three components.

6. Thrust-bearing support





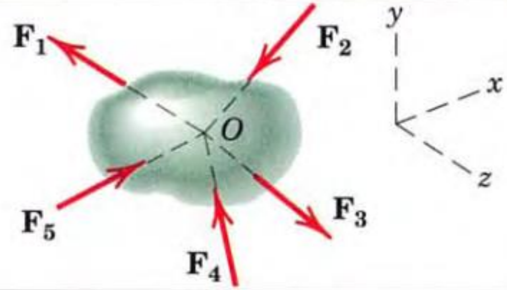
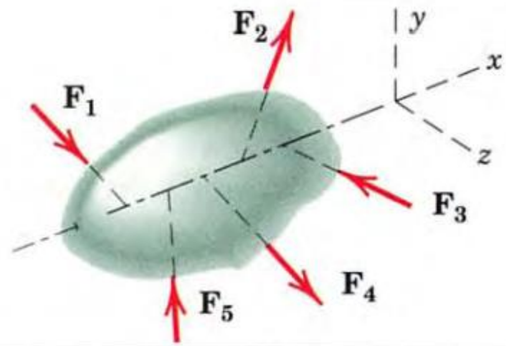
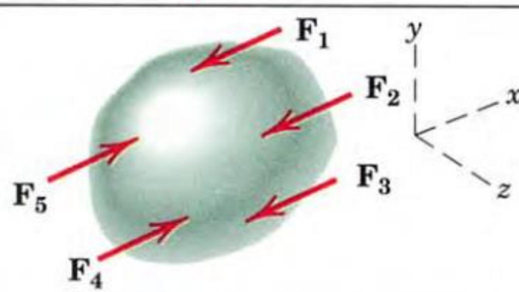
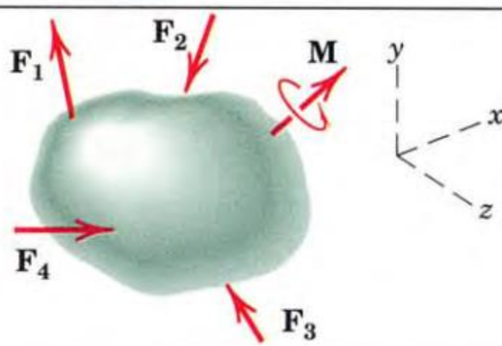
Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z. Couples M_x and M_z must, in some cases, be assumed zero in order to provide statical determinacy.

Equilibrium Conditions

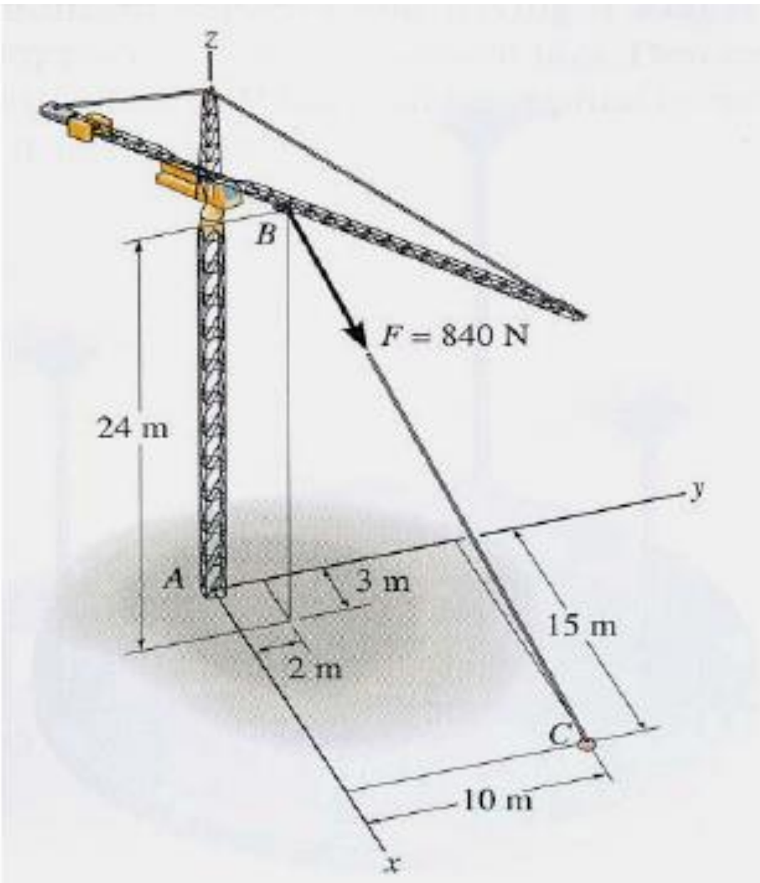
- All resultant *forces* and *moments* on the each *three axes* must be *zero*.

$$\sum F_x = 0 \;,\; \sum F_y = 0 \;,\; \sum F_z = 0$$

$$\sum M_x = 0 \;,\; \sum M_y = 0 \;,\; \sum M_z = 0$$

<table>
<tr><th colspan="3">CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS</th></tr>
<tr><th>Force System</th><th>Free-Body Diagram</th><th>Independent Equations</th></tr>
<tr><td>1. Concurrent at a point</td><td>
</td><td>$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma F_z = 0$</td></tr>
<tr><td>2. Concurrent with a line</td><td>
</td><td>$\Sigma F_x = 0$ $\Sigma M_y = 0$
$\Sigma F_y = 0$ $\Sigma M_z = 0$
$\Sigma F_z = 0$</td></tr>
<tr><td>3. Parallel</td><td>
</td><td>$\Sigma F_x = 0$ $\Sigma M_y = 0$
$\Sigma M_z = 0$</td></tr>
<tr><td>4. General</td><td>
</td><td>$\Sigma F_x = 0$ $\Sigma M_x = 0$
$\Sigma F_y = 0$ $\Sigma M_y = 0$
$\Sigma F_z = 0$ $\Sigma M_z = 0$</td></tr>
</table>

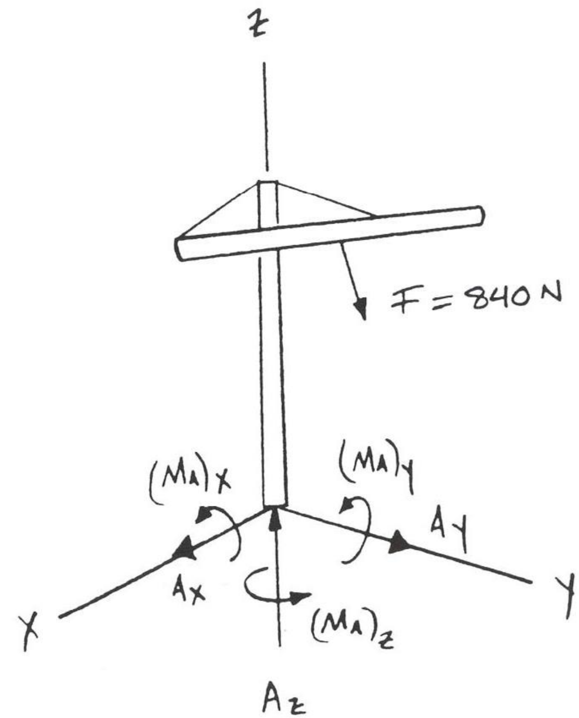
Example



Given: The cable of the tower crane is subjected to 840 N force. A fixed base at A supports the crane.

Find: Reactions at the fixed base A.

Solution



$\boldsymbol{r}_{BC} = \{12\,\boldsymbol{i} + 8\,\boldsymbol{j} - 24\,\boldsymbol{k}\}$ m

$\boldsymbol{F} = F\,[\boldsymbol{n}_{BC}]$ N

$= 840\,[12\,\boldsymbol{i} + 8\,\boldsymbol{j} - 24\,\boldsymbol{k}] \,/\, (12^2 + 8^2 + (-24^2))^{1/2}$

$= \{360\,\boldsymbol{i} + 240\,\boldsymbol{j} - 720\,\boldsymbol{k}\}$ N

$\boldsymbol{F}_A = \{A_X\,\boldsymbol{i} + A_Y\,\boldsymbol{j} + A_Z\,\boldsymbol{k}\}$ N

Solution

- From Equilibrium equation we get,

  $\boldsymbol{F} + \boldsymbol{F_A} = 0$

$\{(360 + A_X)\,\boldsymbol{i} + (240 + A_Y)\,\boldsymbol{j} + (-720 + A_Z)\,\boldsymbol{k}\} = 0$

- Solving each component equation yields
- $A_X = -360$ N ,
- $A_Y = -240$ N , and $A_Z = 720$ N.

Sum the moments acting at point A.

$$\sum \mathbf{M} = \mathbf{M_A} + \mathbf{r_{AC}} \times \mathbf{F} = 0$$

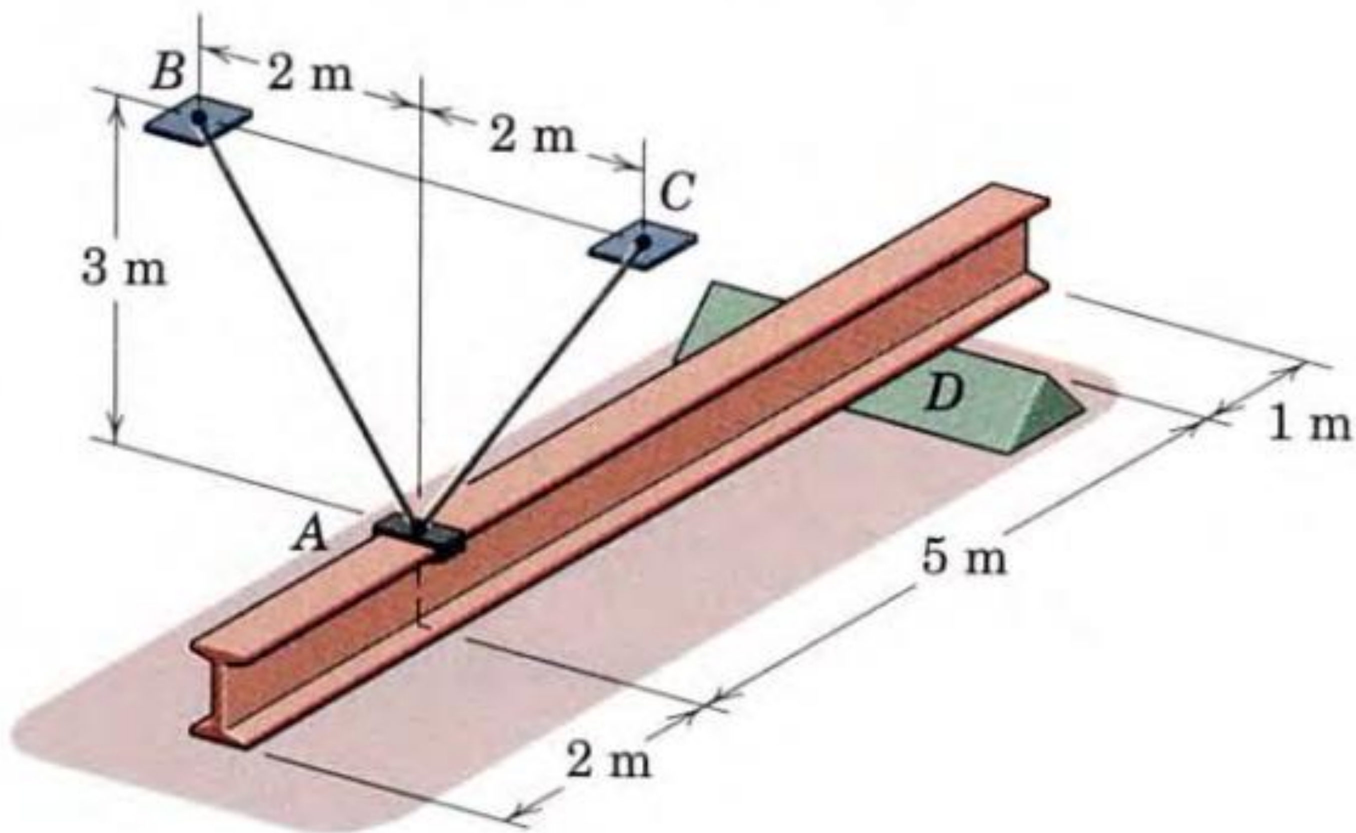
$$= M_{AX}\ \mathbf{i} + M_{AY}\ \mathbf{j} + M_{AZ}\ \mathbf{k} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 15 & 10 & 0 \\ 360 & 240 & -720 \end{vmatrix} = 0$$

$$= M_{AX}\ \mathbf{i} + M_{AY}\ \mathbf{j} + M_{AZ}\ \mathbf{k} - 7200\ \mathbf{i} + 10800\ \mathbf{j} = 0$$

$M_{AX} = 7200\ N \cdot m$, $M_{AY} = -10800\ N \cdot m$, and $M_{AZ} = 0$

Note: For simpler problems, one can directly use three scalar moment equations, $\sum M_X = \sum M_Y = \sum M_Z = 0$

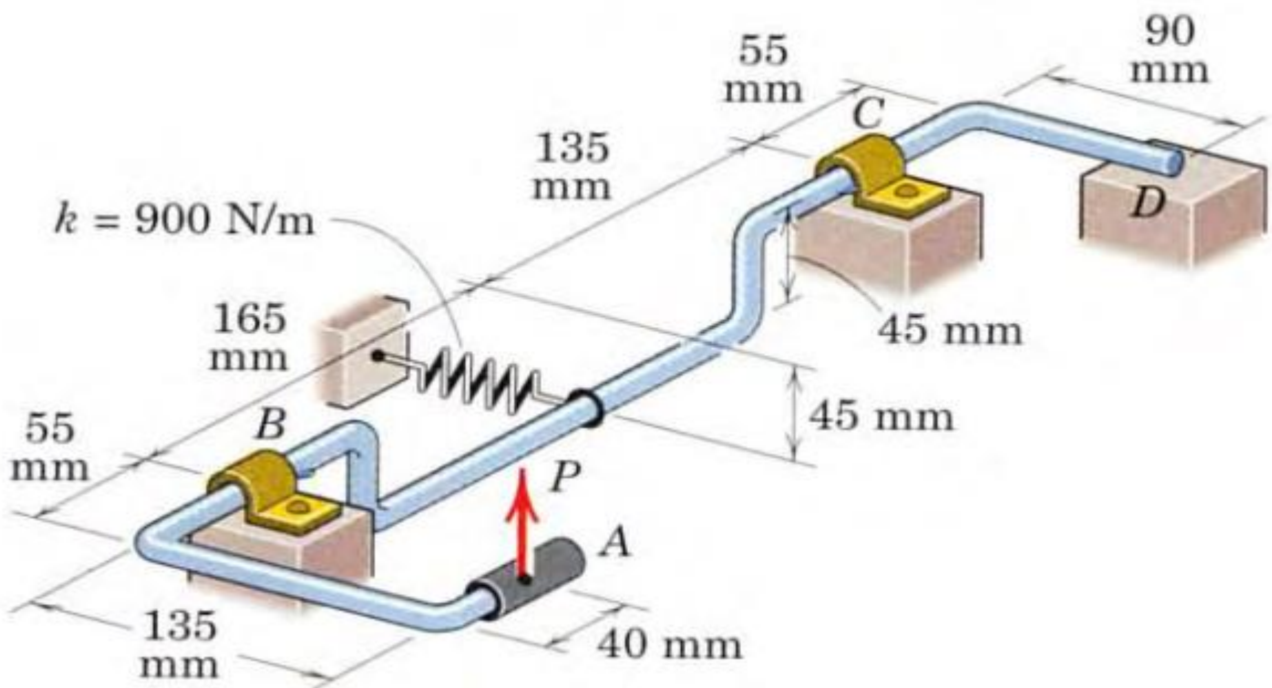
3/64 The uniform I-beam has a mass of 60 kg per meter of its length. Determine the tension in the two supporting cables and the reaction at D.



Problem 3/64

Results: T=1698N, D=1884N

3/80 The spring of modulus $k = 900$ N/m is stretched a distance $\delta = 60$ mm when the mechanism is in the position shown. Calculate the force P_{min} required to initiate rotation about the hinge axis BC, and determine the corresponding magnitudes of the bearing forces which are perpendicular to BC. What is the normal reaction force at D if $P = P_{min}/2$?



Problem 3/80

Results: Pmin=18N, B=30.8, C=29.7N, D=13.5N example: A uniform pipe cover of radius 240 mm and mass 30 kg is held in a horizontal position by the cable *CD*. The bearings at *A* and *B* are self-aligning and bearing *A* resists thrust. Determine the cable tension and the reactions at *A* and *B*.

$T = 343.4$ N
$A_x = 49.1$ N
$A_y = 73.6$ N
$A_z = 98.1$ N
$B_x = 245$ N
$B_y = 73.5$ N