CHAPTER 3

EQUILIBRIUM in 2D

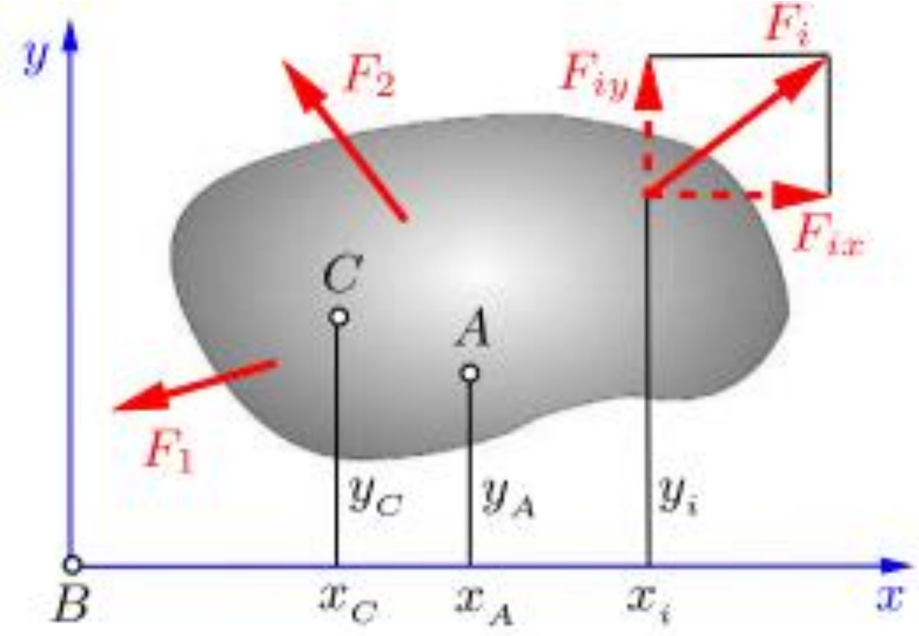
Introduction

- In this chapter, we shall make continuous use of the concepts developed in chapter 2 involving *forces*, *moments*, *couple*, and *resultants* as we apply the principles of *equilibrium*.
- A *particle* is said to be in *equilibrium* if it remains at *rest* if originally at rest, or has a *constant velocity* if originally in motion.

- When a body is in *equilibrium*, the resultant of all *forces* acting on it is *zero*. Thus, the resultant force **R** and the resultant couple **M** are both *zero*, and the equilibrium eqn's:

$$\mathbf{R} = \sum \mathbf{F} = 0$$

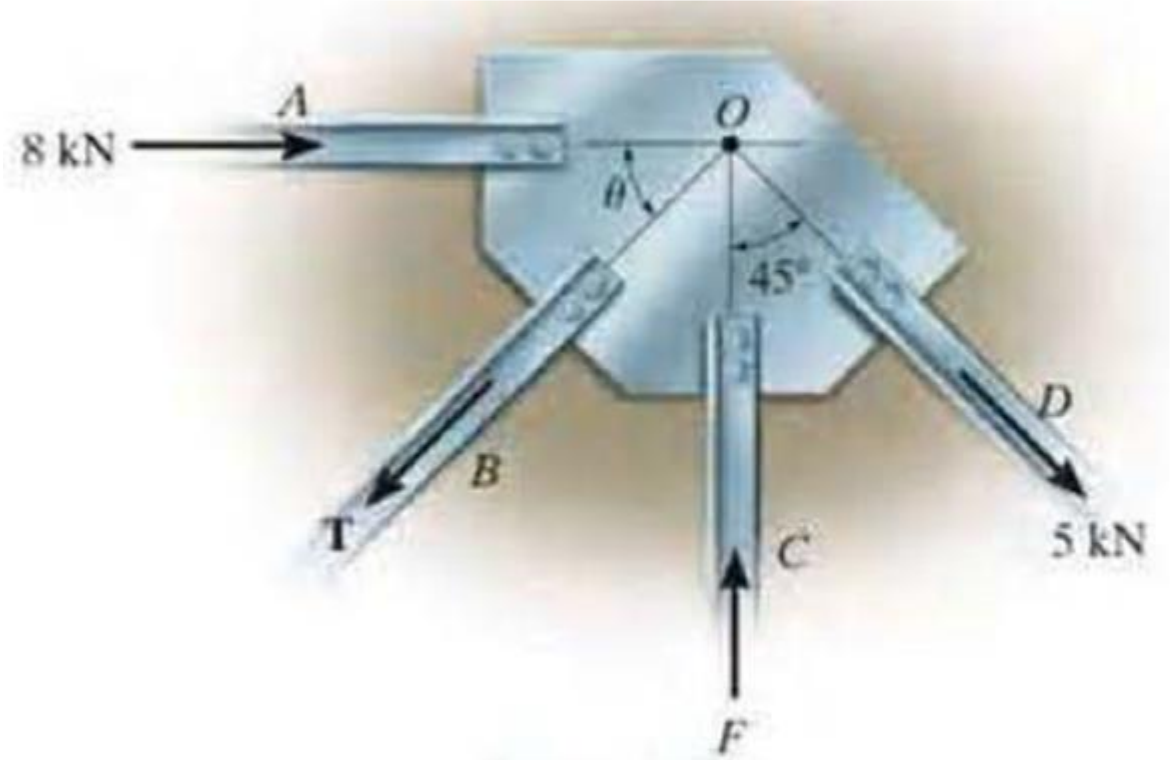
$$\mathbf{M} = \sum \mathbf{M} = 0$$



$$\sum F_{ix} = 0\,, \quad \sum M_i^{(A)} = 0\,, \quad \sum M_i^{(B)} = 0$$

$$\sum F_{iy} = 0\,, \quad \sum M_i^{(A)} = 0\,, \quad \sum M_i^{(B)} = 0$$

- ***Example***: The gusset plate is subjected to the forces of four members. Determine the force in member ***B*** and its proper orientation **θ** for equilibrium. The forces are concurrent at point ***O***. Take ***F*** = 12 kN. (T=14.3kN, θ=36.27˚)

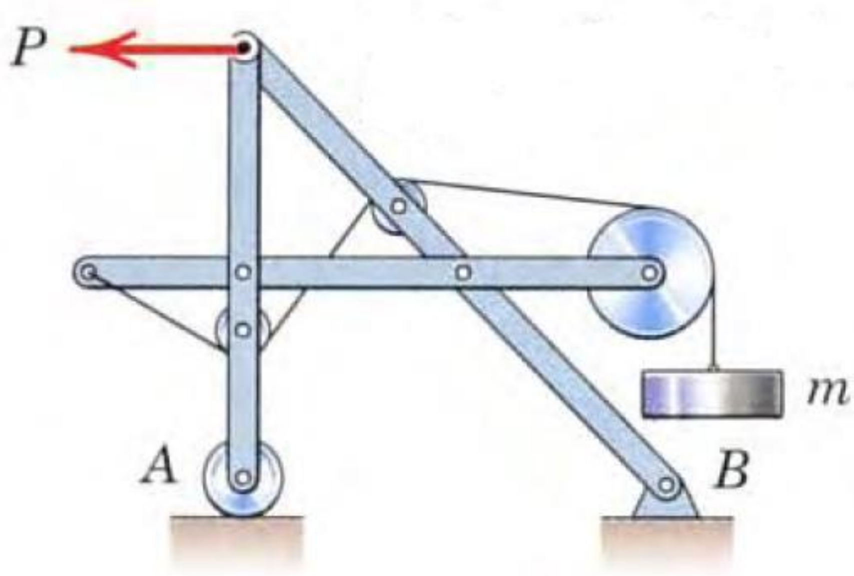


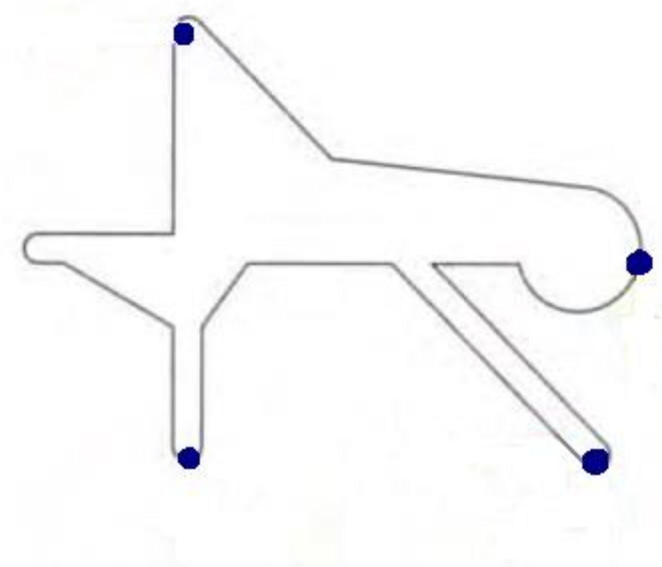
Mechanical System Isolation

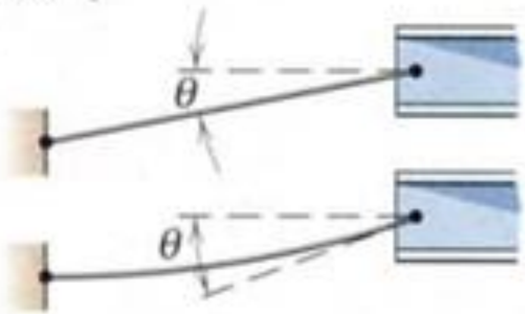
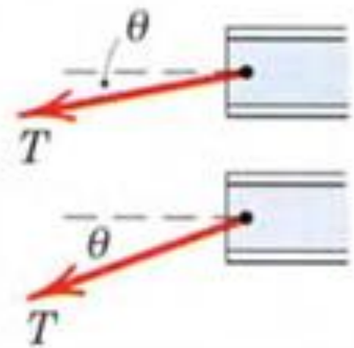
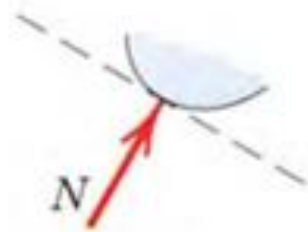
- A mechanical system is defined as *a body* or *group of bodies* which can be *isolated* from all *other bodies*. This isolation is accomplished by means of the *free-body diagram*.

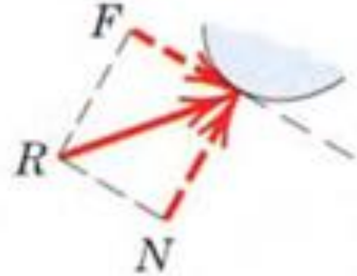
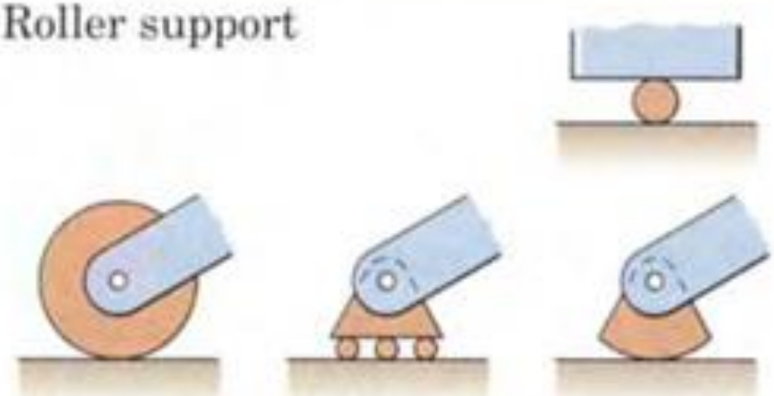
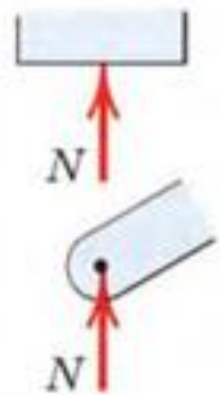
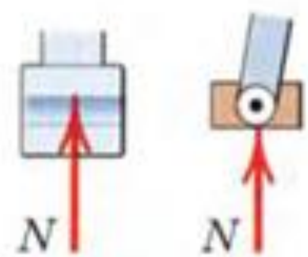
➢ The *free-body diagram* is the *most important* single step in the *solution of problems* in mechanics.

- An example of system isolation

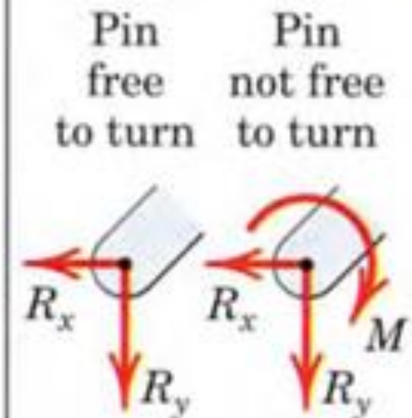
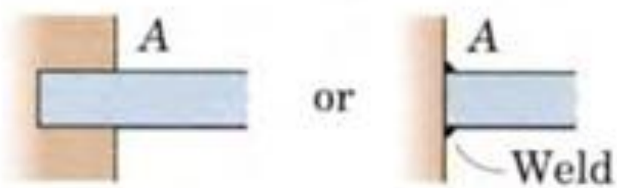
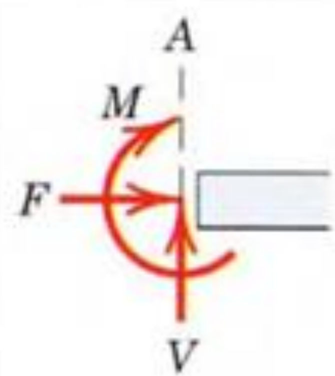


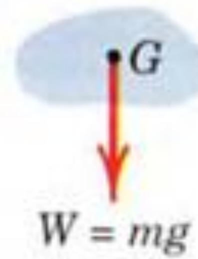
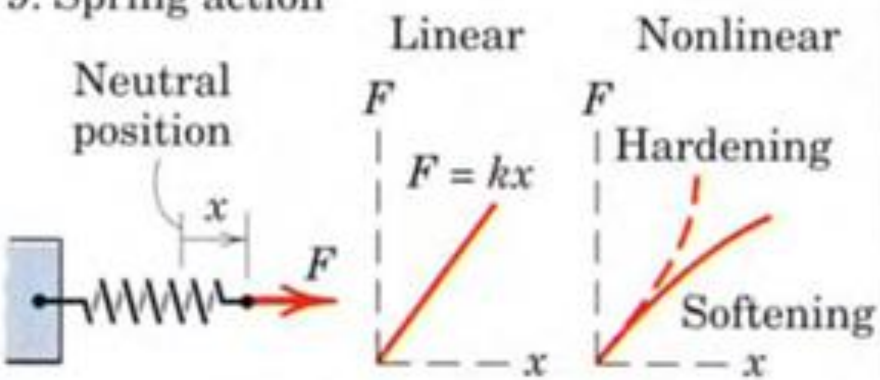


MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.

3. Rough surfaces 		Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.
4. Roller support 		Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 		Collar or slider free to move along smooth guides; can support force normal to guide only.

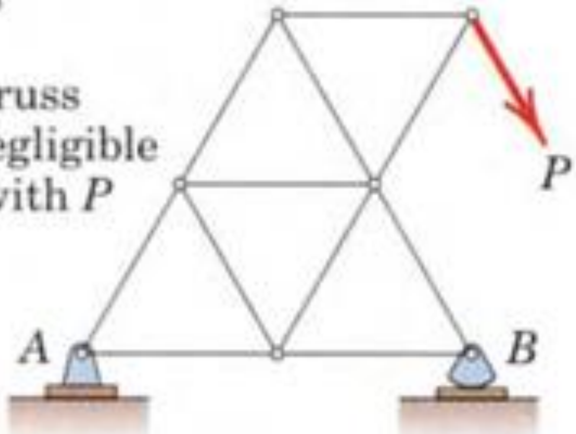
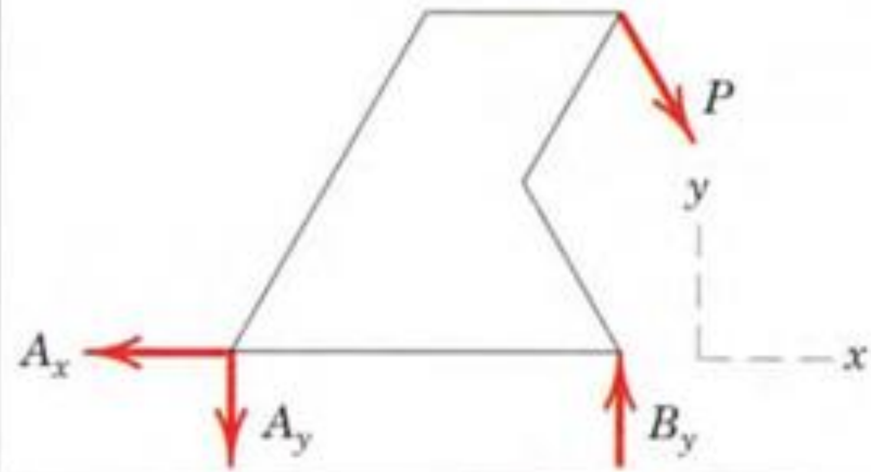
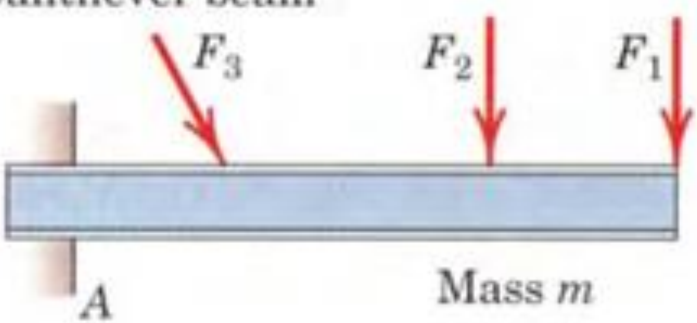
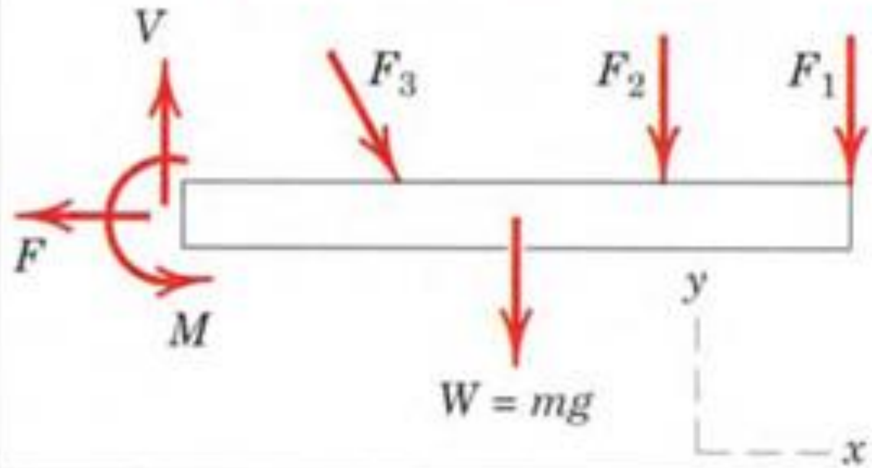
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)

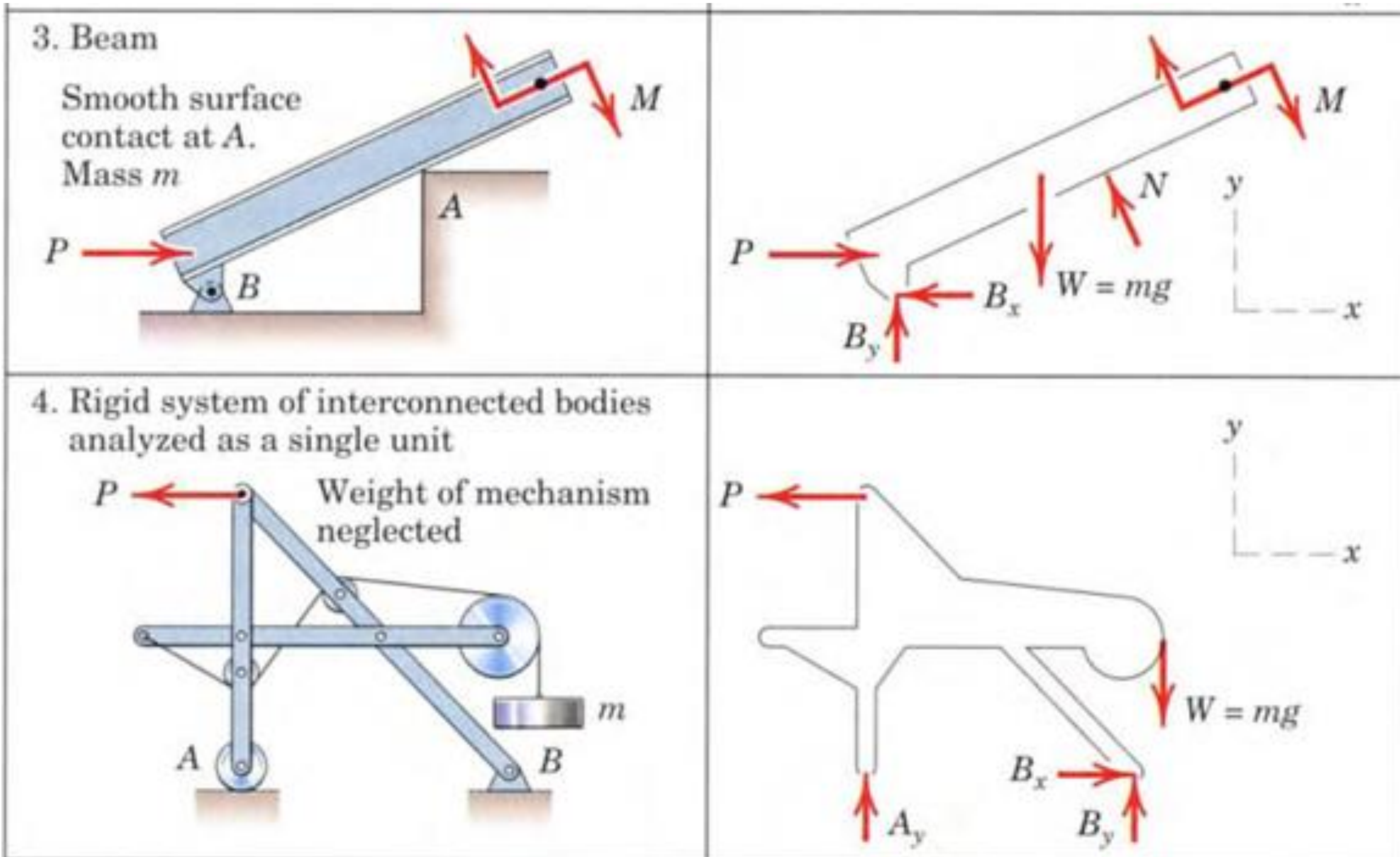
Type of Contact and Force Origin	Action on Body to Be Isolated
6. Pin connection 	 A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the axis; usually shown as two components R_x and R_y. A pin not free to turn may also support a couple M.
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.

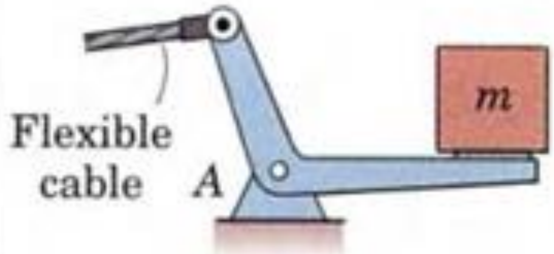
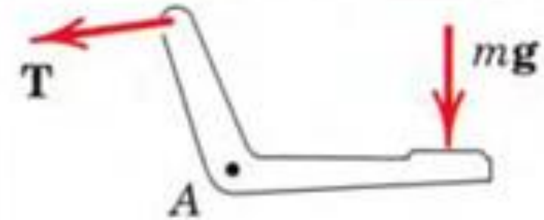
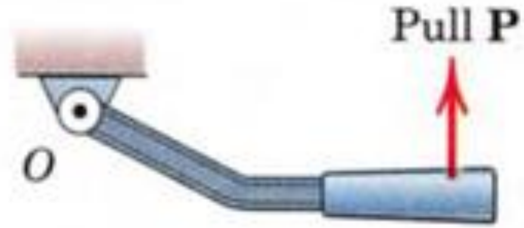
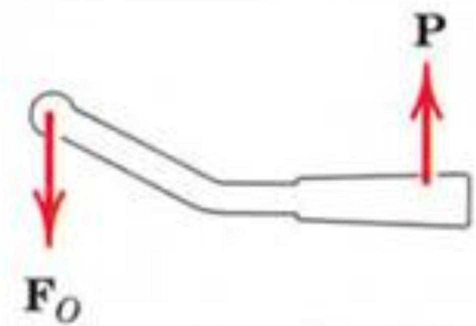
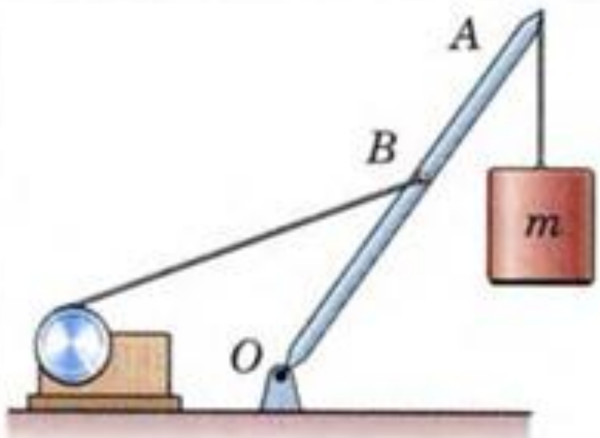
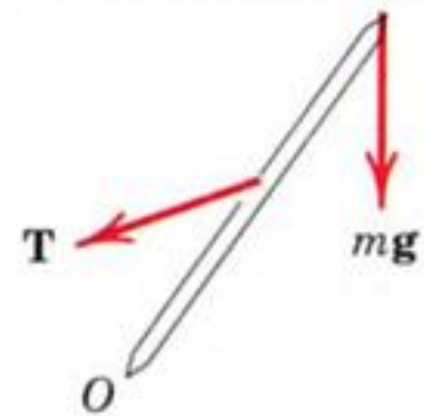
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 	 Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

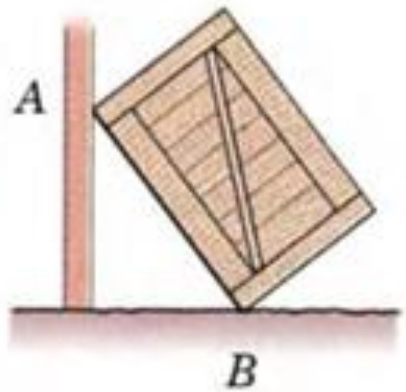
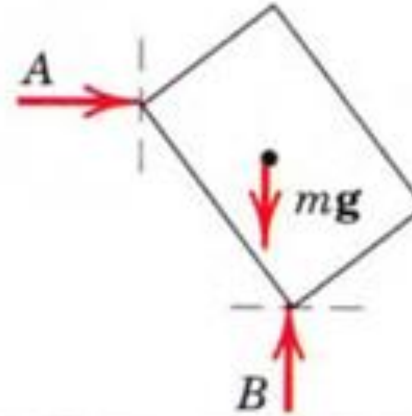
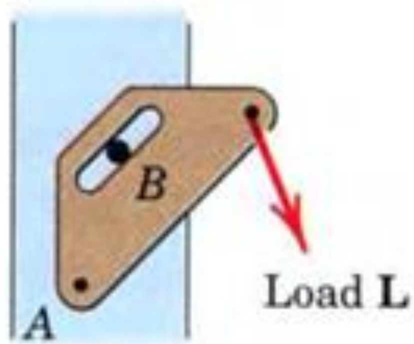
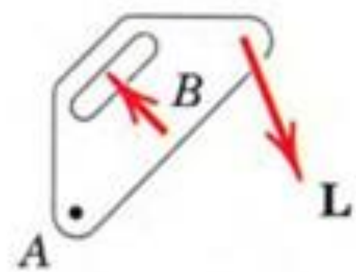
Construction of Free-Body Diagrams

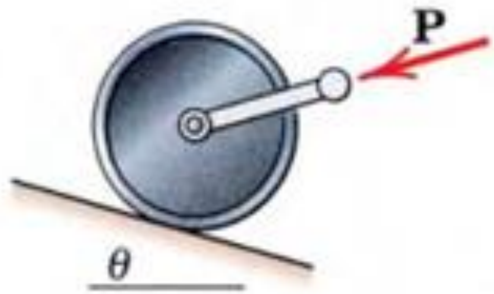
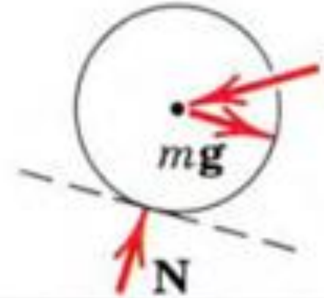
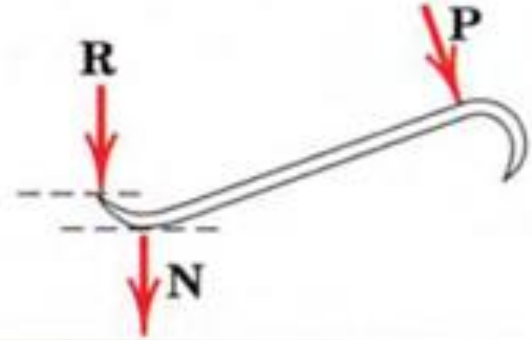
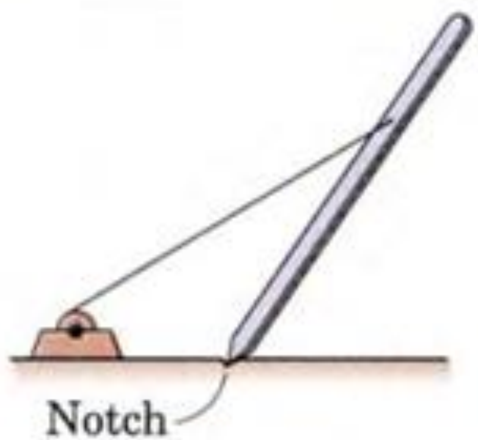
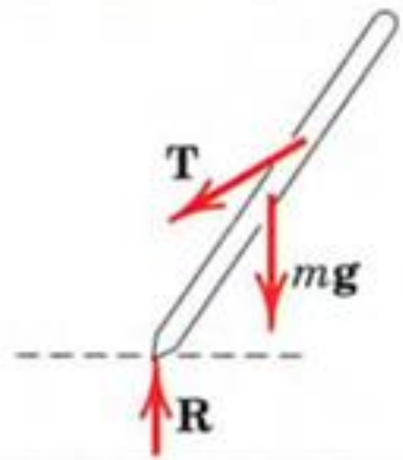
- *Step 1*. *Decide* which *system* to isolate.
- *Step 2*. Next *isolate* the chosen *system* by drawing a *diagram* which represents its complete *external boundary*.
- *Step 3*. *Identify* all *forces* which act on the *isolated system* as applied by the removed contacting and attracting bodies.
- *Step 4*. *Show* the choice of *coordinate axes* directly on the diagram.

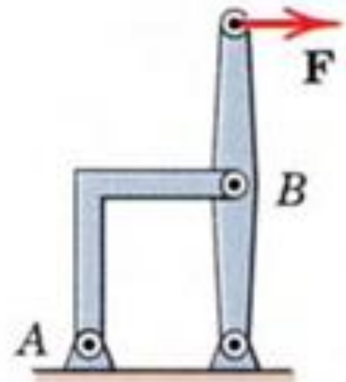
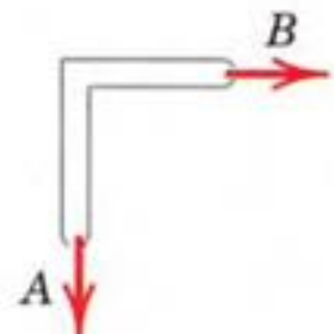
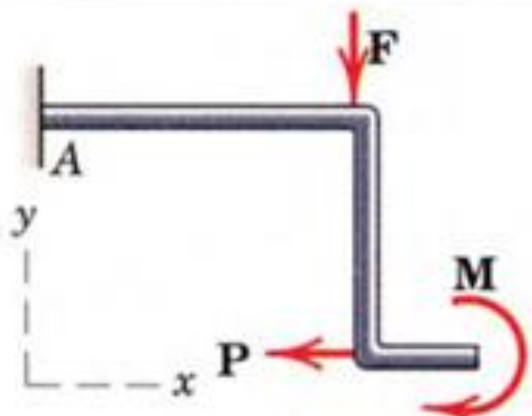
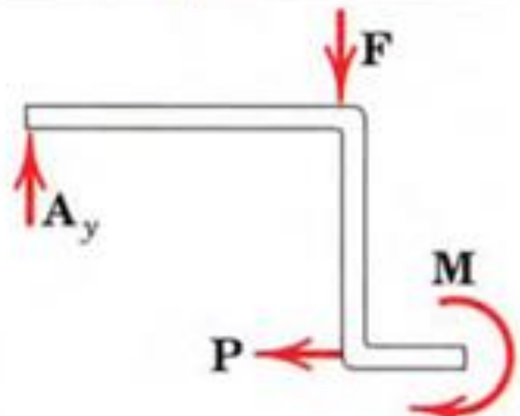
SAMPLE FREE-BODY DIAGRAMS	
Mechanical System	Free-Body Diagram of Isolated Body
1. Plane truss Weight of truss assumed negligible compared with P 	
2. Cantilever beam 	


3. Beam
Smooth surface contact at A. Mass m
P
B
A
M
N
y
x
W = mg
B_x
B_y
4. Rigid system of interconnected bodies analyzed as a single unit
Weight of mechanism neglected
P
A
B
m
W = mg
A_y
B_x
B_y

	Body	Incomplete *FBD*
1. Bell crank supporting mass m with pin support at A.		
2. Control lever applying torque to shaft at O.		
3. Boom OA, of negligible mass compared with mass m. Boom hinged at O and supported by hoisting cable at B.		

4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B.		

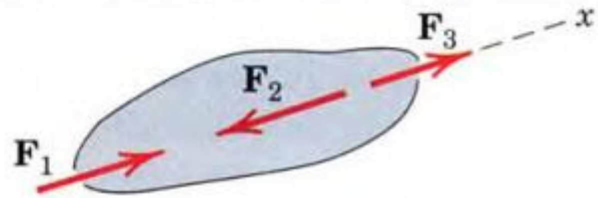
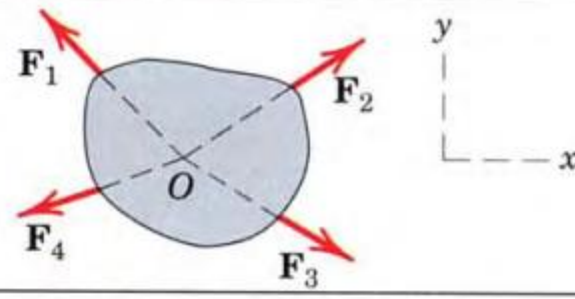
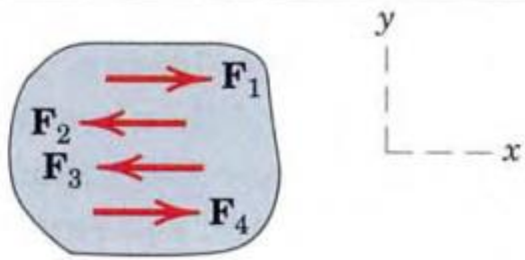
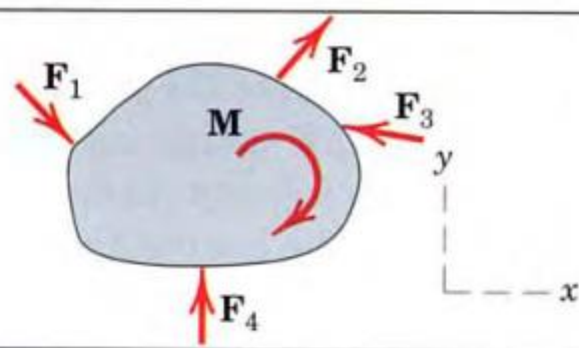
	Body	Wrong or Incomplete *FBD*
1. Lawn roller of mass m being pushed up incline θ.		
2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass m being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		

4. Supporting angle bracket for frame; Pin joints.		
5. Bent rod welded to support at A and subjected to two forces and couple.		

Equilibrium Conditions

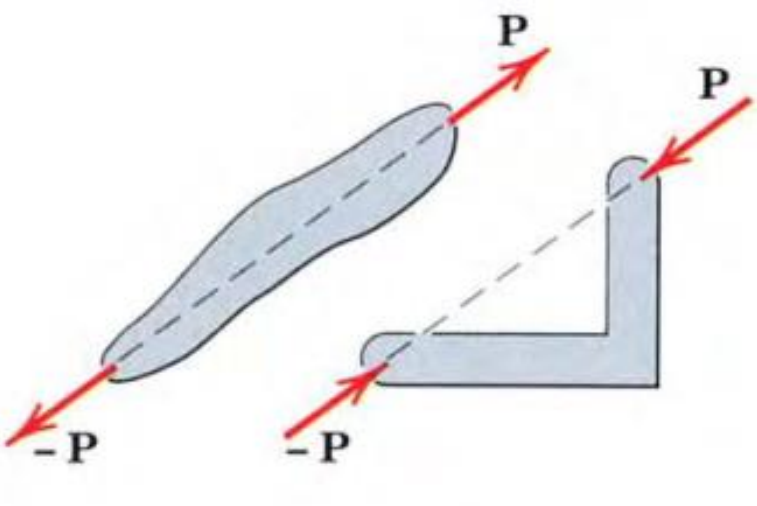
- We defined *equilibrium* as the condition in which the *resultant* of *all forces* acting on a body is *zero*. Stated in another way, a body is in *equilibrium* if *all forces* and *moments* applied to it are *in balance*.

$$\boxed{\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0}$$

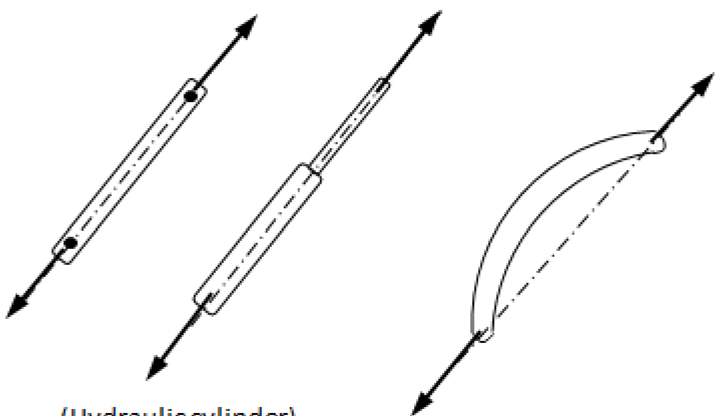
<table>
<tr><th colspan="3">CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS</th></tr>
<tr><th>Force System</th><th>Free-Body Diagram</th><th>Independent Equations</th></tr>
<tr><td>1. Collinear</td><td>
</td><td>$\Sigma F_x = 0$</td></tr>
<tr><td>2. Concurrent at a point</td><td>
</td><td>$\Sigma F_x = 0$
$\Sigma F_y = 0$</td></tr>
<tr><td>3. Parallel</td><td>
</td><td>$\Sigma F_x = 0 \qquad \Sigma M_z = 0$</td></tr>
<tr><td>4. General</td><td>
</td><td>$\Sigma F_x = 0 \qquad \Sigma M_z = 0$
$\Sigma F_y = 0$</td></tr>
</table>

Two-Force Member

- You should be alert to two frequently occurring equilibrium situations. The first situation is the *equilibrium* of a *body* under the *action* of *two forces* only.

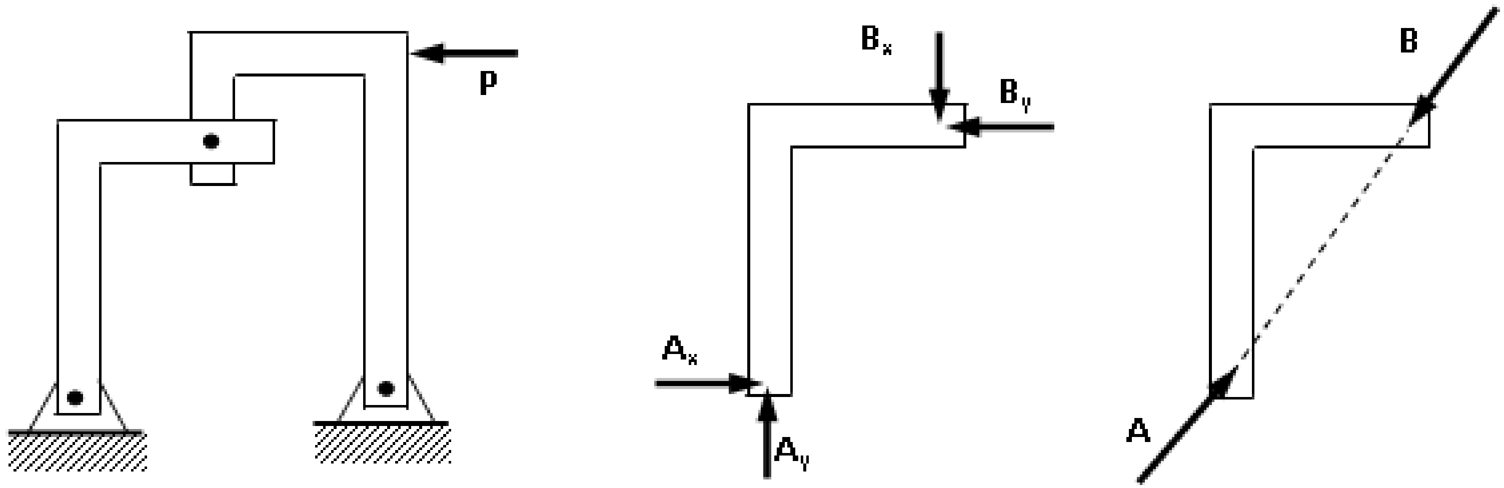


Two-force members



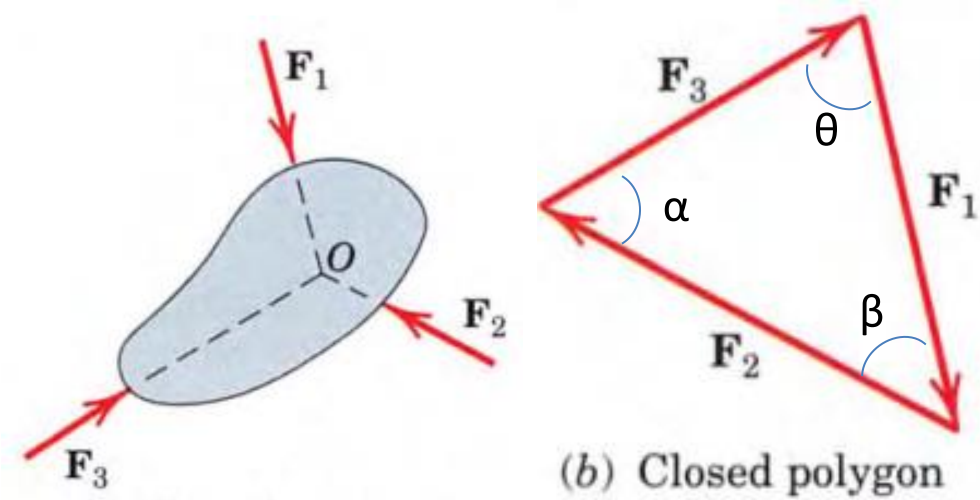
(Hydraulic cylinder)

Two-force member



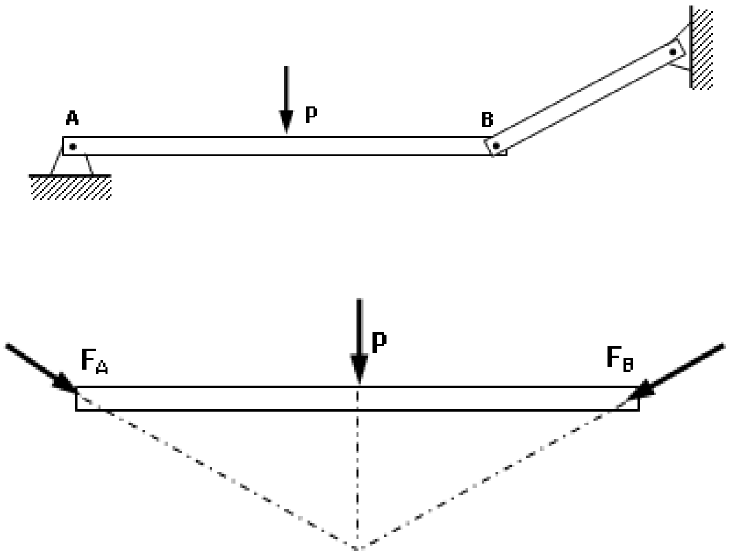
Three-Force Member

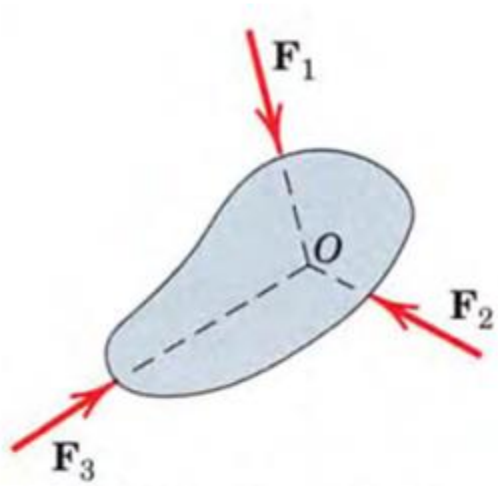
- The second situation is a *three-force member*, which is a *body* under the *action* of *three forces*.

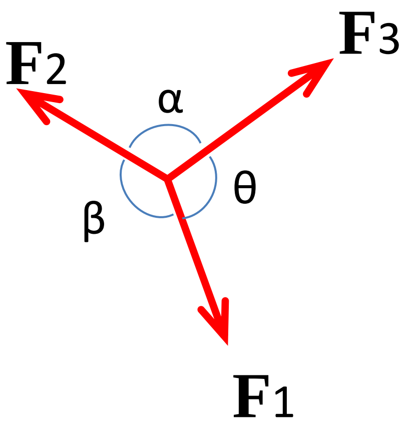


(a) Three-force member

(b) Closed polygon satisfies $\Sigma \mathbf{F} = 0$







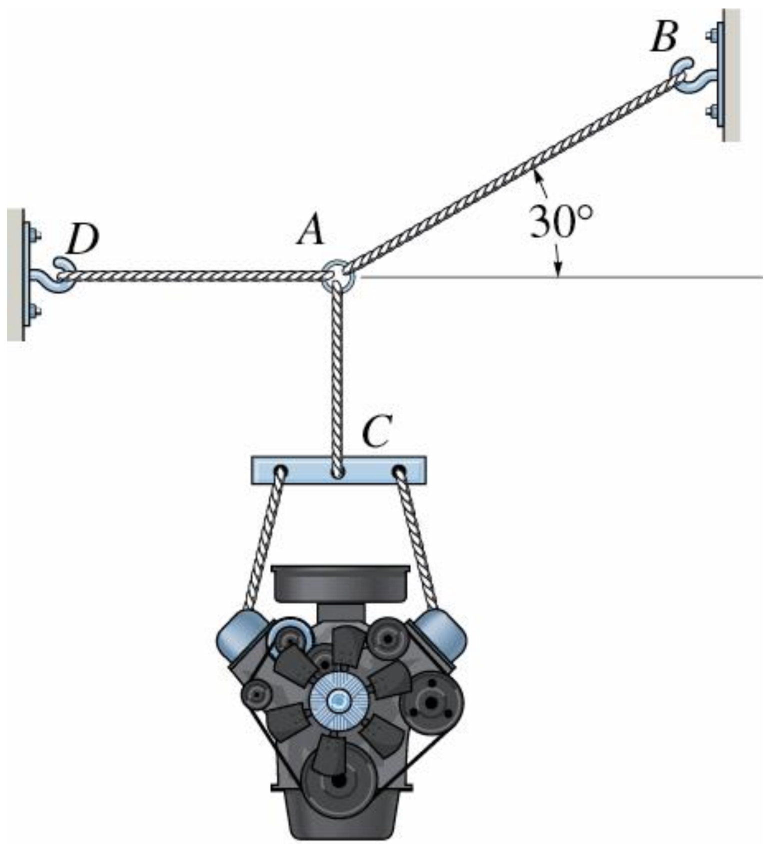
$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\theta} = \frac{F_3}{\sin\beta}$$

EXAMPLES

EQUILIBRIUM in 2D

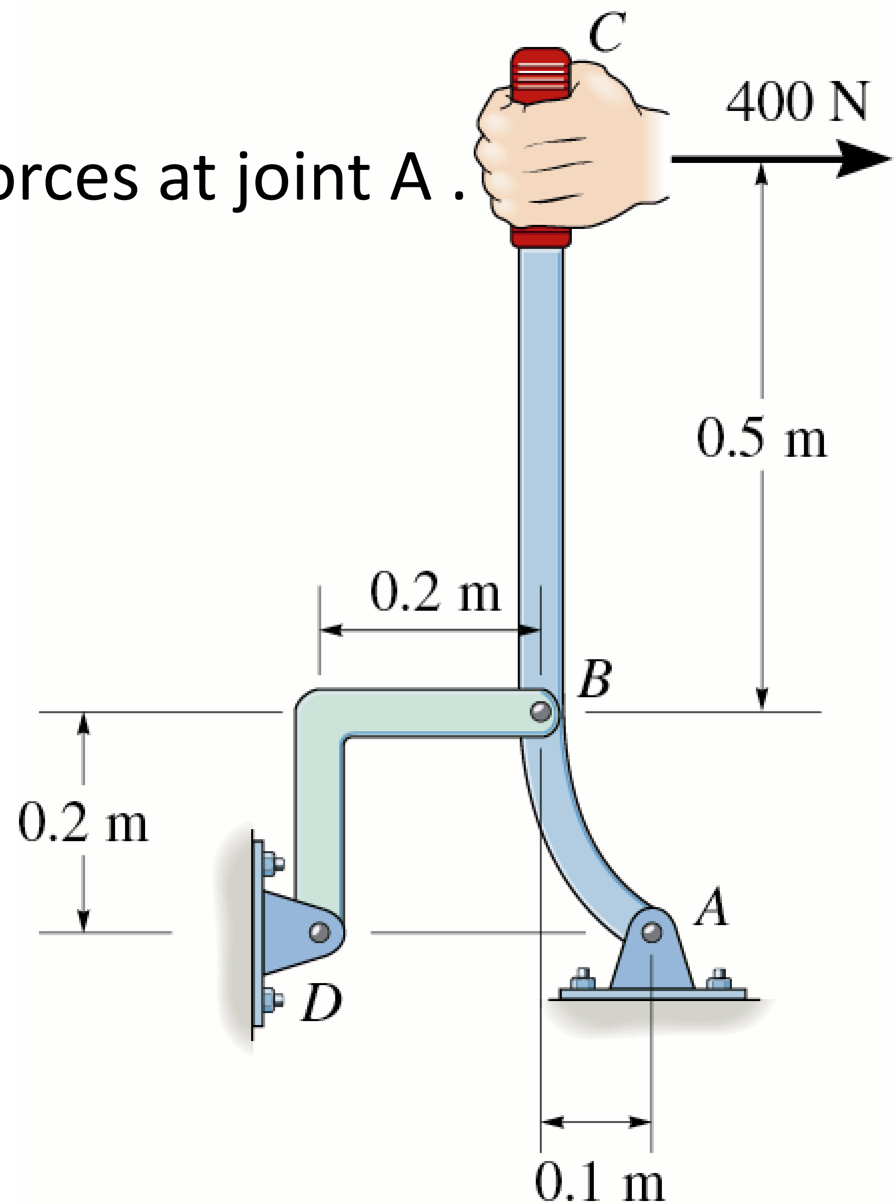
Example 1

- Find the forces on the ropes AD and AB if the engine Block is 250 kg.



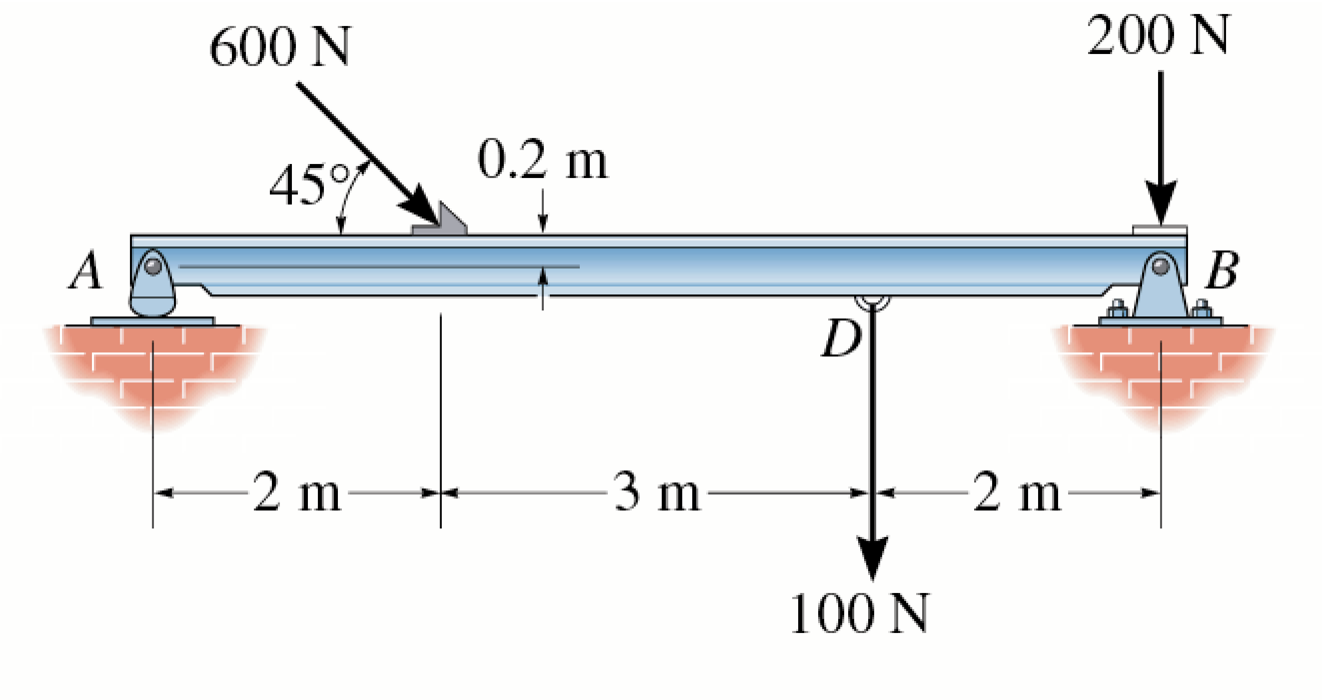
Example 2

- Determine the reaction forces at joint A .



Example 3

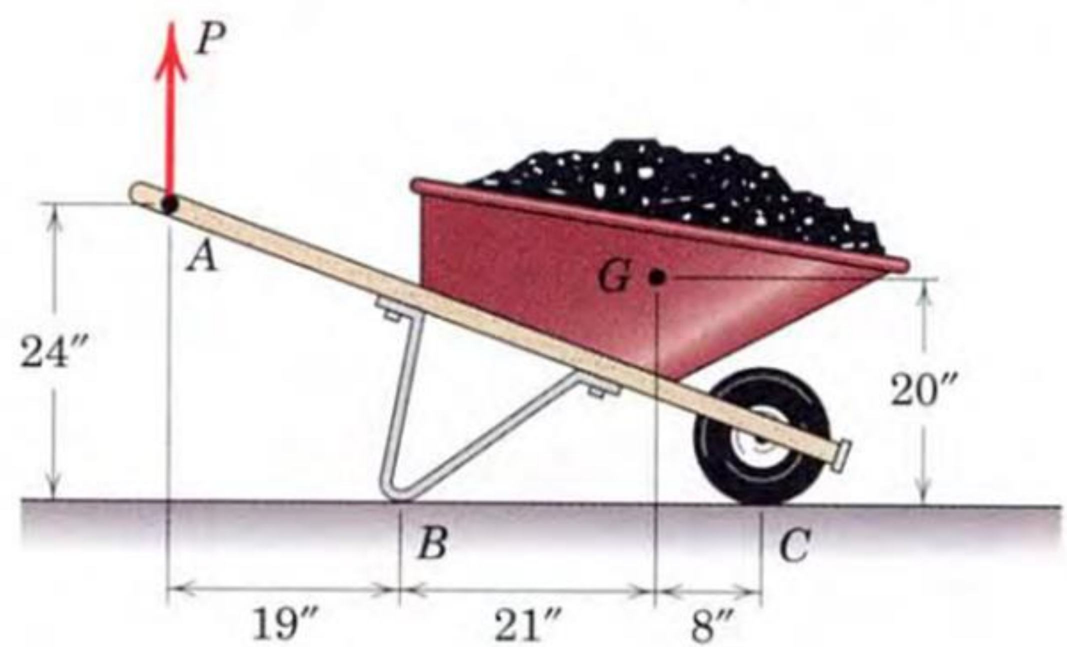
- Determine the reaction forces at joints.



Example 4

- Determine the magnitude P of the vertical force required to lift the wheelbarrow free of the ground at point B. The combined weight of the wheelbarrow and its load is 240 lb with center of gravity at G.

P=40 lb



Example 5

- The pin A, which connects the 200 kg steel beam with center of gravity at G to the vertical column, is welded both to the beam and to the column. To test the weld, the 80 kg man loads the beam by exerting a 300 N force on the rope which passes through a hole as shown. Calculate the moment M supported by the pin.

  - **Answer:** **M**=4.94 kNm (CCW)