AE 104

STATICS

CHAPTER 2

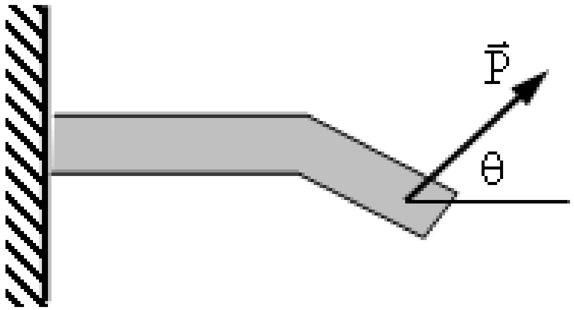
FORCE SYSTEMS

Introduction

- In this chapter, the *properties* and *effects* of various kinds of *forces* which *act on* engineering *structures* and *mechanisms* will be examined.

Force

- A *force* has been defined as the *action* of one body on another. We find that *force is a vector* quantity.

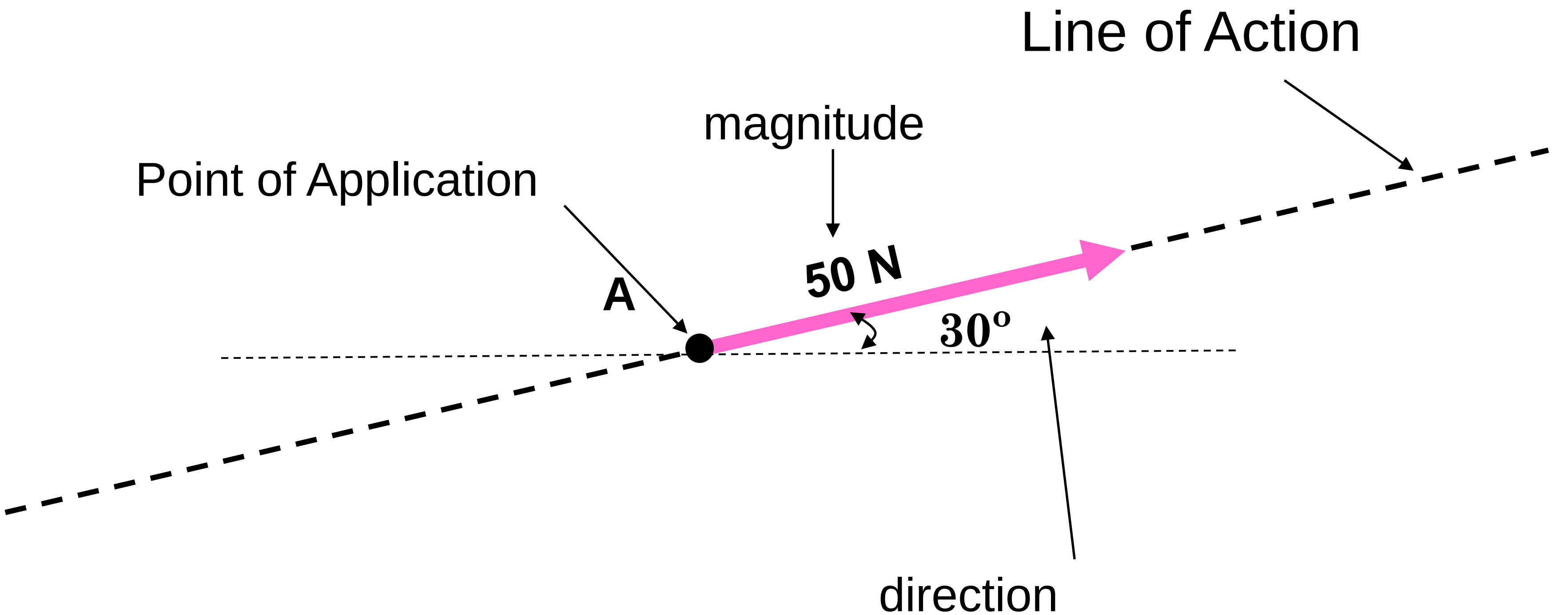


- The action of a force on a body can be separated into two effects,

External effect:

Internal effect:

- The complete *specification* of the action of a *force* must include its *magnitude*, *direction*, and *point of application*, in which case it is treated as a fixed vector.

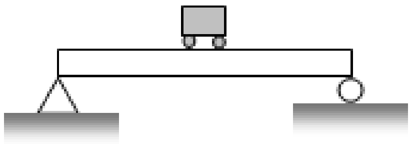


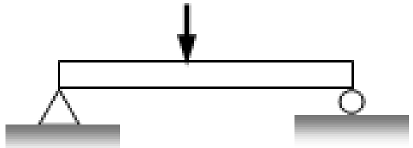
Since the vector has a well defined point of application, it is a fixed vector, therefore can not be moved without modifications

Classification of Forces

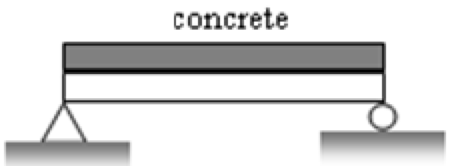
- Body force: Forces that are applied on hole body.
- Surface Force: Forces that are apllied on surfaces of body.
- Concentrated (single) Force: When the dimensions of the area on which forces are applied are very small compared with the other dimensions of the body, The forces can be considered as concentrated at a point with negligible loss of accuracy.
- Distributed forces: Actually every contact force is applied over a finite area and is therefore a distributed force.

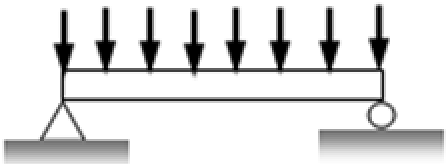
Single Load





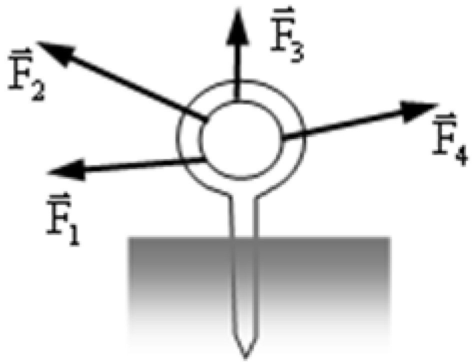
Distributed Load



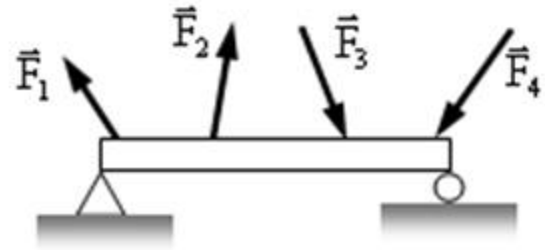


Forces due to application

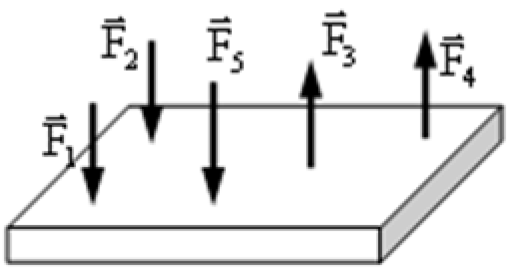
concurrent



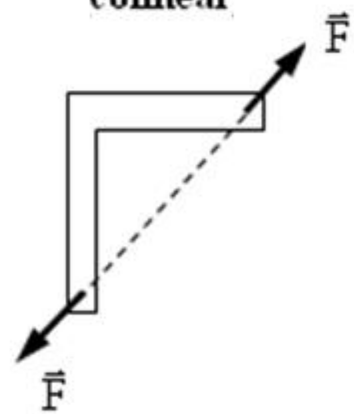
coplanar



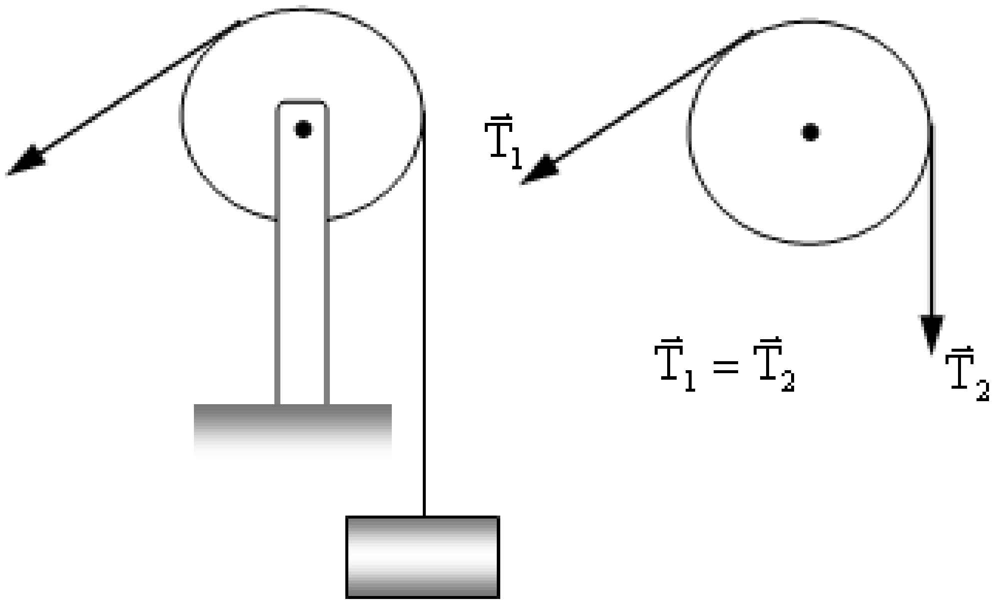
parallel



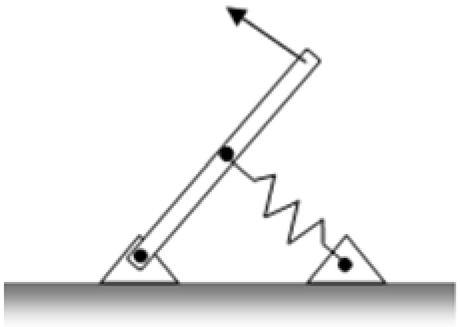
colinear

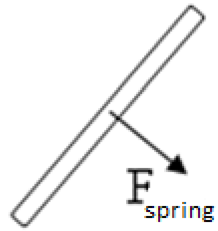


Forces on Pulley

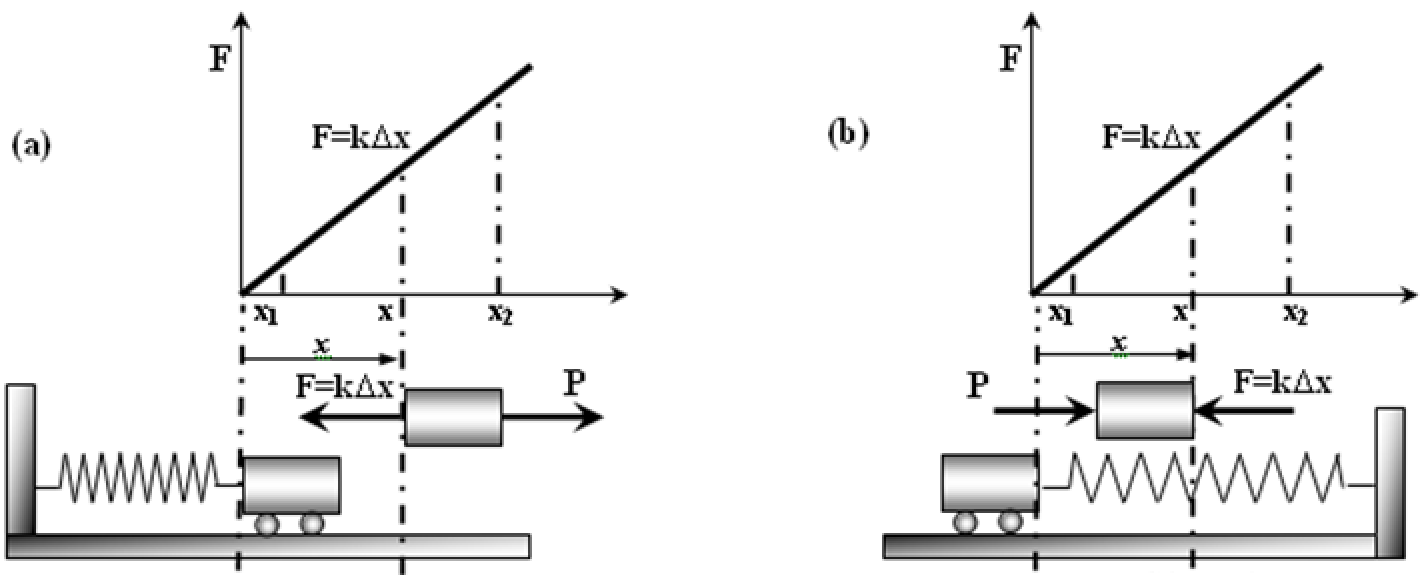


Forces on springs

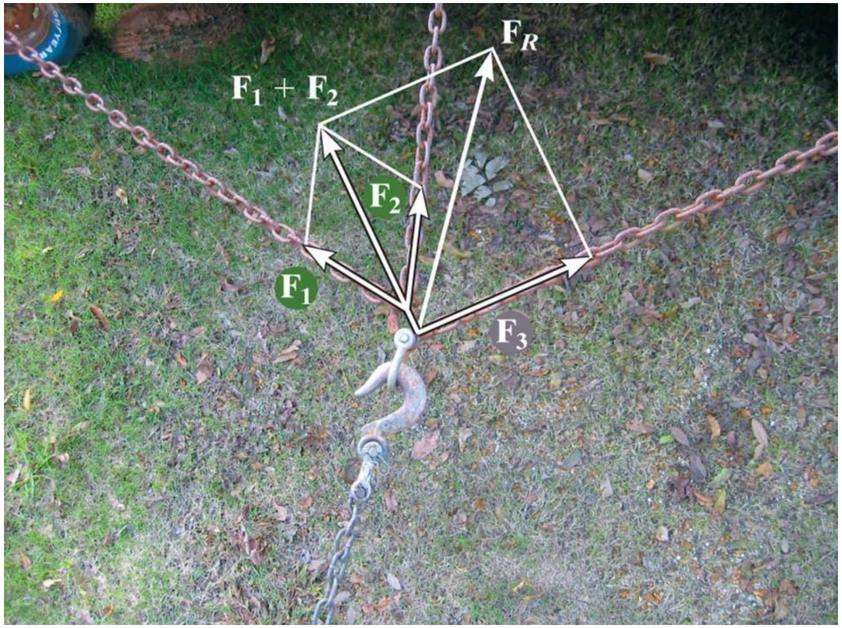


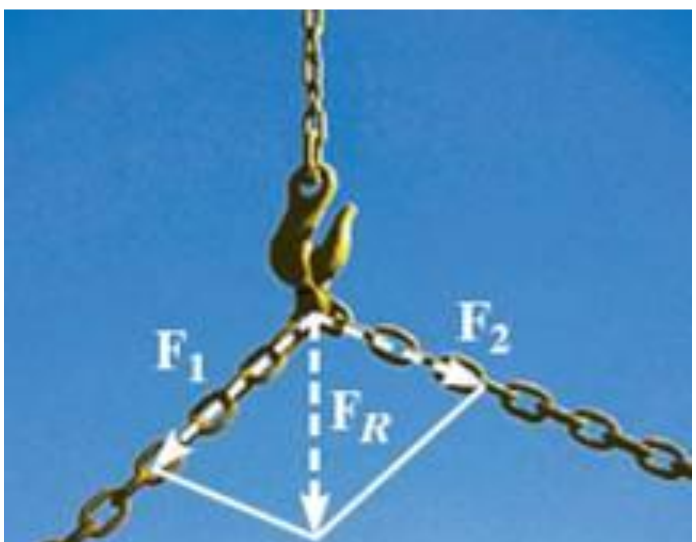


$$F_{spring} = k\Delta x$$



Addition of Forces in a Plane



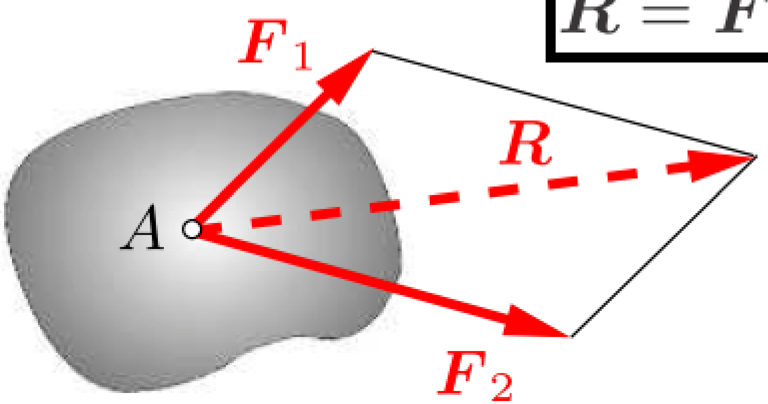


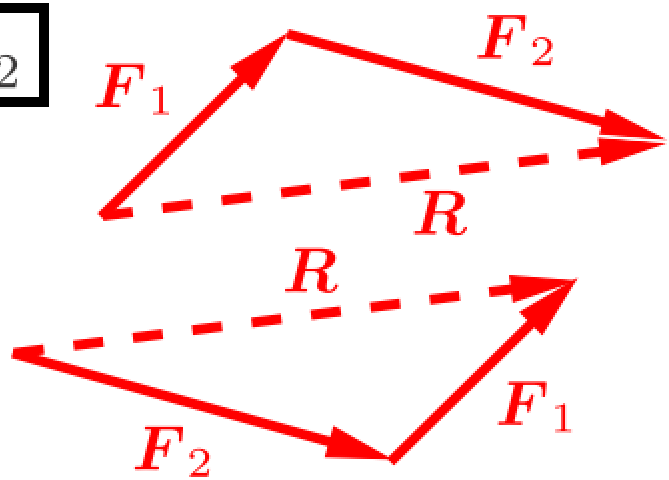
- There are some concurrent forces acting on the hook due to the chains.
- We need to decide if the hook will fail (bend or break)?
- To do this, we need to know the resultant force acting on the hook.

Addition of Forces in a Plane

- The *effect* of two nonparallel *forces* F1 and F2 acting at a *point A* of a body is the same as the effect of the *single force R* acting at the same point and *obtained* as the *diagonal* of the *parallelogram* formed by F1 and F2 or it can be *obtained* by using *triangle method*.

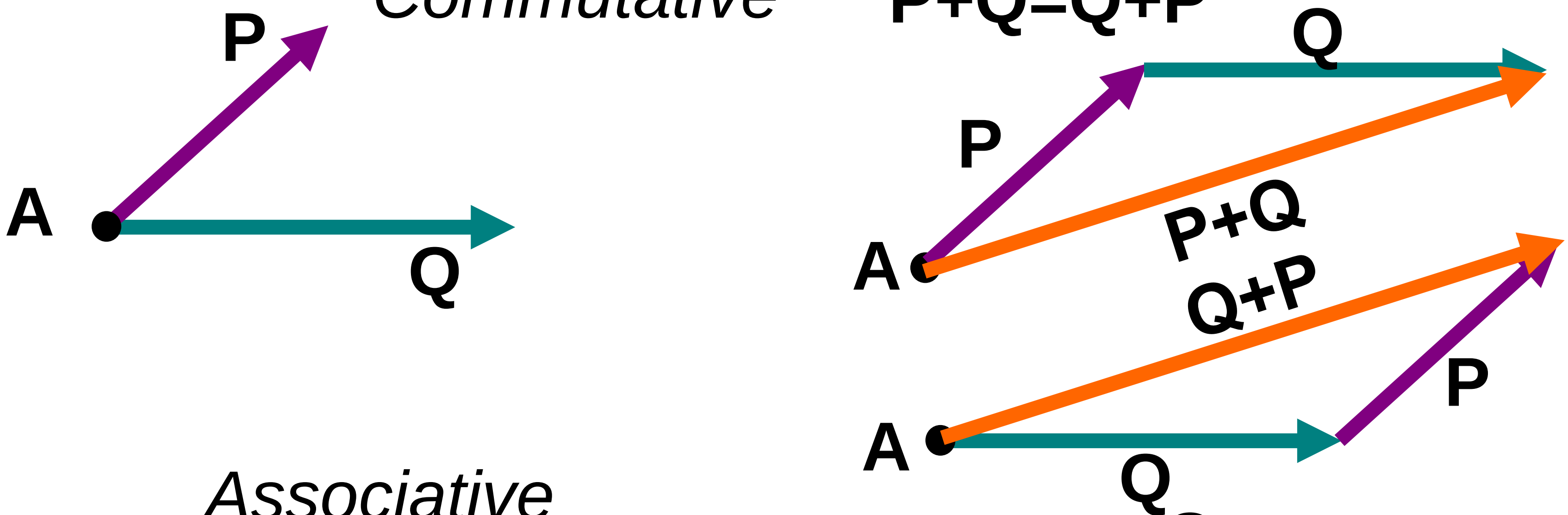
$$\boldsymbol{R} = \boldsymbol{F}_1 + \boldsymbol{F}_2$$



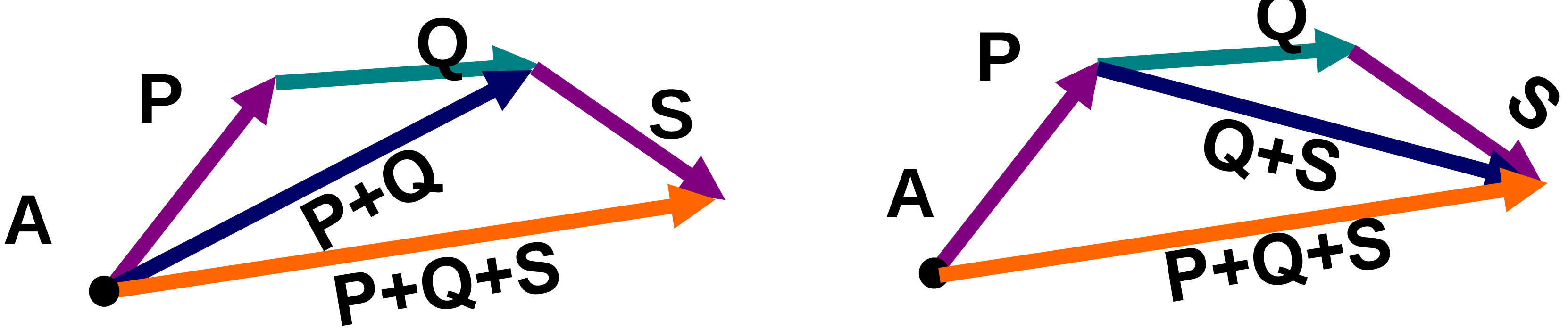


Properties of Vector Addition

Commutative **P+Q=Q+P**

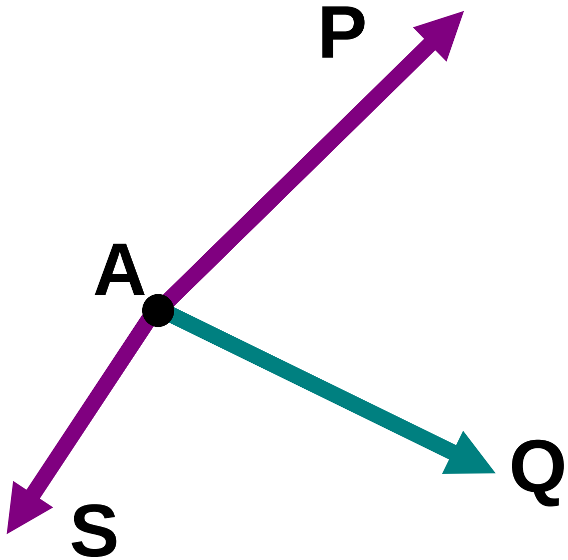


Associative

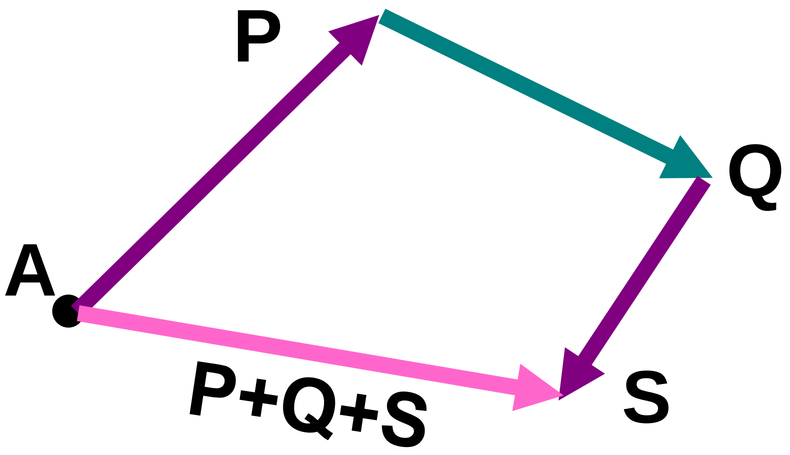


P+Q+S=(P+Q)+S=P+(Q+S)

Resultant of several concurrent forces

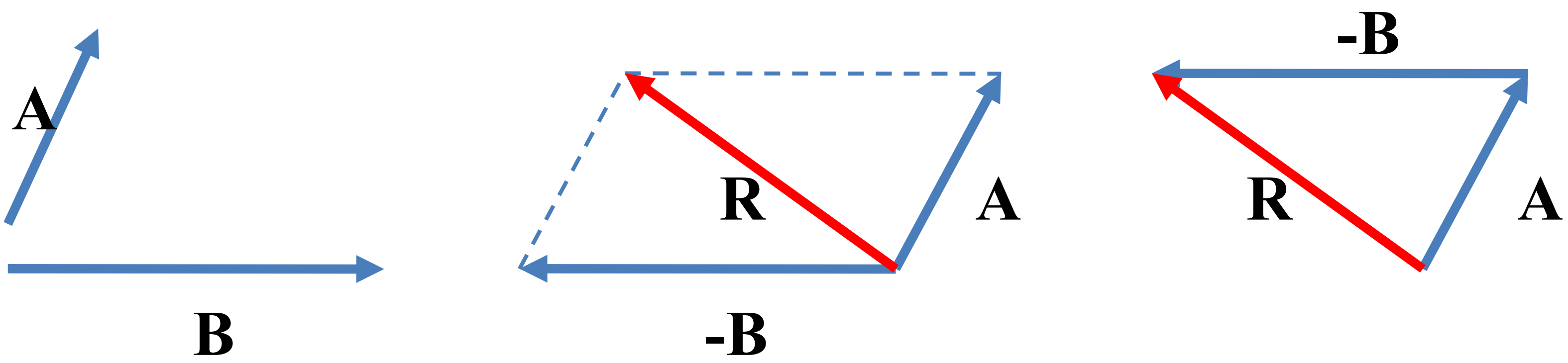


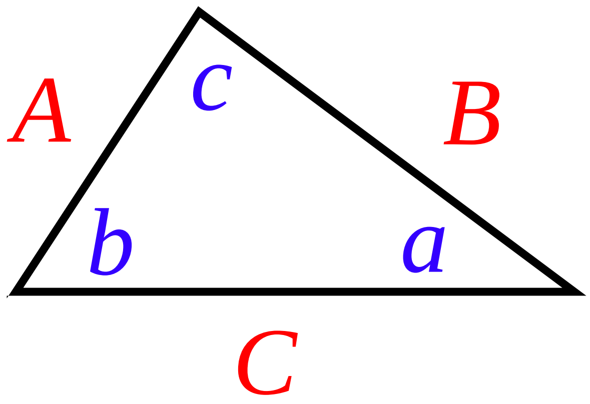
Using polygon rule



Force Subtraction

$$R = A - B = A + (-B)$$





$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

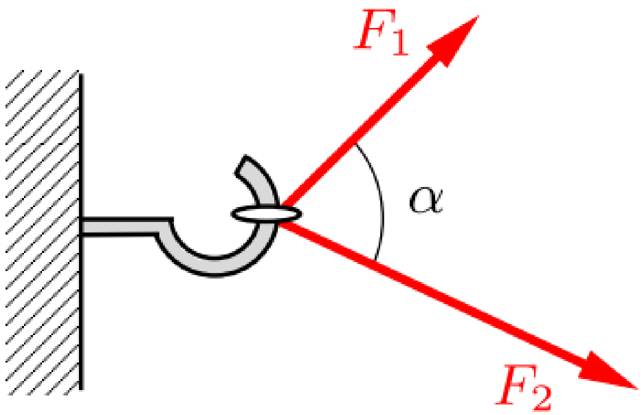
law of sines

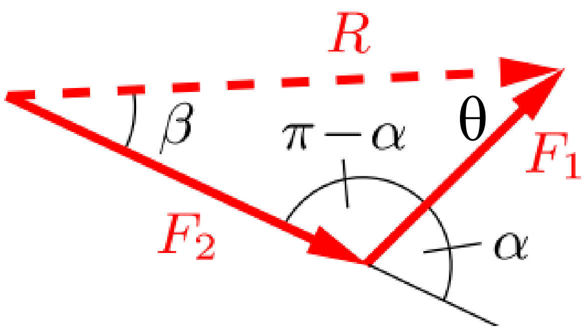
$$C = \sqrt{\mathrm{A}^2 + B^2 - 2AB\cos c}$$

law of cosines

Example

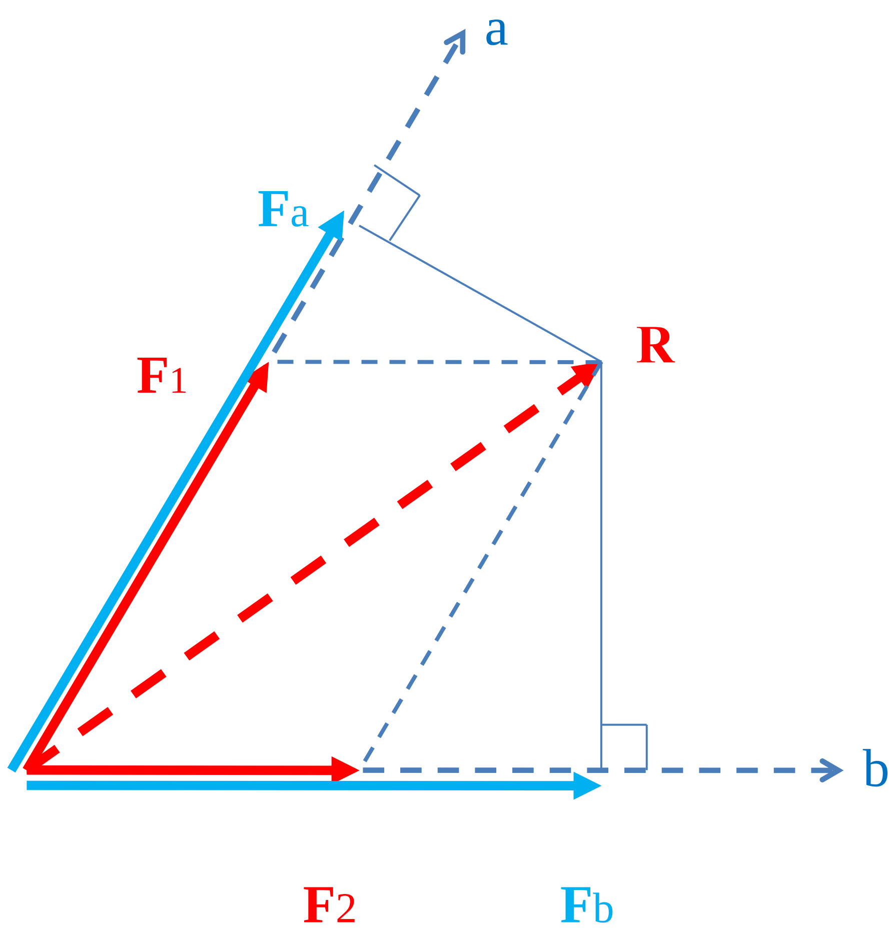
- A hook carries two forces F_1 and F_2, which define the angle α. Determine the magnitude and direction of the resultant.





$R^2 = F_1^2 + F_2^2 - 2\,F_1\,F_2 \cos(\pi - \alpha)$ or $\dfrac{R}{\sin(\pi - \alpha)} = \dfrac{F_1}{\sin\beta} = \dfrac{F_2}{\sin\theta}$

Projection and Component

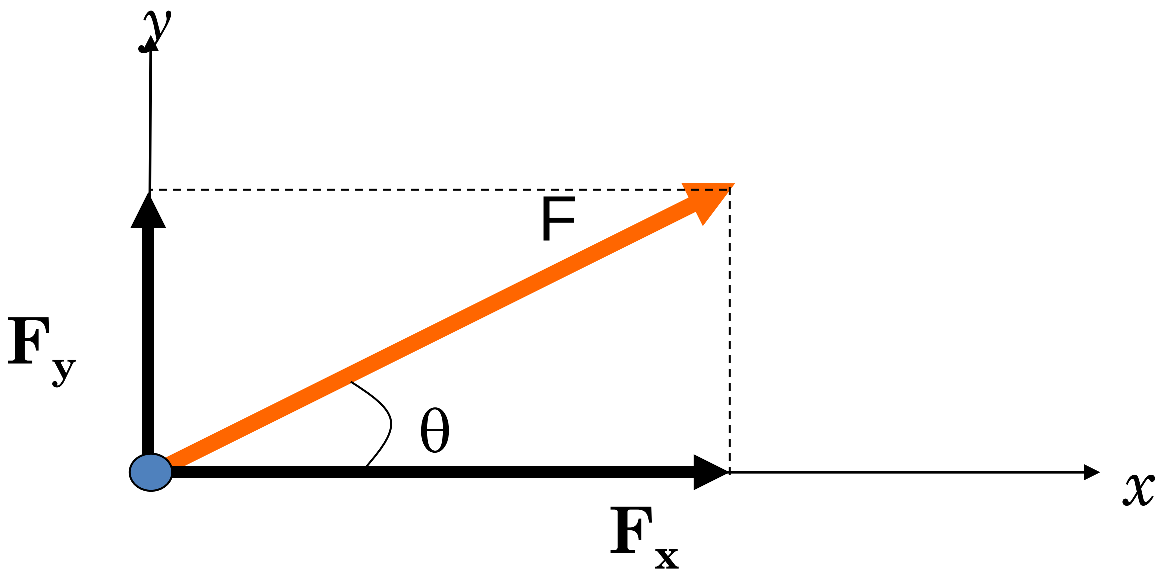


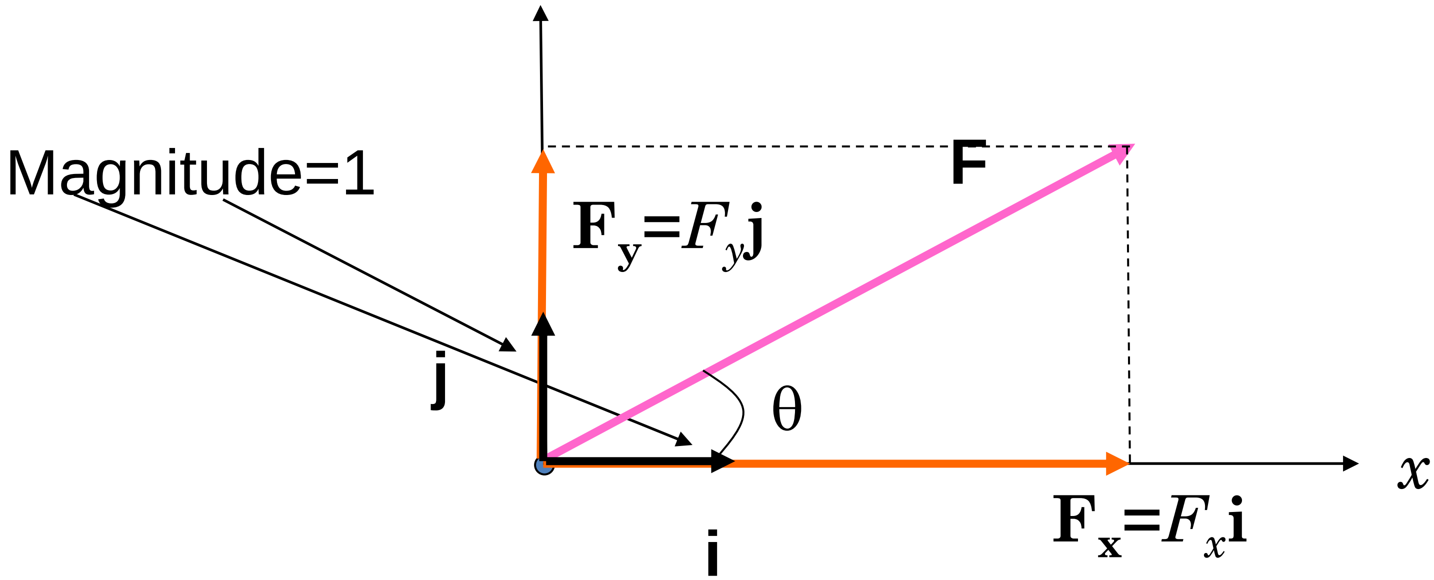
$\mathbf{F}_a$ and $\mathbf{F}_b$ are projections of $\mathbf{R}$

$\mathbf{F}_1$ and $\mathbf{F}_2$ components of $\mathbf{R}$

Rectangular Components

- The most common two-dimensional resolution of a force vector is into rectangular components.



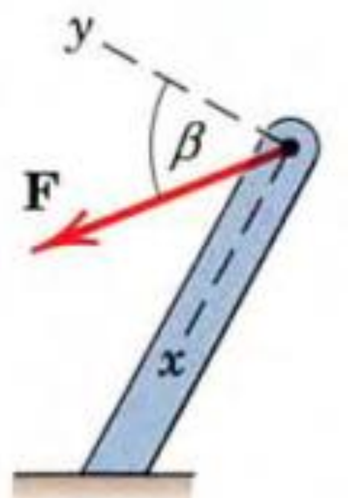


- **F** = **F**x + **F**y where **F**x and **F**y are vector components of **F.**
- **F** = Fx**i** + Fy**j** where Fx and Fy are the x and y scalar components of the vector **F. i** and **j** are the **unit vectors** along the x and y axes,

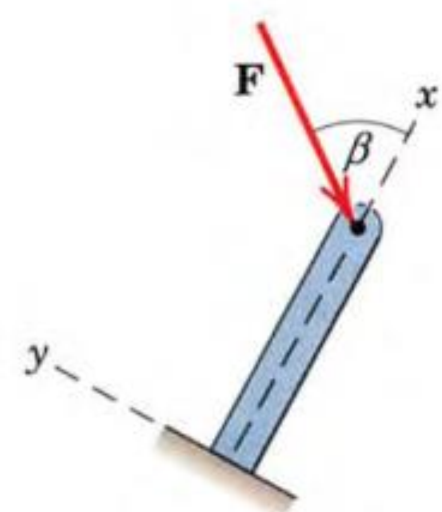
$$F_x = F\cos\theta \qquad F_y = F\sin\theta$$

$$F = \sqrt{F_x^2 + F_y^2} \qquad \tan\theta = \frac{F_y}{F_x}$$

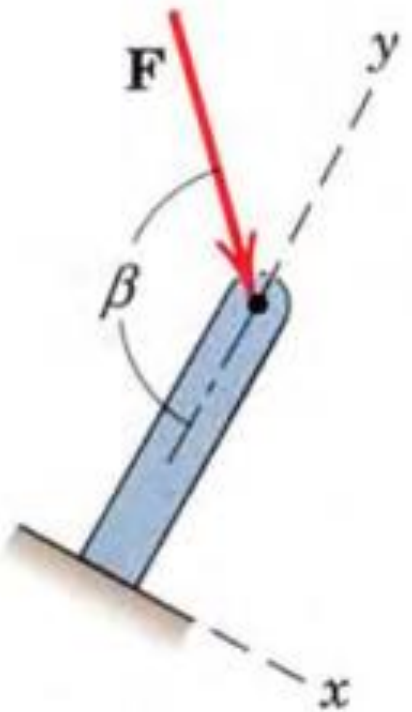
Rectangular Components of Forces



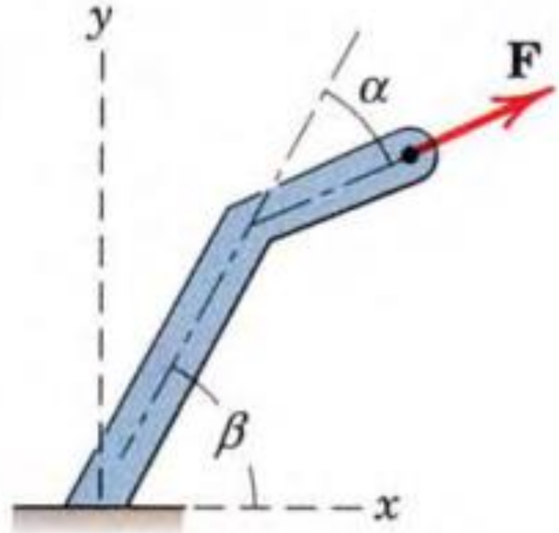
$$F_x = F \sin \beta$$
$$F_y = F \cos \beta$$



$$F_x = -F \cos \beta$$
$$F_y = -F \sin \beta$$



$$F_x = F\sin(\pi - \beta)$$
$$F_y = -F\cos(\pi - \beta)$$



$$F_x = F\cos(\beta - \alpha)$$
$$F_y = F\sin(\beta - \alpha)$$

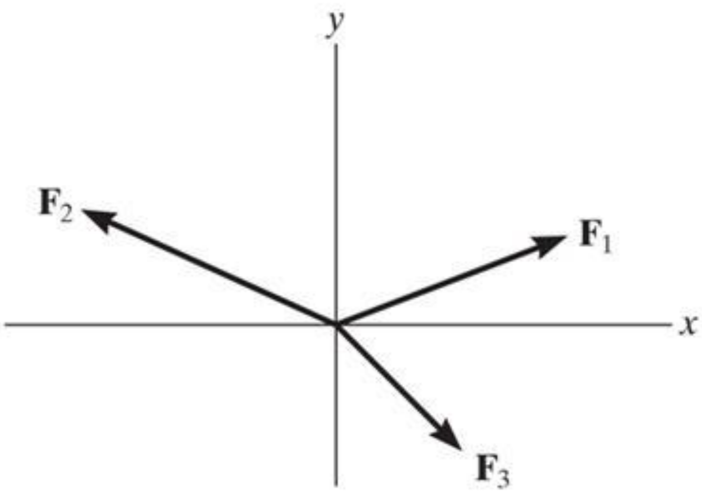
When **three or more coplanar forces** act on a particle, the rectangular components of their resultant **R** can be obtained by adding algebraically the corresponding components of the given forces.

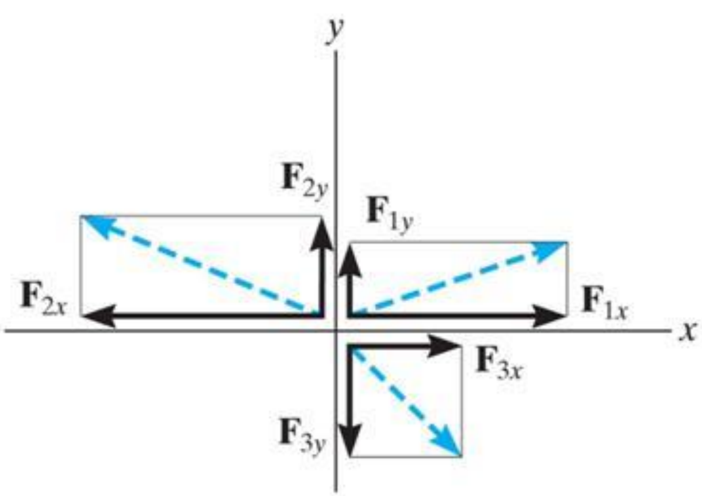
$$R_{\mathbf{x}} = \Sigma R_{\mathbf{x}} \qquad R_{\mathbf{y}} = \Sigma R_{\mathbf{y}}$$

The magnitude and direction of **R** can be determined from

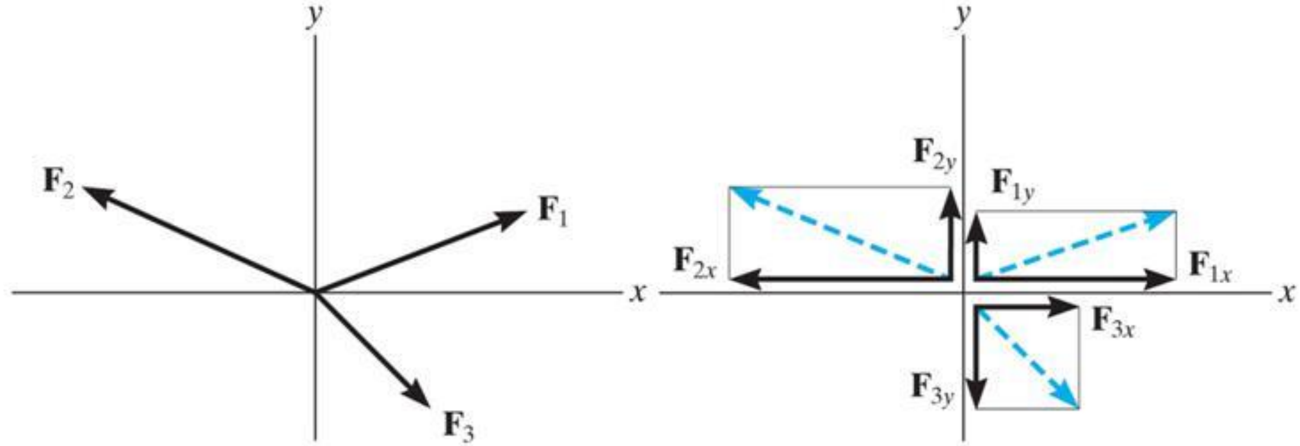
$$\tan\theta = \frac{R_{\mathbf{y}}}{R_{\mathbf{x}}} \qquad R = \sqrt{R_{\mathbf{x}}^2 + R_{\mathbf{y}}^2}$$

Addition of Several Vectors





- Step 1 is to *resolve* each force into its *components*.
- Step 2 is to *add* all the *x-components* together, followed by adding all the *y- components* together. These two totals are the x and y components of the resultant vector.
- Step 3 is to *find* the *magnitude* and *angle* of the *resultant vector*.



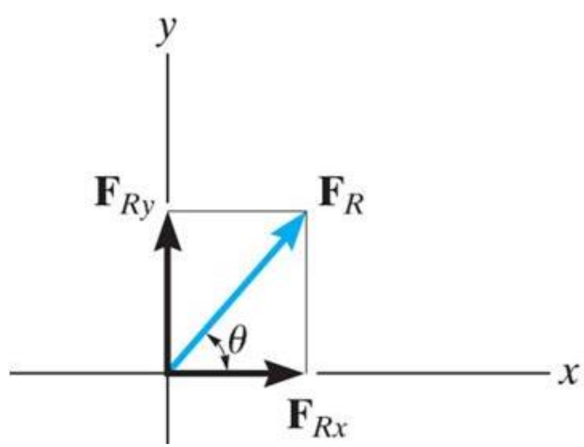
- Break the three vectors into components, then add them.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= F_{1x}\,\boldsymbol{i} + F_{1y}\,\boldsymbol{j} - F_{2x}\,\boldsymbol{i} + F_{2y}\,\boldsymbol{j} + F_{3x}\,\boldsymbol{i} - F_{3y}\,\boldsymbol{j}$$

$$= (F_{1x} - F_{2x} + F_{3x})\,\boldsymbol{i} + (F_{1y} + F_{2y} - F_{3y})\,\boldsymbol{j}$$

$$= (F_{Rx})\,\boldsymbol{i} + (F_{Ry})\,\boldsymbol{j}$$



$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

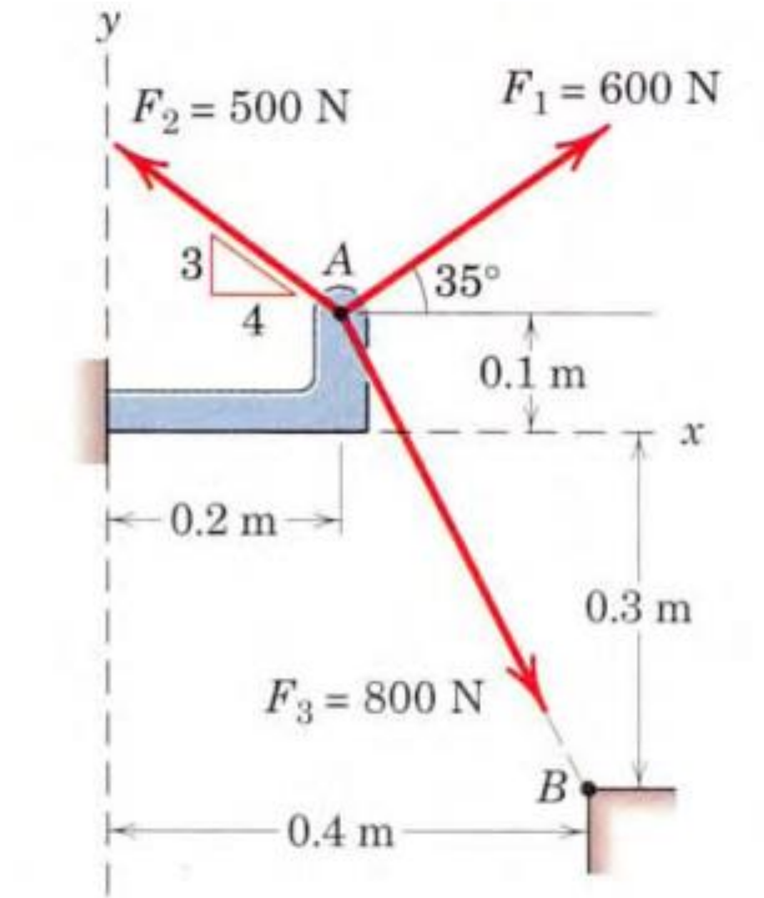
$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

Examples

FORCES SYSTEMS

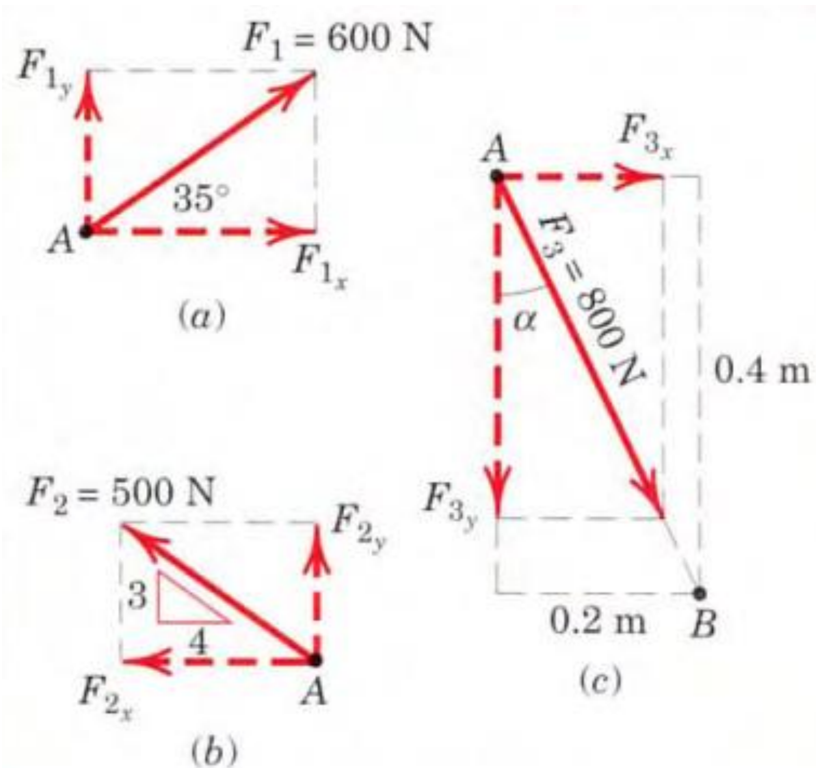
Example 1

- The forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ all of which act on point A of the bracket, are specified in three different ways. Determine the x and y scalar components of each of the forces.



- $\mathbf{F}_{1x}$=491 N $\mathbf{F}_{1y}$=344 N
- $\mathbf{F}_{2x}$=-400 N $\mathbf{F}_{2y}$=300 N
- $\mathbf{F}_{3x}$=358 N $\mathbf{F}_{3y}$=-716 N

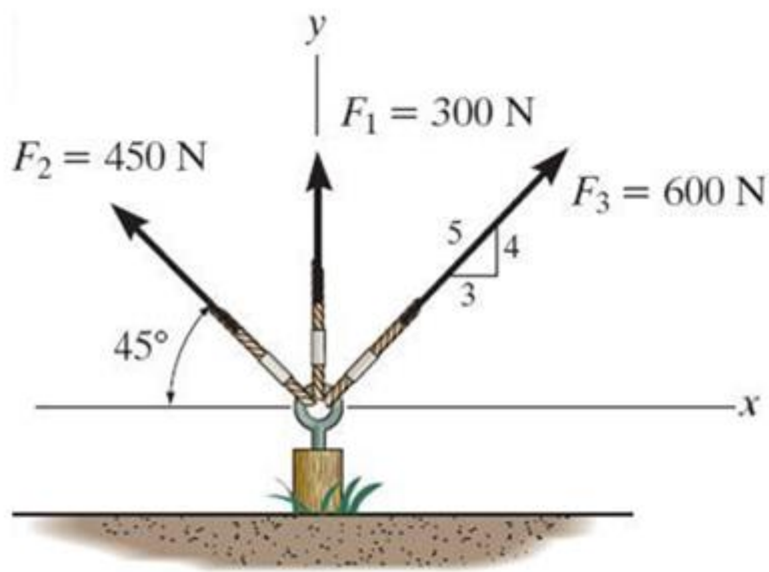
SOL. 1



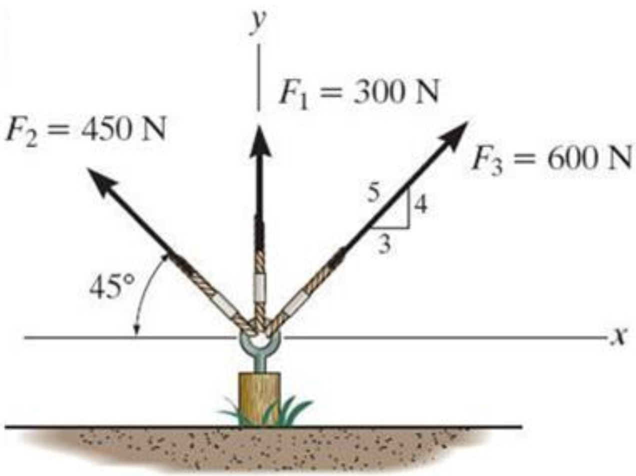
- $\mathbf{F}_{1x}$=491 N $\mathbf{F}_{1y}$=344 N
- $\mathbf{F}_{2x}$=-400 N $\mathbf{F}_{2y}$=300 N
- $\mathbf{F}_{3x}$=358 N $\mathbf{F}_{3y}$=-716 N

Example 2

- Determine the Resultant Forces and angle of this forces.



SOL. 2



$\mathbf{F}_1 = \{0\,\boldsymbol{i} + 300\,\boldsymbol{j}\}$ N

$\mathbf{F}_2 = \{-\,450\cos(45°)\,\boldsymbol{i} + 450\sin(45°)\,\boldsymbol{j}\}$ N

$= \{-\,318.2\,\boldsymbol{i} + 318.2\,\boldsymbol{j}\}$ N

$\mathbf{F}_3 = \{(3/5)\,600\,\boldsymbol{i} + (4/5)\,600\,\boldsymbol{j}\}$ N

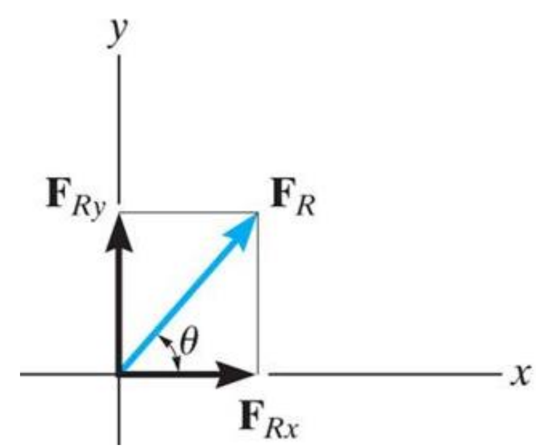
$= \{360\,\boldsymbol{i} + 480\,\boldsymbol{j}\}$ N

Summing up all the ***i*** and ***j*** components respectively, we get:

$\mathbf{F}_R = \{(0 - 318.2 + 360)\,\boldsymbol{i} + (300 + 318.2 + 480)\,\boldsymbol{j}\}$ N $= \underline{\{41.80\,\boldsymbol{i} + 1098\,\boldsymbol{j}\}\text{ N}}$

$F_R = ((41.80)^2 + (1098)^2)^{1/2} = \underline{1099\text{ N}}$

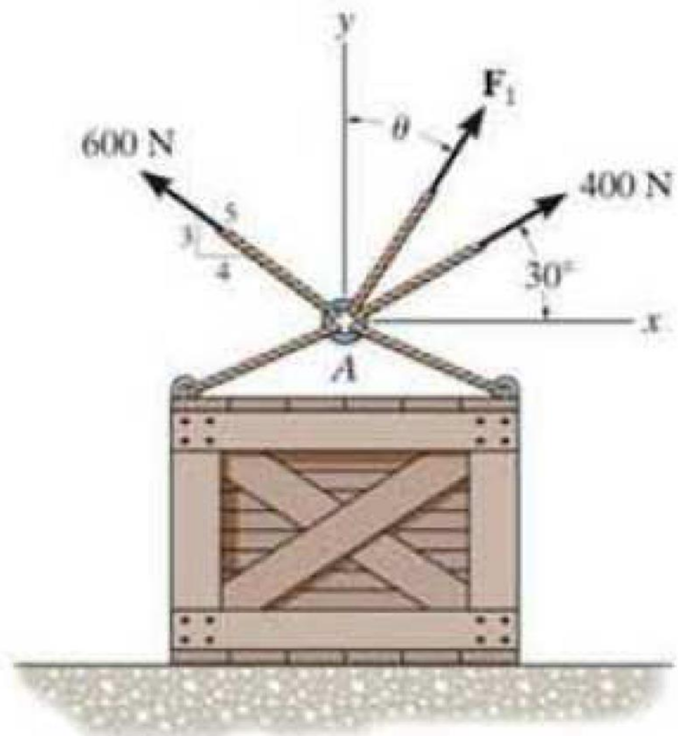
$\phi = \tan^{-1}(1098/41.80) = \underline{87.8°}$



Example 3

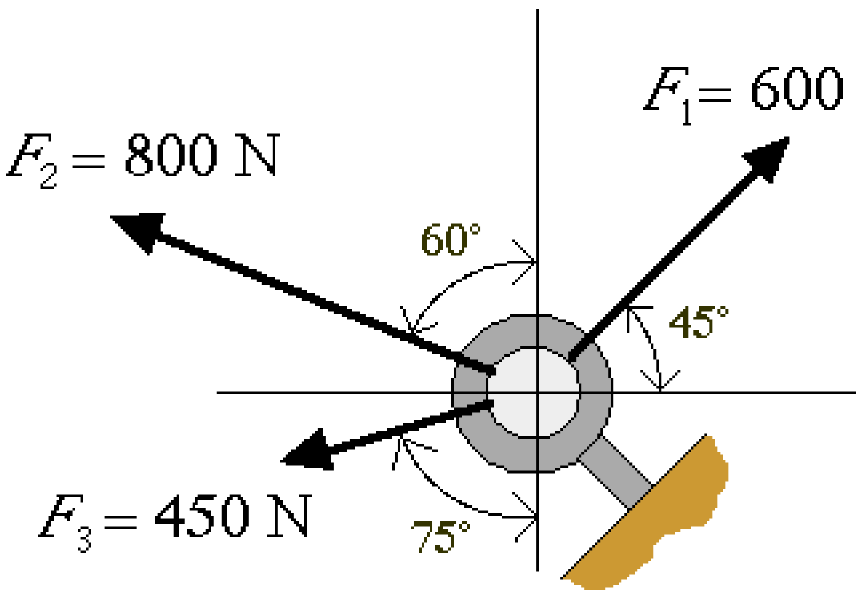
To have resultant Force in y direction with a magnitude of 800 N. Find the magnitude of F_1 and value of angle θ.

- Answer: F_1=275, θ=29,1°



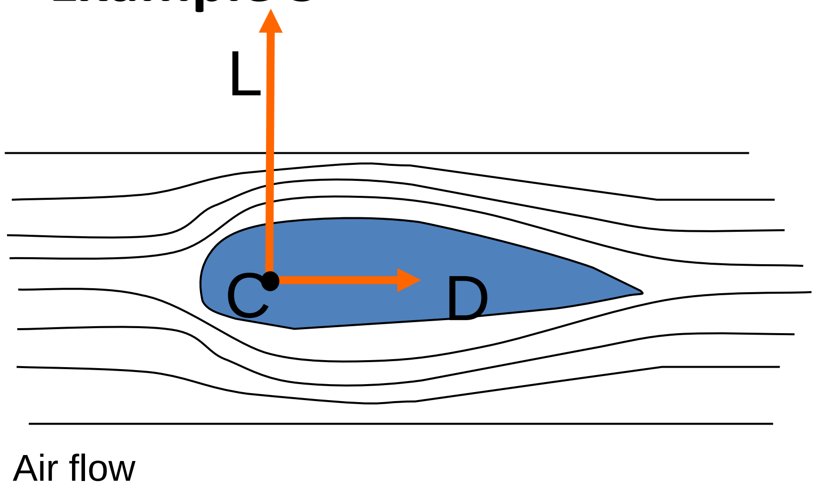
Example 4

A ring supports three forces. What is the resultant force applied to the ring?



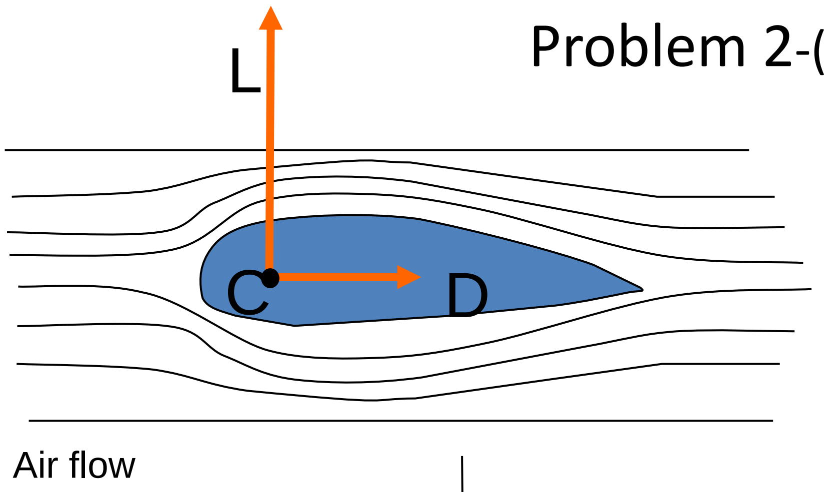
A: **R** = 998 N @ 134.9°

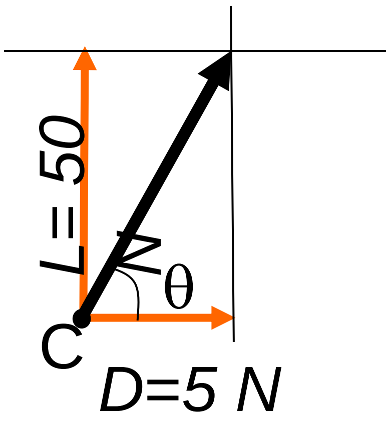
Example 5



The ratio of the lift force **L** to the drag force **D** for the simple airfoil is **L**/**D** = 10. If the lift force on a short section of the airfoil is *50 N*, compute the magnitude of the resultant force **R** and the angle θ which it makes with the horizontal.

Problem 2-(Meriam and Kraige)





$\tan\theta = 50/5$

$\theta = \tan^{-1}10$
$= 84.3^o$

$F = (50^2+5^2)^{0.50} =$
50.2 N