CHAPTER 5

Composite Bodies and Figures

- A *composite body* consists of a series of connected "*simpler*" shaped *bodies*, which may be *rectangular*, *triangular*, *semicircular*, etc. Such a *body* can often be *sectioned* or *divided* into its composite parts.

$$\bar{X} = \frac{\sum m\bar{x}}{\sum m} \quad , \quad \bar{Y} = \frac{\sum m\bar{y}}{\sum m} \quad , \quad \bar{Z} = \frac{\sum m\bar{z}}{\sum m}$$

TABLE D/3 PROPERTIES OF PLANE FIGURES

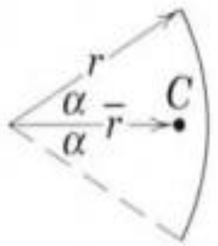
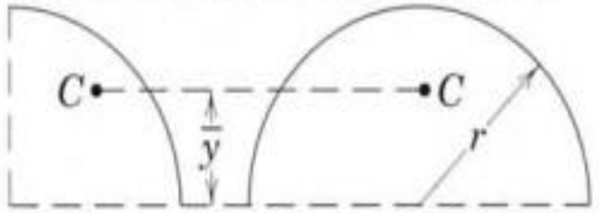
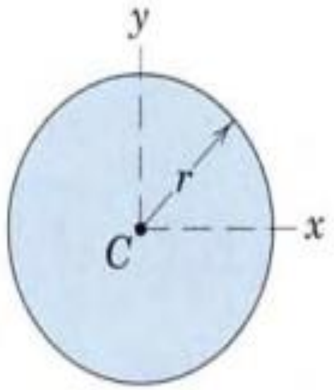
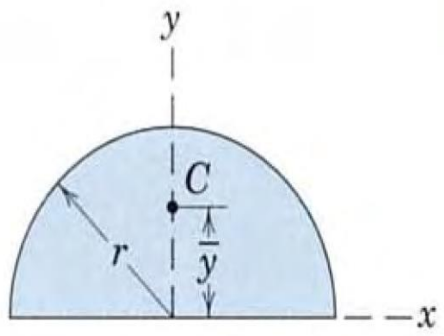
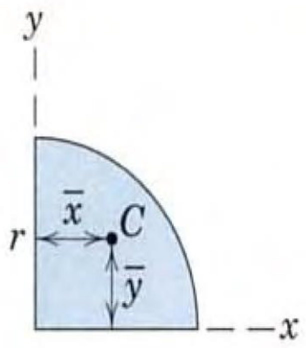
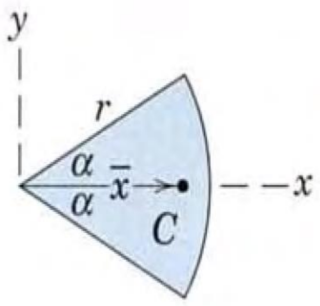
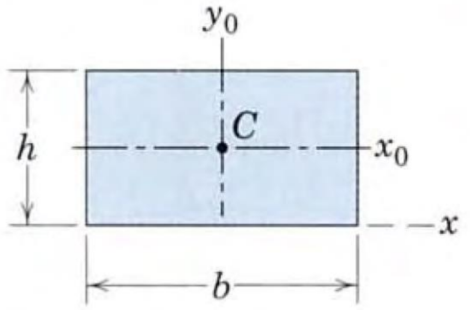
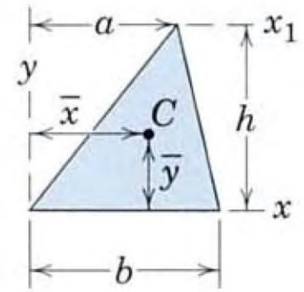
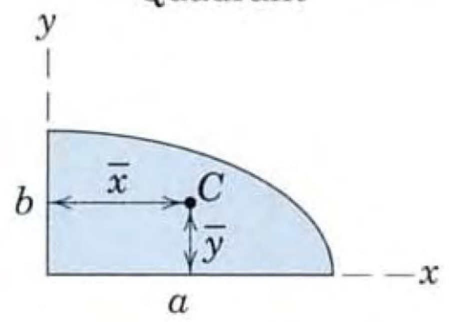
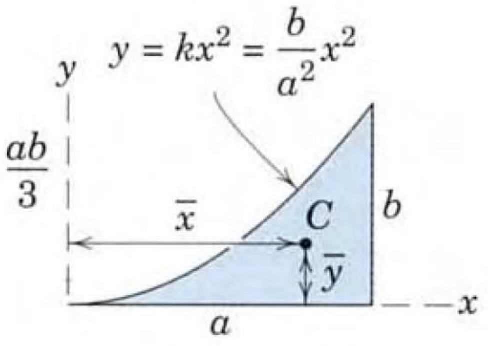
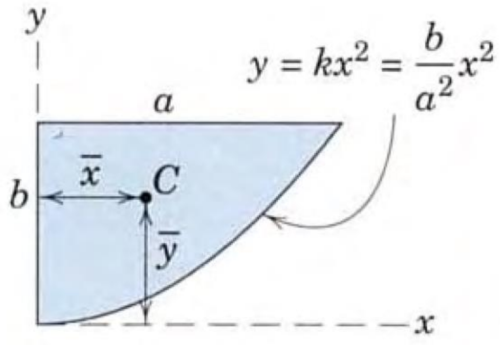
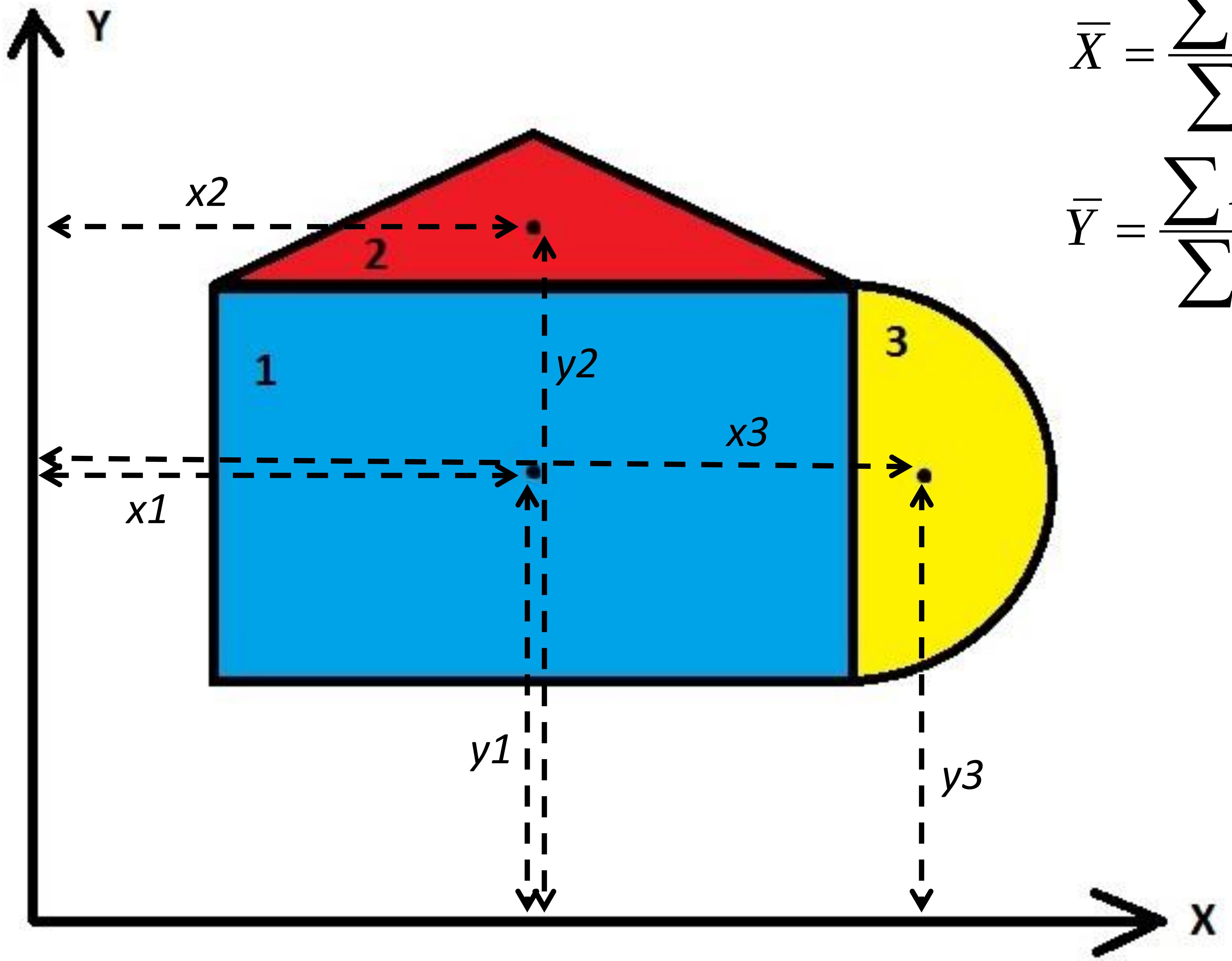
FIGURE	CENTROID	AREA MOMENTS OF INERTIA
Arc Segment 	$\bar{r} = \frac{r \sin\alpha}{\alpha}$	—
Quarter and Semicircular Arcs 	$\bar{y} = \frac{2r}{\pi}$	—
Circular Area 	—	$I_x = I_y = \frac{\pi r^4}{4}$ $I_z = \frac{\pi r^4}{2}$
Semicircular Area 	$\bar{y} = \frac{4r}{3\pi}$	$I_x = I_y = \frac{\pi r^4}{8}$ $\bar{I}_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$ $I_z = \frac{\pi r^4}{4}$
Quarter-Circular Area 	$\bar{x} = \bar{y} = \frac{4r}{3\pi}$	$I_x = I_y = \frac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) r^4$ $I_z = \frac{\pi r^4}{8}$
Area of Circular Sector 	$\bar{x} = \frac{2}{3} \frac{r \sin\alpha}{\alpha}$	$I_x = \frac{r^4}{4}(\alpha - \frac{1}{2}\sin 2\alpha)$ $I_y = \frac{r^4}{4}(\alpha + \frac{1}{2}\sin 2\alpha)$ $I_z = \frac{1}{2} r^4 \alpha$

TABLE D/3 PROPERTIES OF PLANE FIGURES *Continued*

FIGURE	CENTROID	AREA MOMENTS OF INERTIA
Rectangular Area 	—	$I_x = \frac{bh^3}{3}$ $\bar{I}_x = \frac{bh^3}{12}$ $\bar{I}_z = \frac{bh}{12}(b^2 + h^2)$
Triangular Area 	$\bar{x} = \frac{a+b}{3}$ $\bar{y} = \frac{h}{3}$	$I_x = \frac{bh^3}{12}$ $\bar{I}_x = \frac{bh^3}{36}$ $I_{x_1} = \frac{bh^3}{4}$
Area of Elliptical Quadrant 	$\bar{x} = \frac{4a}{3\pi}$ $\bar{y} = \frac{4b}{3\pi}$	$I_x = \frac{\pi ab^3}{16}, \quad \bar{I}_x = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)ab^3$ $I_y = \frac{\pi a^3 b}{16}, \quad \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^3 b$ $I_z = \frac{\pi ab}{16}(a^2 + b^2)$
Subparabolic Area Area $A = \frac{ab}{3}$ 	$\bar{x} = \frac{3a}{4}$ $\bar{y} = \frac{3b}{10}$	$I_x = \frac{ab^3}{21}$ $I_y = \frac{a^3 b}{5}$ $I_z = ab\left(\frac{a^3}{5} + \frac{b^2}{21}\right)$
Parabolic Area Area $A = \frac{2ab}{3}$ 	$\bar{x} = \frac{3a}{8}$ $\bar{y} = \frac{3b}{5}$	$I_x = \frac{2ab^3}{7}$ $I_y = \frac{2a^3 b}{15}$ $I_z = 2ab\left(\frac{a^2}{15} + \frac{b^2}{7}\right)$

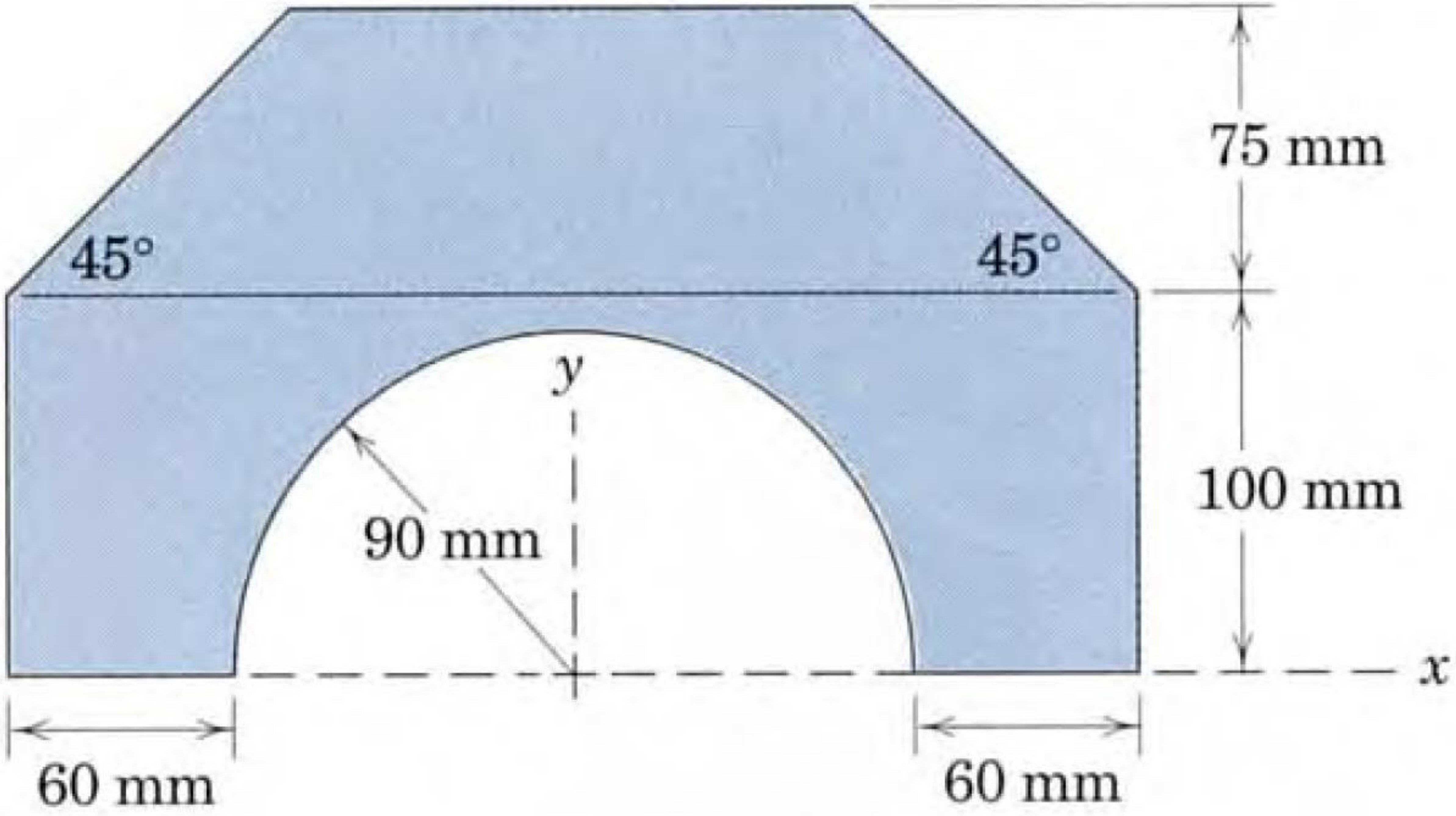


$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \frac{A_1 \times x_1 + A_2 \times x_2 + A_3 \times x_3}{A_1 + A_2 + A_3}$$

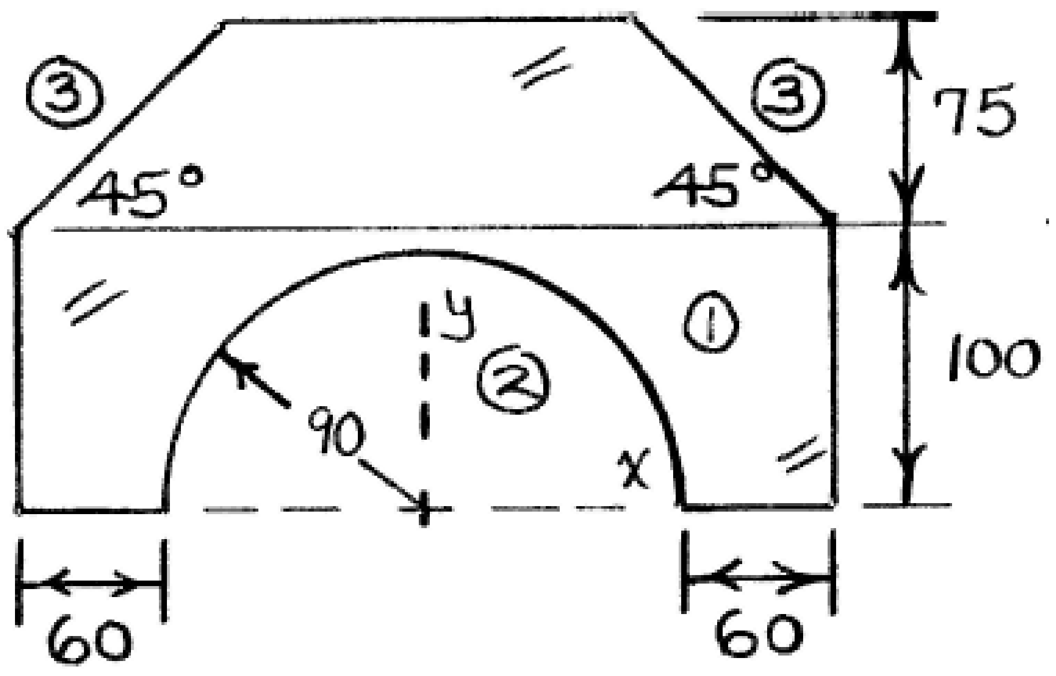
$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{A_1 \times y_1 + A_2 \times y_2 + A_3 \times y_3}{A_1 + A_2 + A_3}$$

Example 1

- Determine the centroid $(\overline{x}, \overline{y})$ of the composite area.



Sol 1



① 175 x 210 rectangle

② semi circle

③ 2 triangle

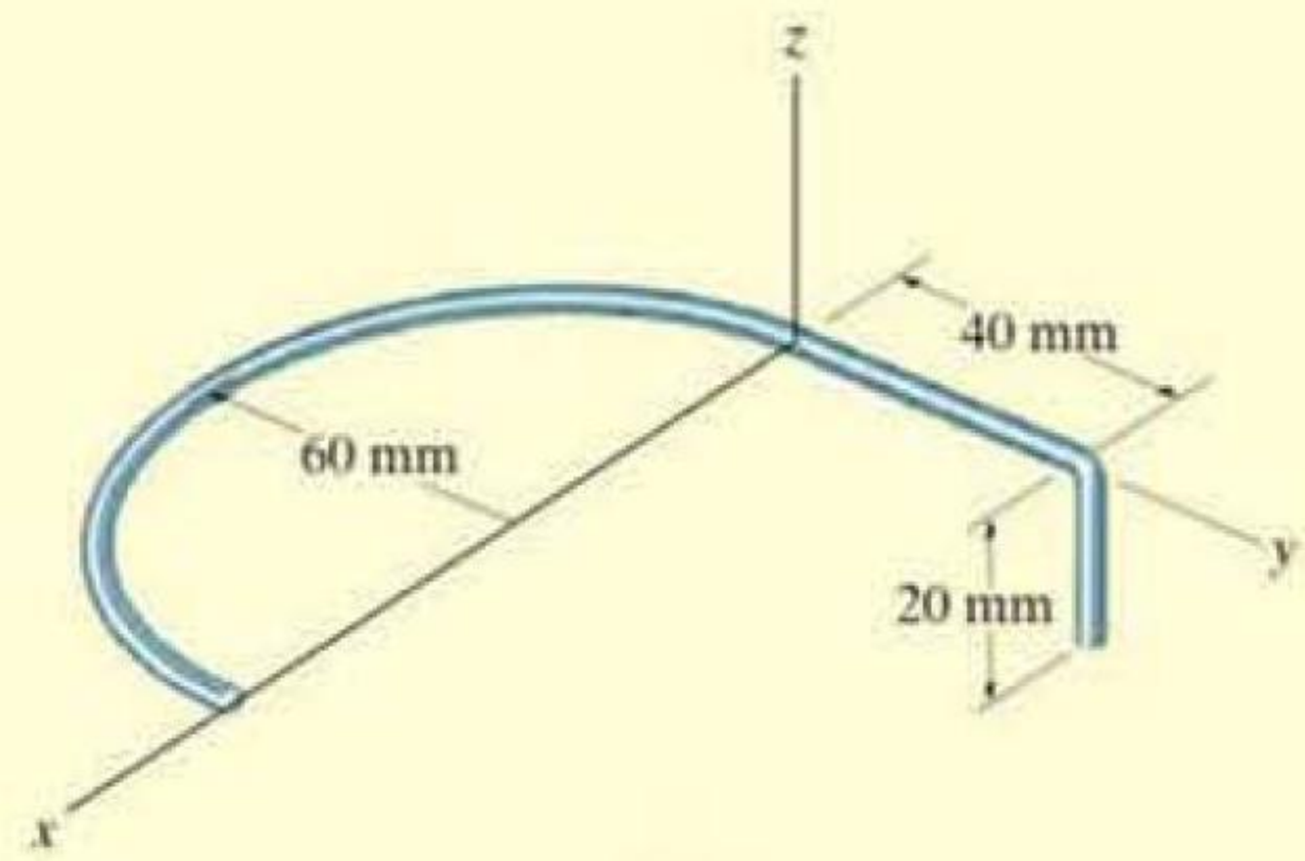
	$A\ (mm^2)$	$\bar{y}\ (mm)$	$A\bar{y}\ (mm^3)$
①	$175(210)$	$\frac{175}{2}$	3 215 625
②	$-\frac{\pi(90^2)}{2}$	$\frac{4(90)}{3\pi}$	− 486 000
③	$-2\frac{1}{2}(75)(75)$	$(100+\frac{2}{3}75)$	− 843 750

$\Sigma A = 18\,400$ $\Sigma A\bar{y} = 1\,886\,000$

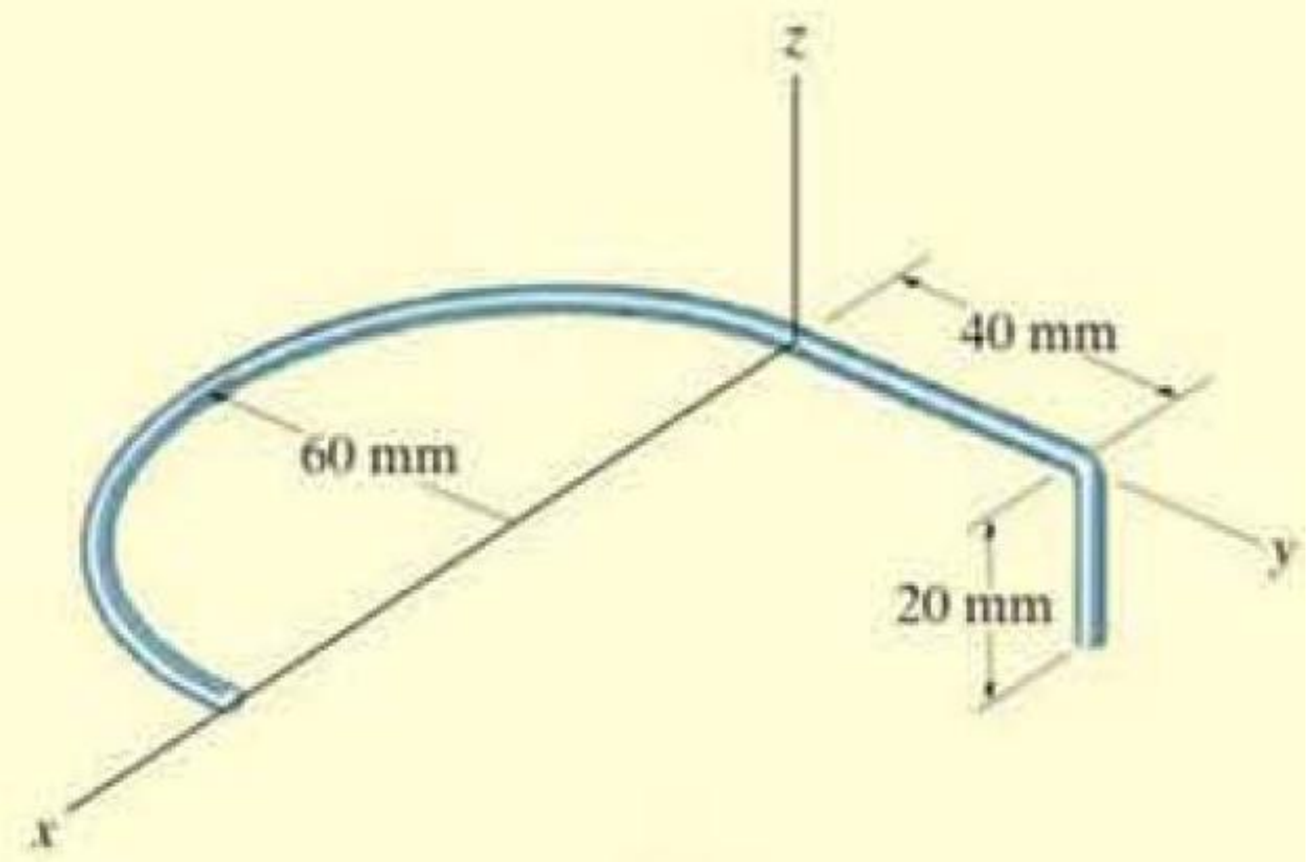
$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \underline{102.5\ mm}$$

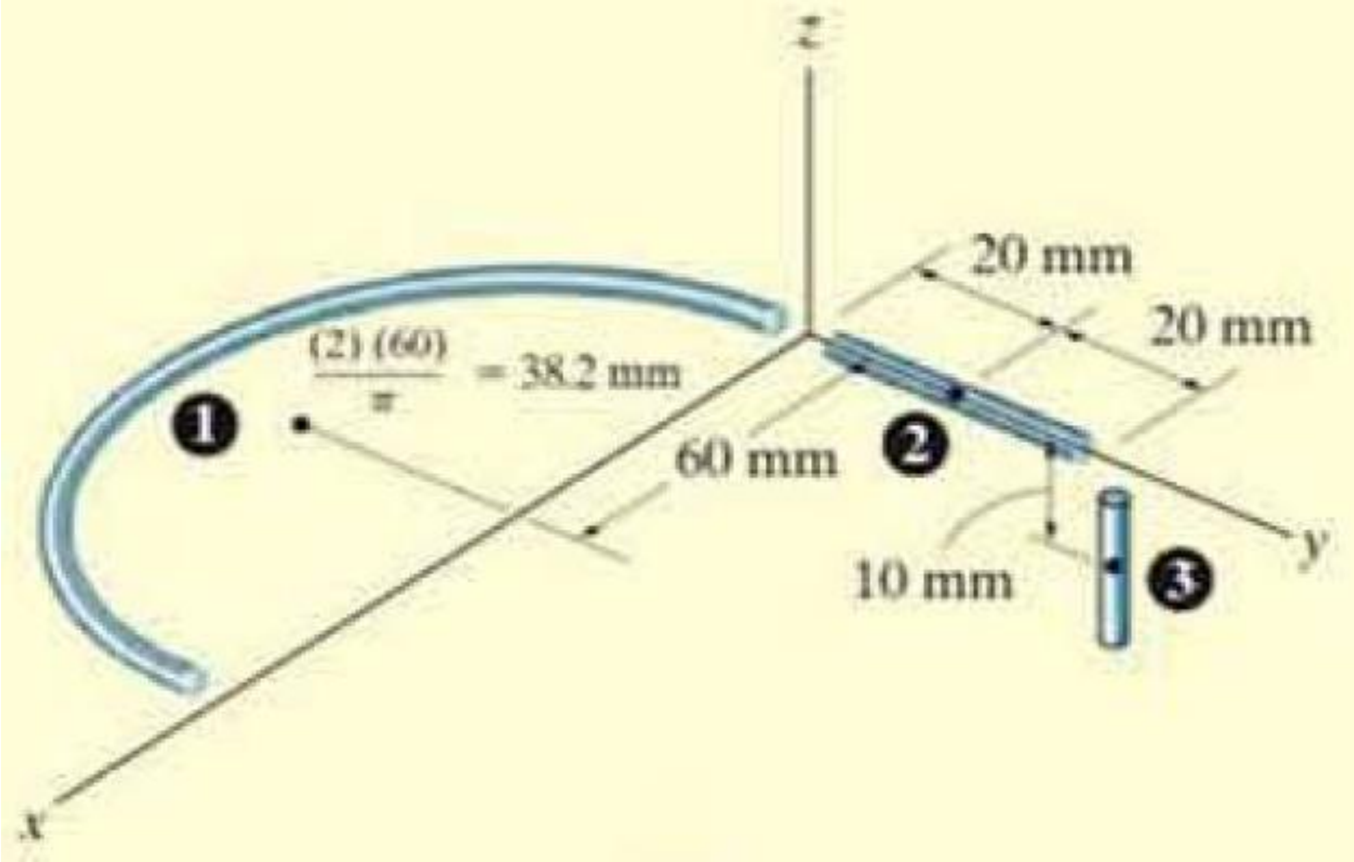
Example 2

- Determine the centroid $(\overline{x}, \overline{y})$ of the body.



sol 2





Segment	L (mm)	$\tilde{x}$ (mm)	$\tilde{y}$ (mm)	$\tilde{z}$ (mm)	$\tilde{x}L$ (mm²)	$\tilde{y}L$ (mm²)	$\tilde{z}L$ (mm²)
1	$\pi(60) = 188.5$	60	−38.2	0	11 310	−7200	0
2	40	0	20	0	0	800	0
3	20	0	40	−10	0	800	−200
	$\Sigma L = 248.5$				$\Sigma\tilde{x}L = 11\,310$	$\Sigma\tilde{y}L = -5600$	$\Sigma\tilde{z}L = -200$

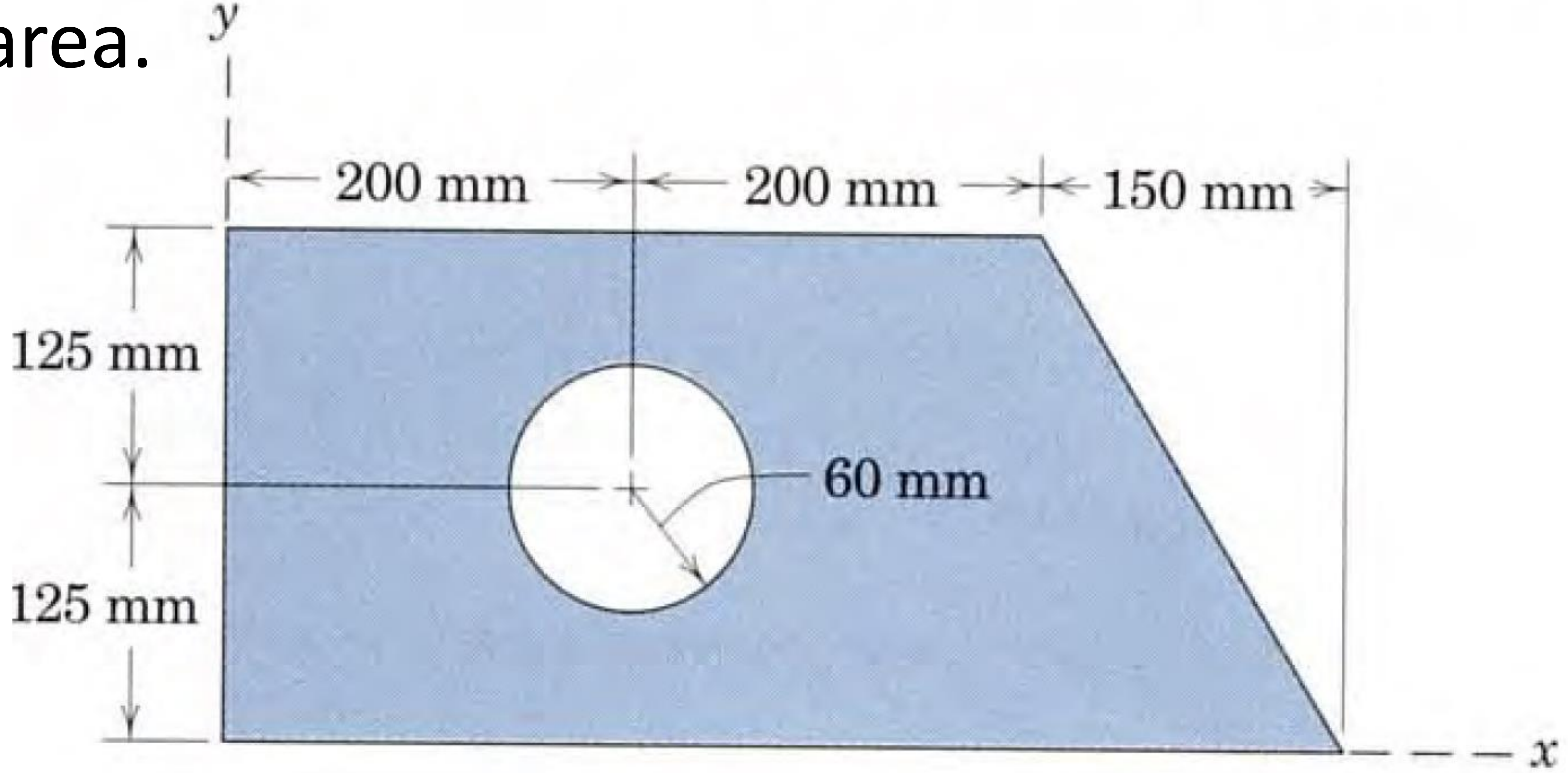
$$\bar{x} = \frac{\Sigma\tilde{x}L}{\Sigma L} = \frac{11\,310}{248.5} = 45.5 \text{ mm}$$

$$\bar{y} = \frac{\Sigma\tilde{y}L}{\Sigma L} = \frac{-5600}{248.5} = -22.5 \text{ mm}$$

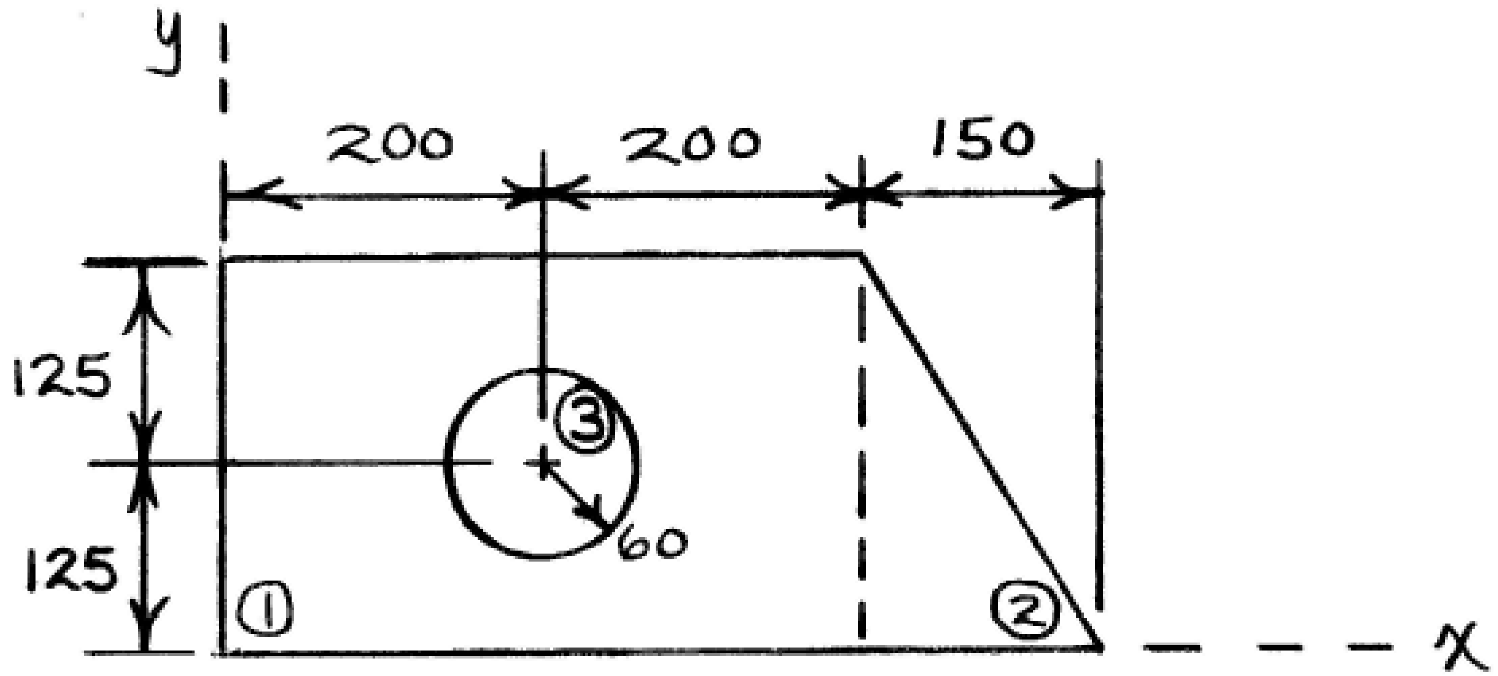
$$\bar{z} = \frac{\Sigma\tilde{z}L}{\Sigma L} = \frac{-200}{248.5} = -0.805 \text{ mm}$$

Example 3

- Determine the centroid $(\overline{x}, \overline{y})$ of the composite area.



Sol. 3



	A mm^2	$\bar{x}$ mm	$\bar{y}$ mm	$A\bar{x}$ mm^3	$A\bar{y}$ mm^3
1	$100\,(10^3)$	200	125	$20\,(10^6)$	$12.50\,(10^6)$
2	$18.75\,(10^3)$	450	$250/3$	$8.44\,(10^6)$	$1.563\,(10^6)$
3	$-11.31\,(10^3)$	200	125	$-2.26\,(10^6)$	$-1.414\,(10^6)$
Total	$107.4\,(10^3)$			$26.2\,(10^6)$	$12.65\,(10^6)$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{26.2\,(10^6)}{107.4\,(10^3)} = \underline{244\text{ mm}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{12.65\,(10^6)}{107.4\,(10^3)} = \underline{117.7\text{ mm}}$$

Example 4

- Determine the centroid $(\overline{x}, \overline{y})$ of the composite area.