CHAPTER 5

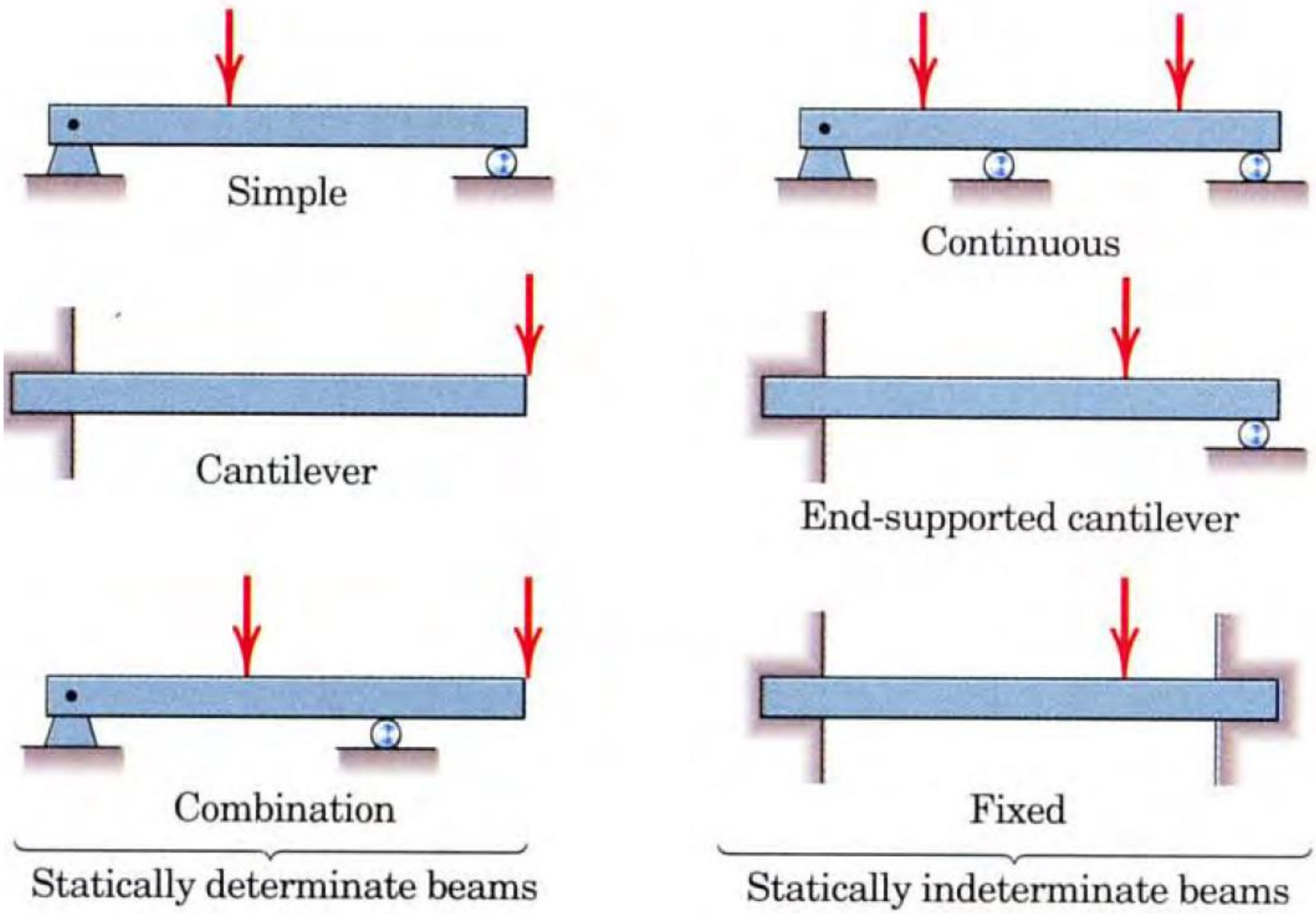
Beams

Introduction

- Beams are slender structural members that offer resistance to bending. They are among the most important elements in engineering.

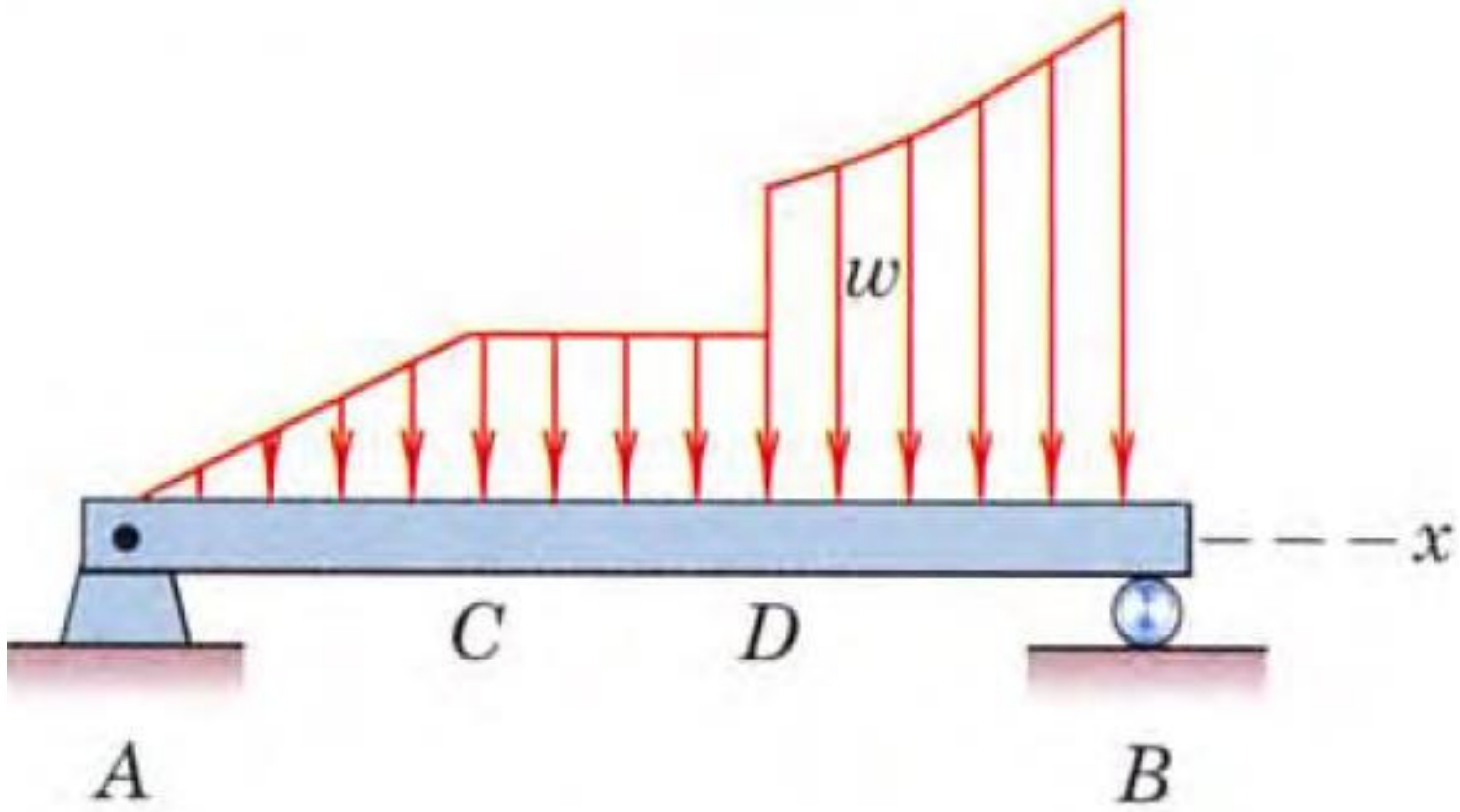
Types of Beams

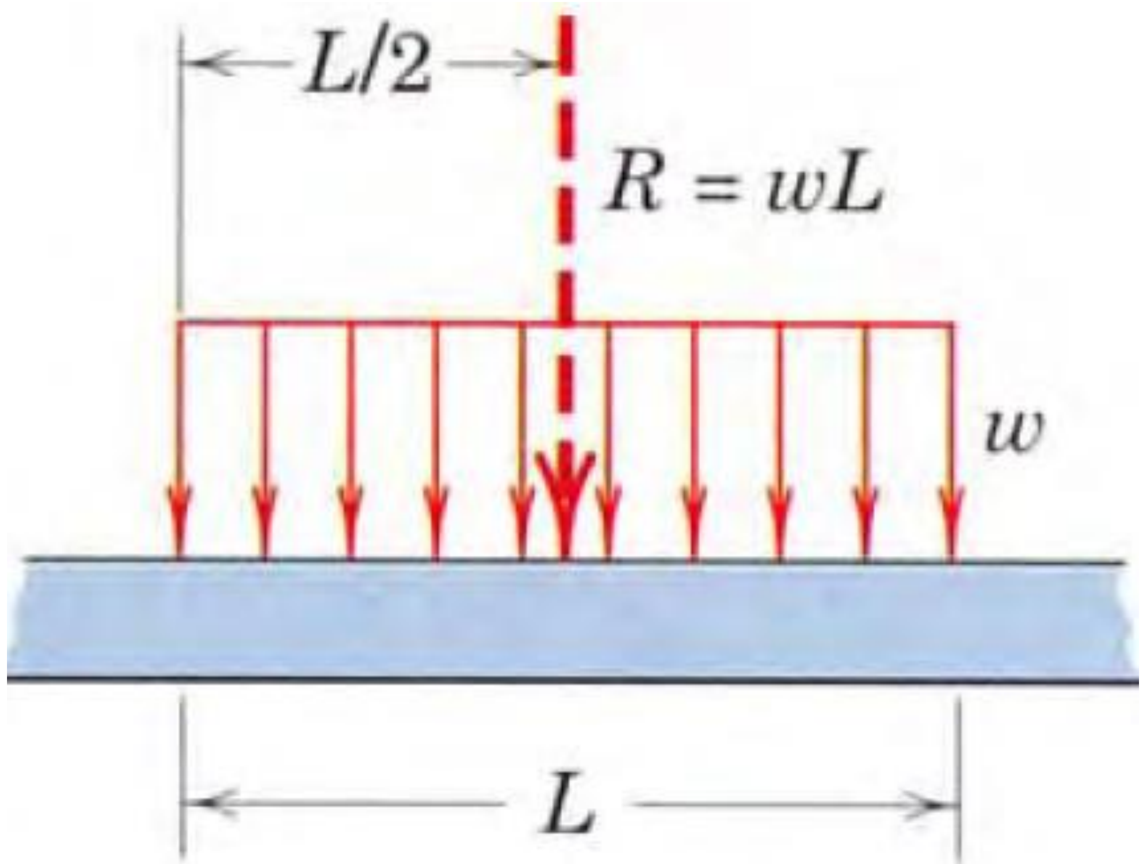
- Beams supported in such a way that their external support reactions can be calculated by the methods of statics alone are called statically determinate beams. In this article we will analyze only statically determinate beams.
- Beams may also be identified by the type of external loading they support.

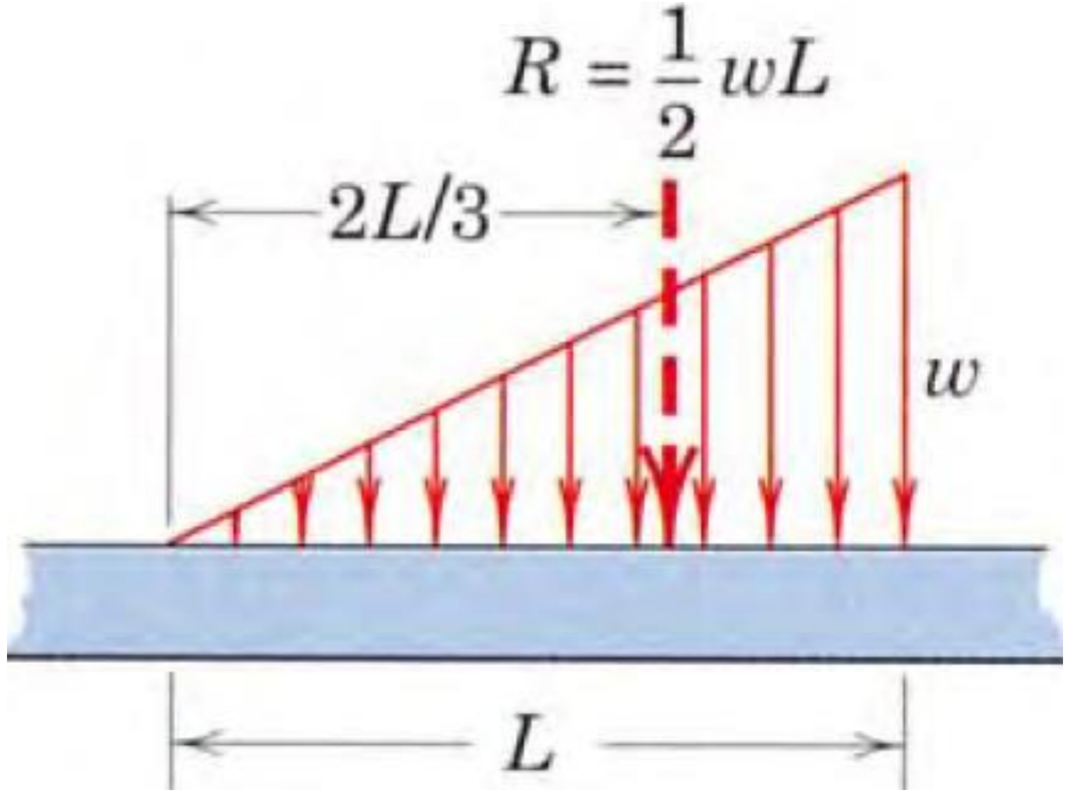

Simple
Continuous
Cantilever
End-supported cantilever
Combination
Fixed
Statically determinate beams
Statically indeterminate beams

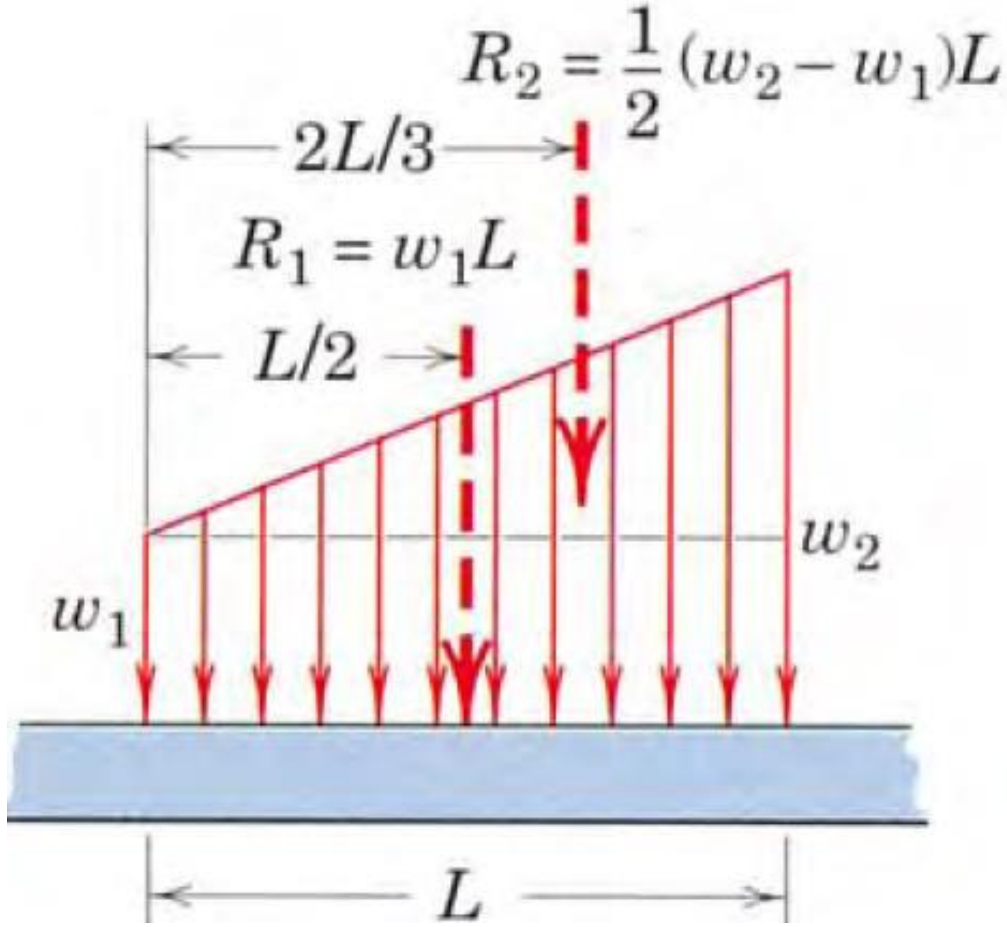
Distributed Loads

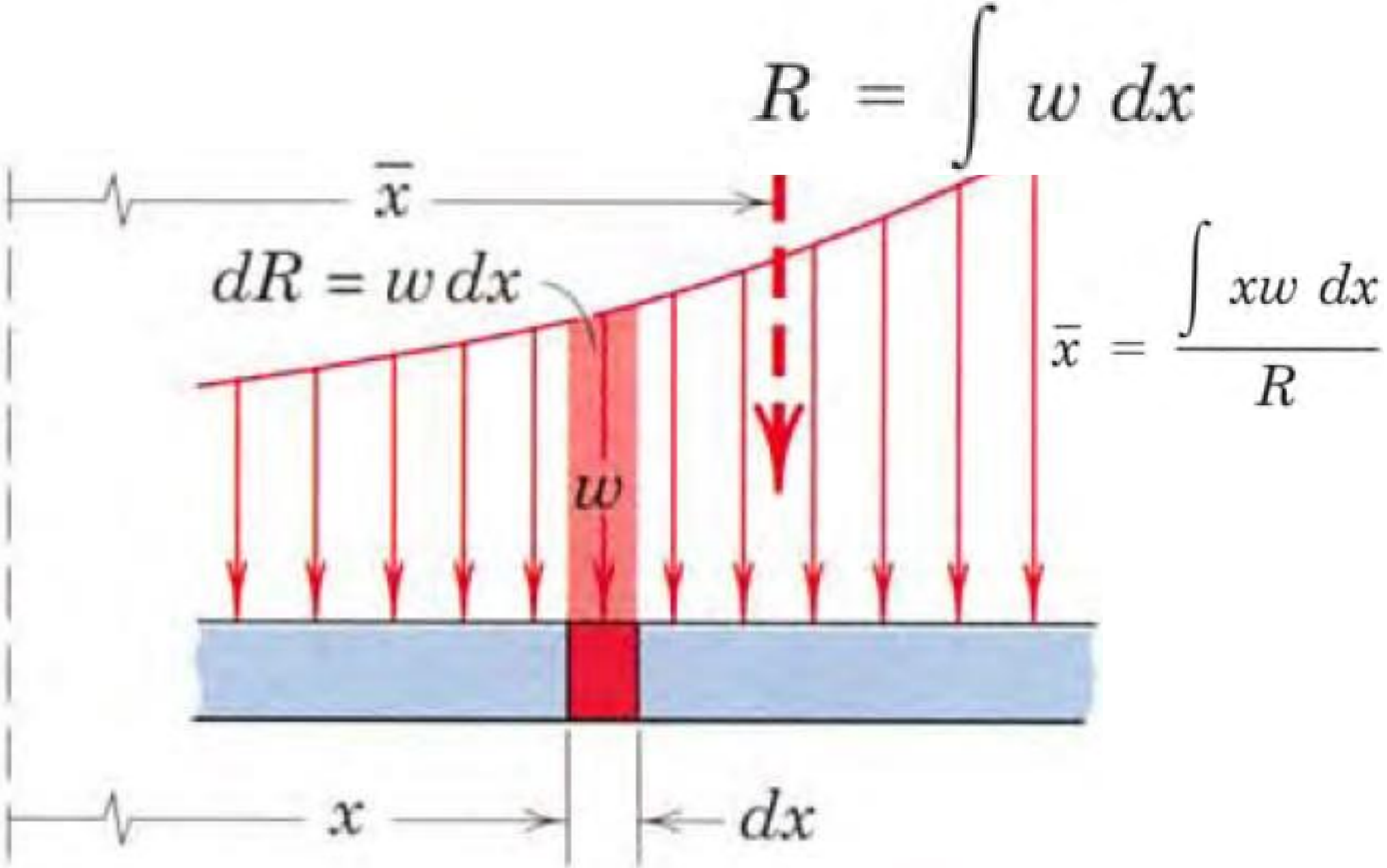
- The intensity of a distributed load may be expressed as force per unit length of beam. The intensity may be constant or variable, continuous or discontinuous.




$L/2$
$R = wL$
w
L

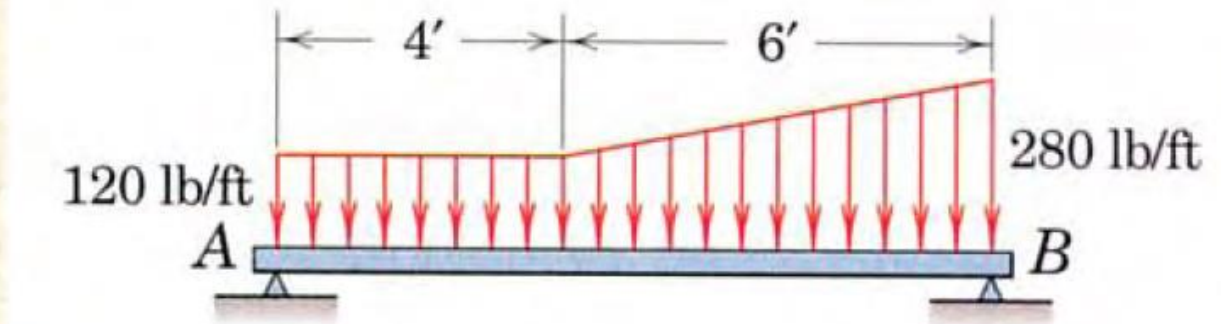

$R = \frac{1}{2} wL$
$2L/3$
w
L


$R_2 = \frac{1}{2}(w_2 - w_1)L$
$2L/3$
$R_1 = w_1 L$
$L/2$
w_2
w_1
L


$R = \int w \, dx$
$\bar{x}$
$dR = w\,dx$
$\bar{x} = \frac{\int xw \, dx}{R}$
w
x
dx

Sample Problem 5/11

Determine the equivalent concentrated load(s) and external reactions for the simply supported beam which is subjected to the distributed load shown.



Solution. The area associated with the load distribution is divided into the rectangular and triangular areas shown. The concentrated-load values are determined by computing the areas, and these loads are located at the centroids of the respective areas.

Once the concentrated loads are determined, they are placed on the free-body diagram of the beam along with the external reactions at A and B. Using principles of equilibrium, we have

$[\Sigma M_A = 0]$ $\quad 1200(5) + 480(8) - R_B(10) = 0$

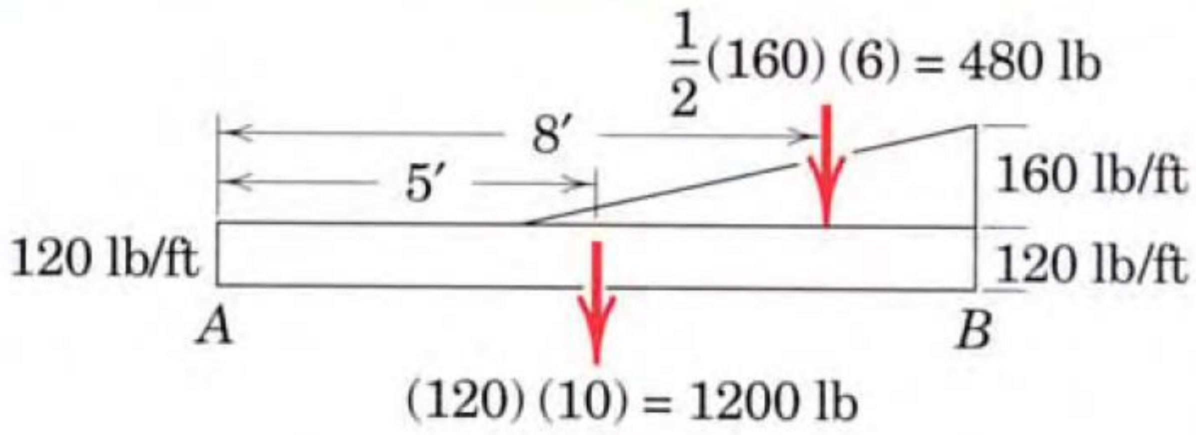
$R_B = 984 \text{ lb}$ *Ans.*

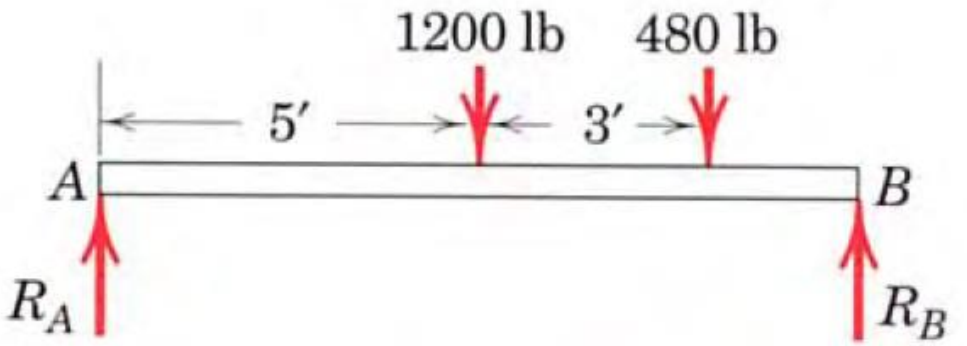
$[\Sigma M_B = 0]$ $\quad R_A(10) - 1200(5) - 480(2) = 0$

$R_A = 696 \text{ lb}$ *Ans.*

Helpful Hint

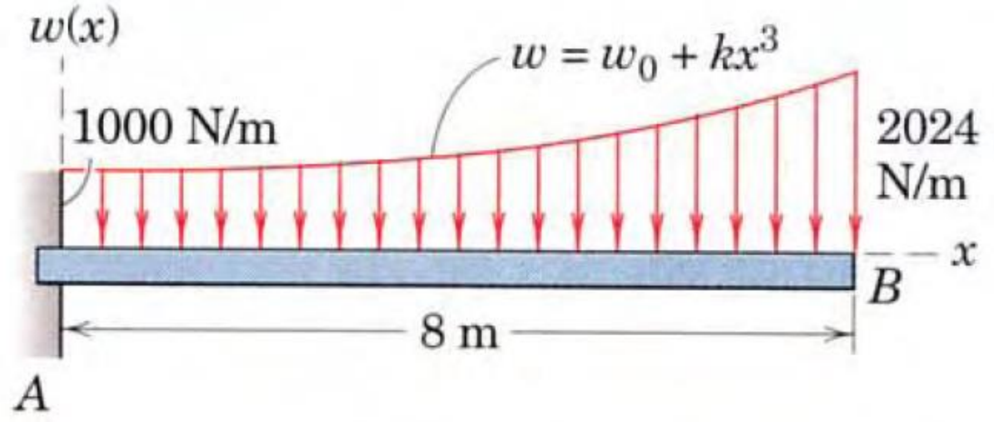
① Note that it is usually unnecessary to reduce a given distributed load to a *single* concentrated load.





Sample Problem 5/12

Determine the reaction at the support A of the loaded cantilever beam.



Solution. The constants in the load distribution are found to be $w_0 = 1000$ N/m and $k = 2$ N/m^4. The load R is then

$$R = \int w\,dx = \int_0^8 (1000 + 2x^3)\,dx = \left(1000x + \frac{x^4}{2}\right)\Bigg|_0^8 = 10\,048 \text{ N}$$

The x-coordinate of the centroid of the area is found by

$$\bar{x} = \frac{\int xw\,dx}{R} = \frac{1}{10\,048}\int_0^8 x(1000 + 2x^3)\,dx$$

$$= \frac{1}{10\,048}(500x^2 + \tfrac{2}{5}x^5)\Big|_0^8 = 4.49 \text{ m}$$

From the free-body diagram of the beam, we have

$[\Sigma M_A = 0]$ $\quad M_A - (10\,048)(4.49) = 0$

$M_A = 45\,100$ N·m *Ans.*

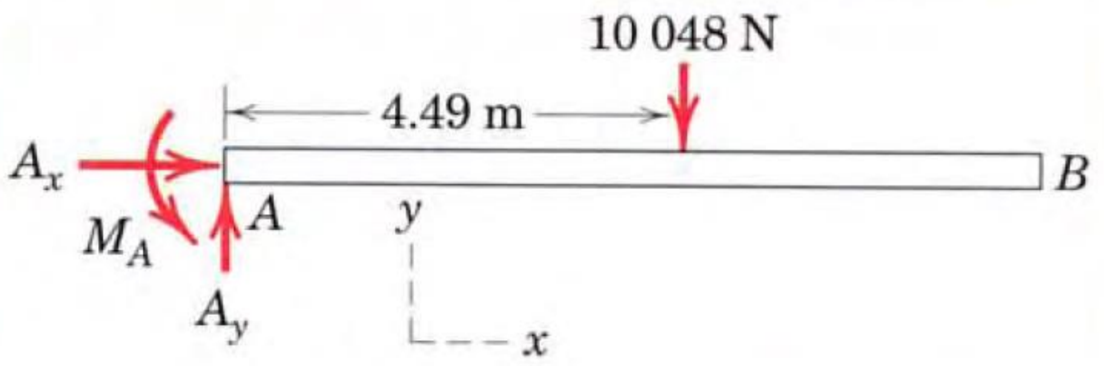
$[\Sigma F_y = 0]$ $\quad A_y = 10\,048$ N *Ans.*

Note that $A_x = 0$ by inspection.

Helpful Hints

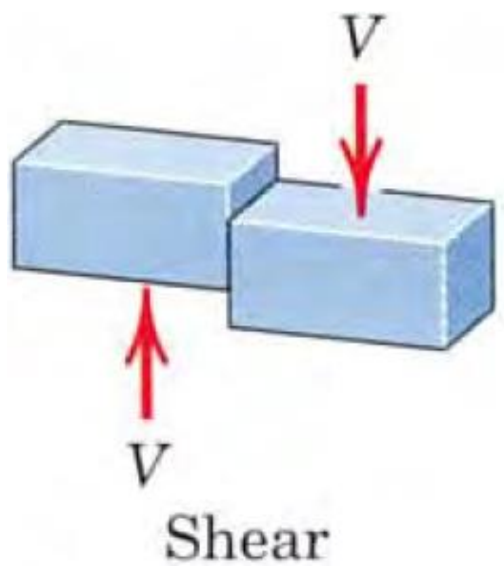
① Use caution with the units of the constants w_0 and k.

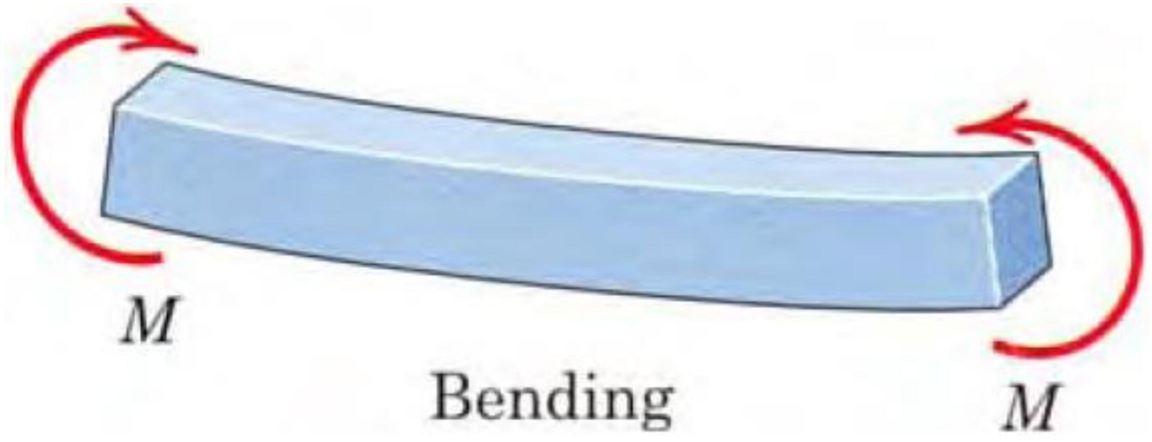
② The student should recognize that the calculation of R and its location $\bar{x}$ is simply an application of centroids as treated in Art. 5/3.

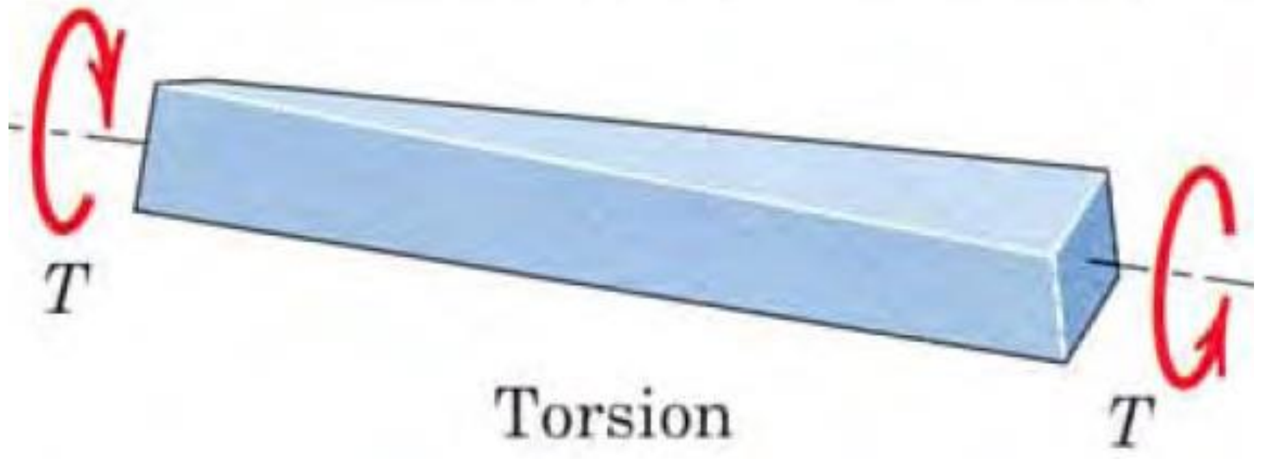


Shear and Bending

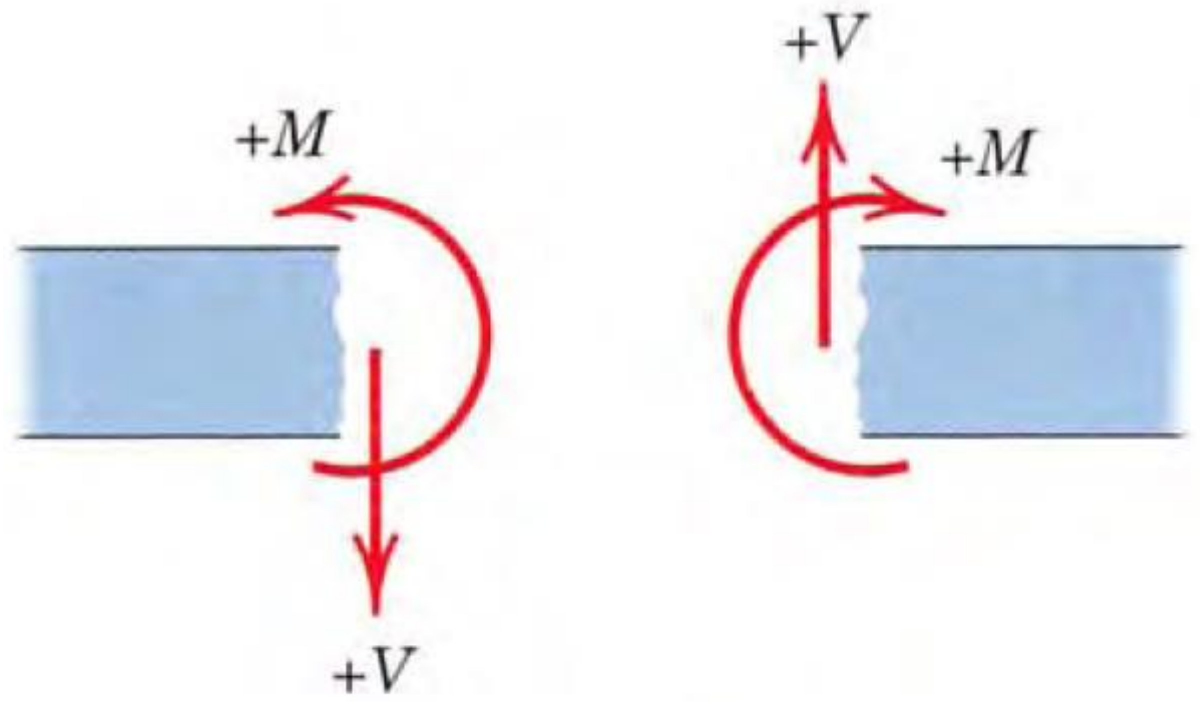
- A beam can resist shear, bending, and torsion. The force V is called the shear force, the couple M is known as the bending moment, and the couple T is called a torsional moment. In this article we will not analyze the torsion.





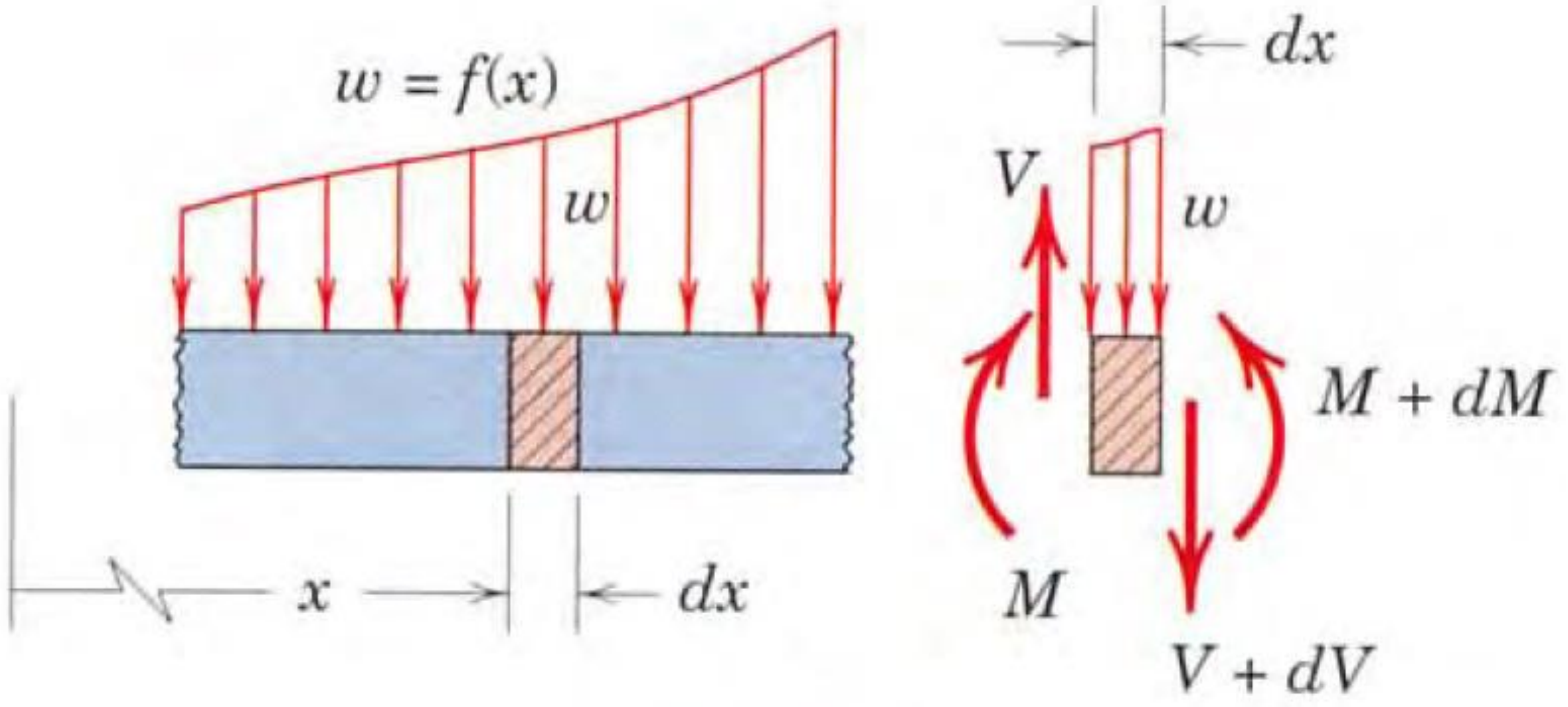


- The shear force V and bending moment M caused by forces applied to the beam in a single plane. The conventions for positive values of shear V and bending moment M shown in figure are the ones generally used.

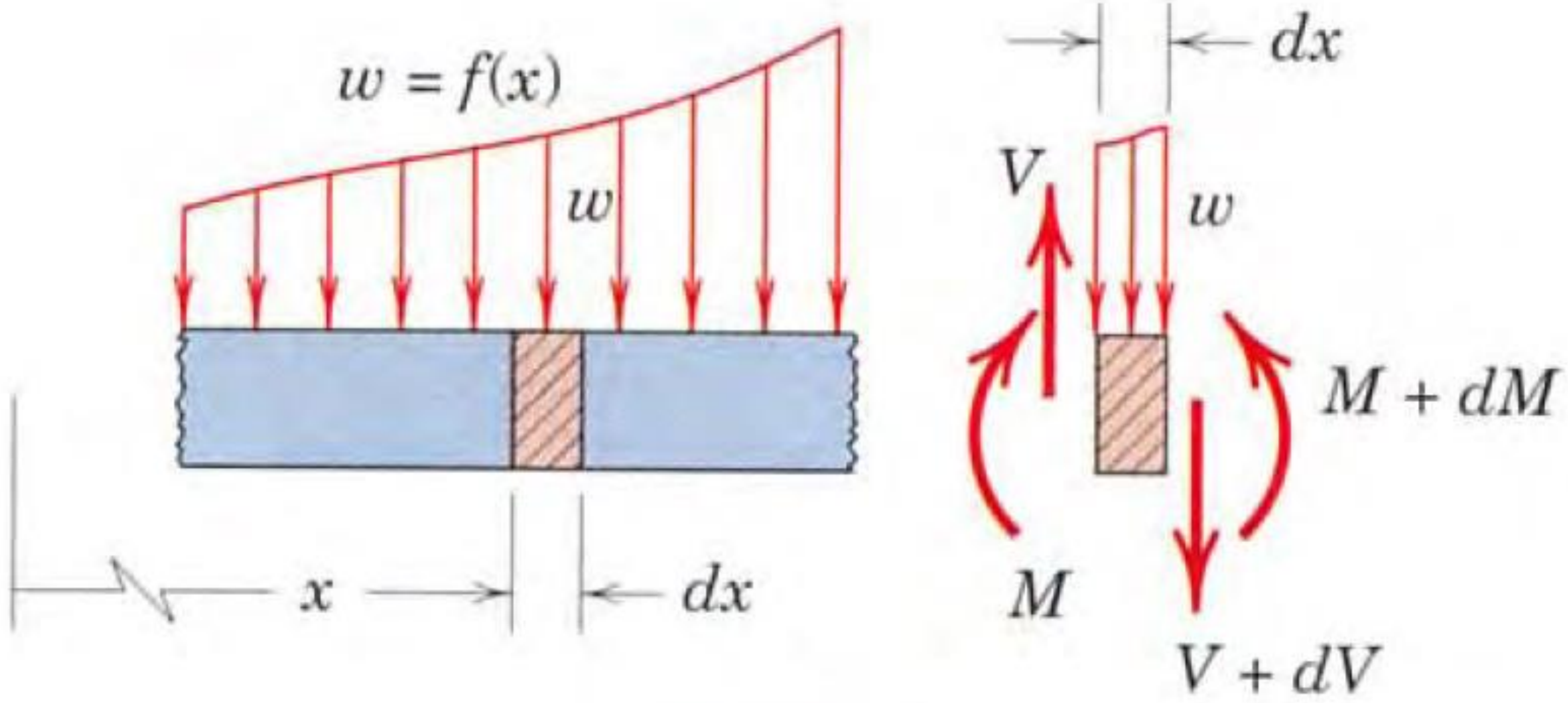


Shear and Moment Relationships

- General relationships may be established for any beam with distributed loads which will aid greatly in the determination of the shear and moment distributions along the beam.



- Equilibrium of the element requires that the sum of the vertical force be zero. Thus, we have



$$V - w\,dx - (V + dV) = 0$$

or

$$dV = -w\,dx$$

- We may now express the shear force V in terms of the loading w

$$dV = -w\,dx \quad , \qquad \int_{V_0}^{V} dV = -\int_{x_0}^{x} w\,dx$$

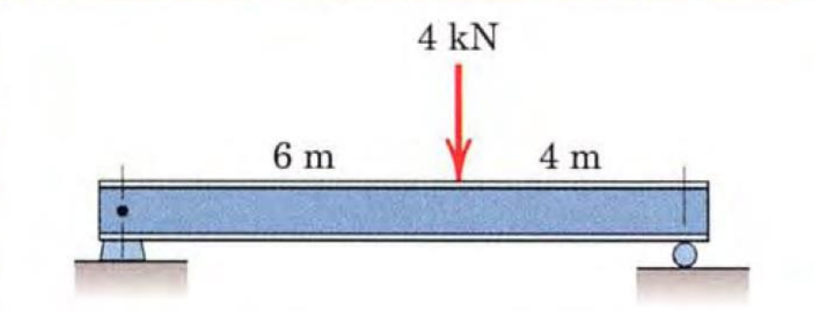
- And the moment equilibrium is

$$M + wdx\frac{dx}{2} + (V + dV)dx - (M + dM) = 0$$

$$dM = Vdx \quad , \qquad \int_{M_0}^{M} dM = \int_{x_0}^{x} Vdx$$

Sample Problem 5/13

Determine the shear and moment distributions produced in the simple beam by the 4-kN concentrated load.



Solution. From the free-body diagram of the entire beam we find the support reactions, which are

$$R_1 = 1.6 \text{ kN} \qquad R_2 = 2.4 \text{ kN}$$

A section of the beam of length x is next isolated with its free-body diagram on which we show the shear V and the bending moment M in their positive directions. Equilibrium gives

$$[\Sigma F_y = 0] \qquad 1.6 - V = 0 \qquad V = 1.6 \text{ kN}$$

$$[\Sigma M_{R_1} = 0] \qquad M - 1.6x = 0 \qquad M = 1.6x$$

These values of V and M apply to all sections of the beam to the left of the 4-kN load.

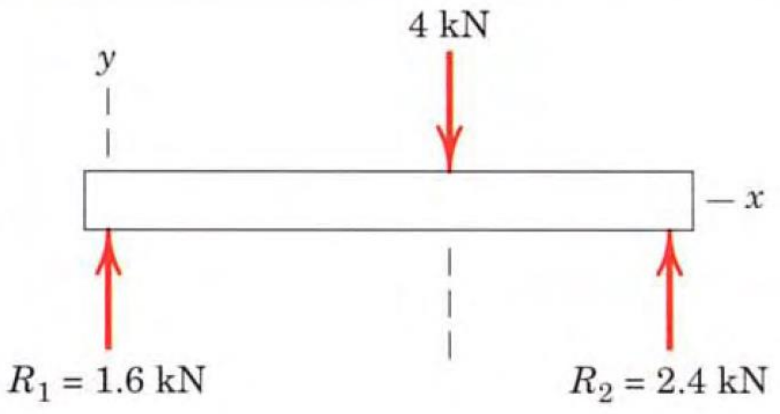
A section of the beam to the right of the 4-kN load is next isolated with its free-body diagram on which V and M are shown in their positive directions. Equilibrium requires

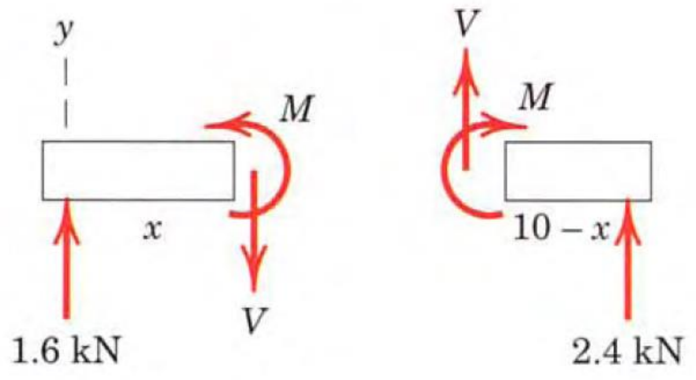
$$[\Sigma F_y = 0] \qquad V + 2.4 = 0 \qquad V = -2.4 \text{ kN}$$

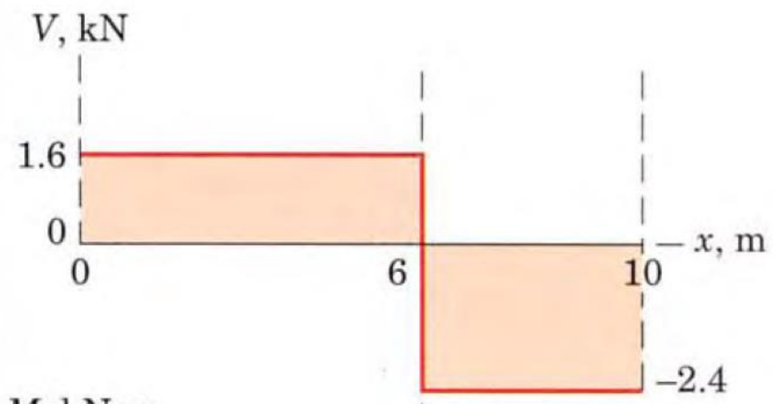
$$[\Sigma M_{R_2} = 0] \qquad -(2.4)(10 - x) + M = 0 \qquad M = 2.4(10 - x)$$

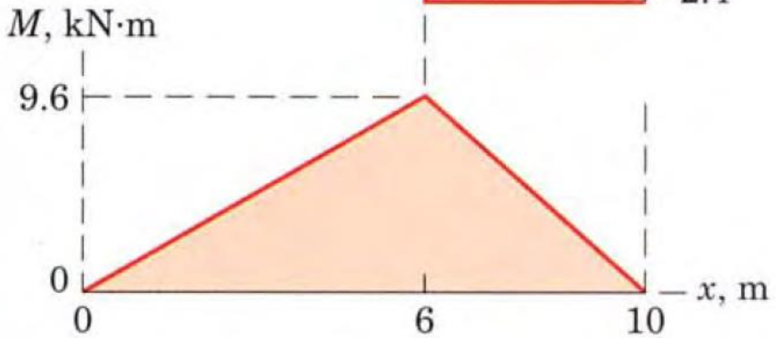
These results apply only to sections of the beam to the right of the 4-kN load.

The values of V and M are plotted as shown. The maximum bending moment occurs where the shear changes direction. As we move in the positive x-direction starting with $x = 0$, we see that the moment M is merely the accumulated area under the shear diagram.





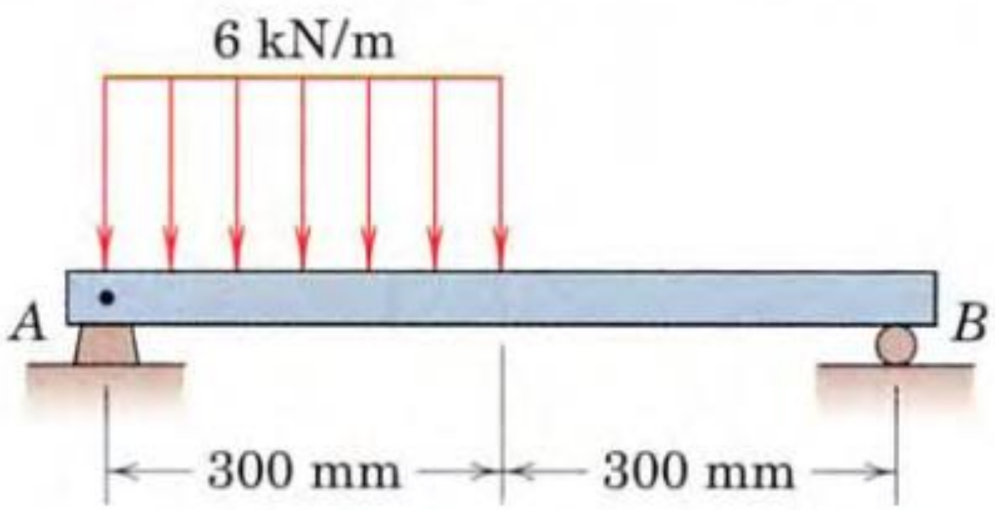




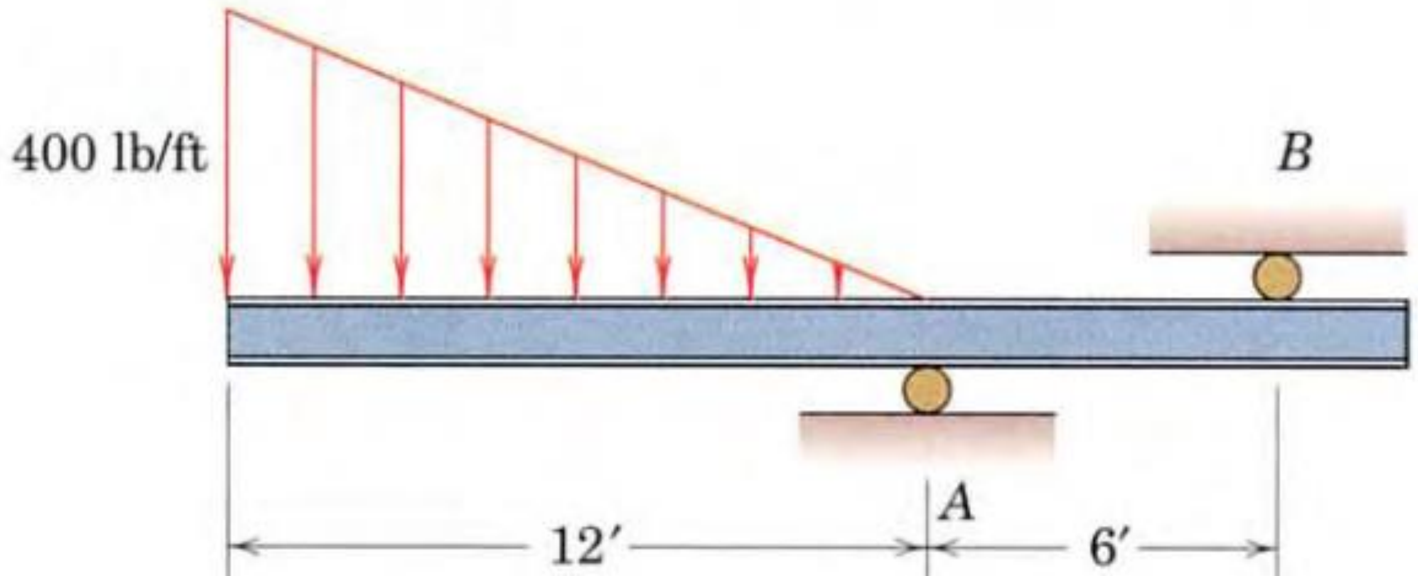
Review Problems

5/93 Determine the reactions at A and B for the beam subjected to the uniform load distribution.

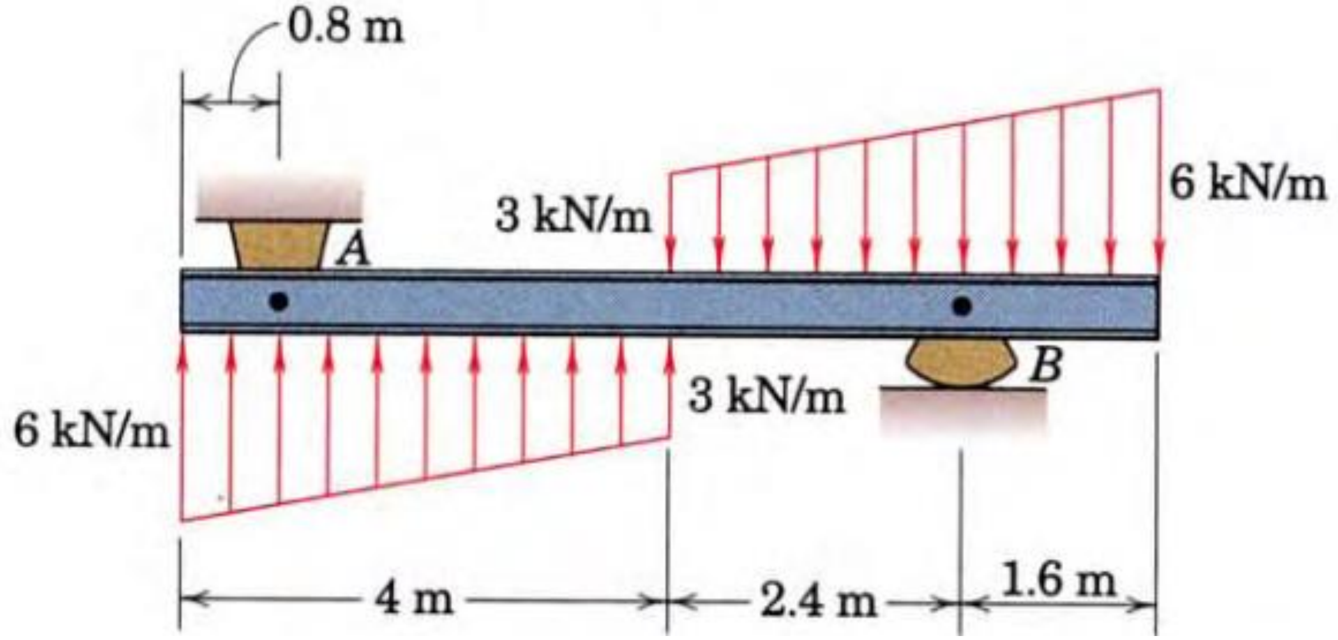
Ans. $R_A = 1.35$ kN, $R_B = 0.45$ kN



5/96 Calculate the reactions at A and B for the beam loaded as shown.

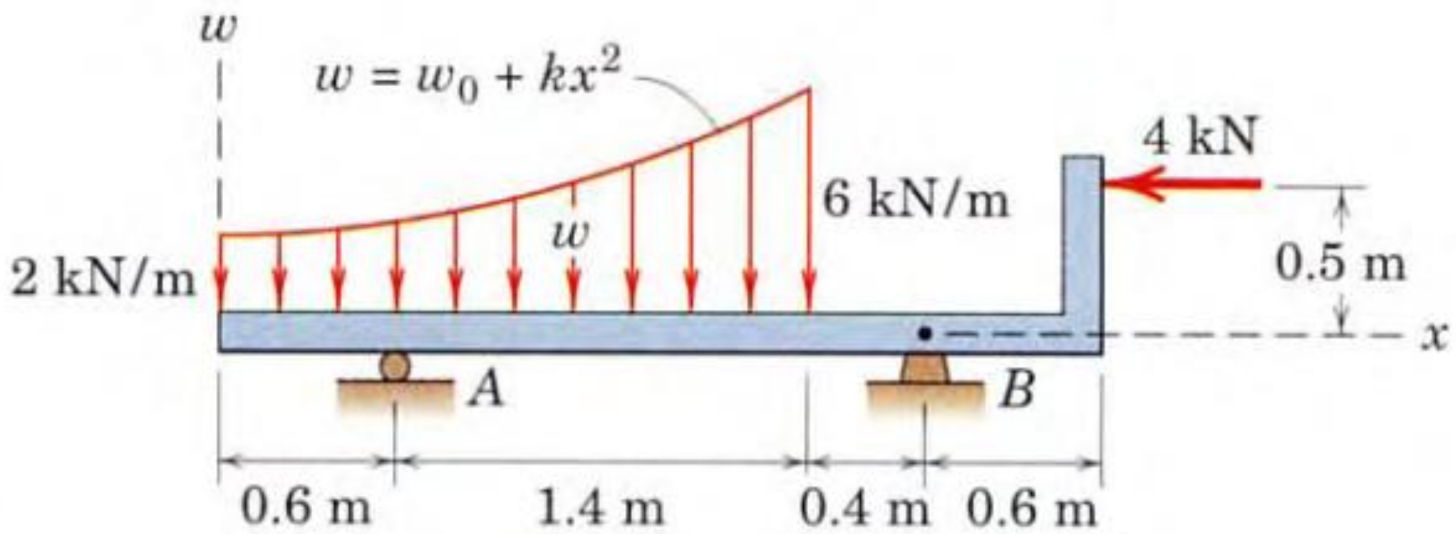


5/102 Calculate the support reactions at A and B for the beam subjected to the two linearly distributed loads.



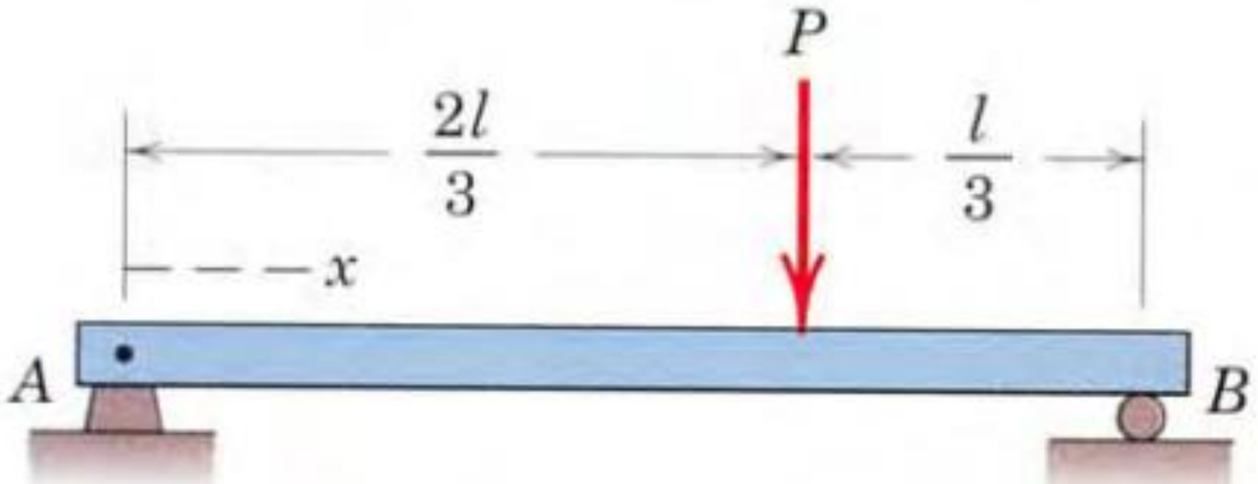
5/109 Determine the reactions at A and B for the beam subjected to the distributed and concentrated loads.

Ans. $A_y = 5.56$ kN, $B_x = 4$ kN, $B_y = 1.111$ kN

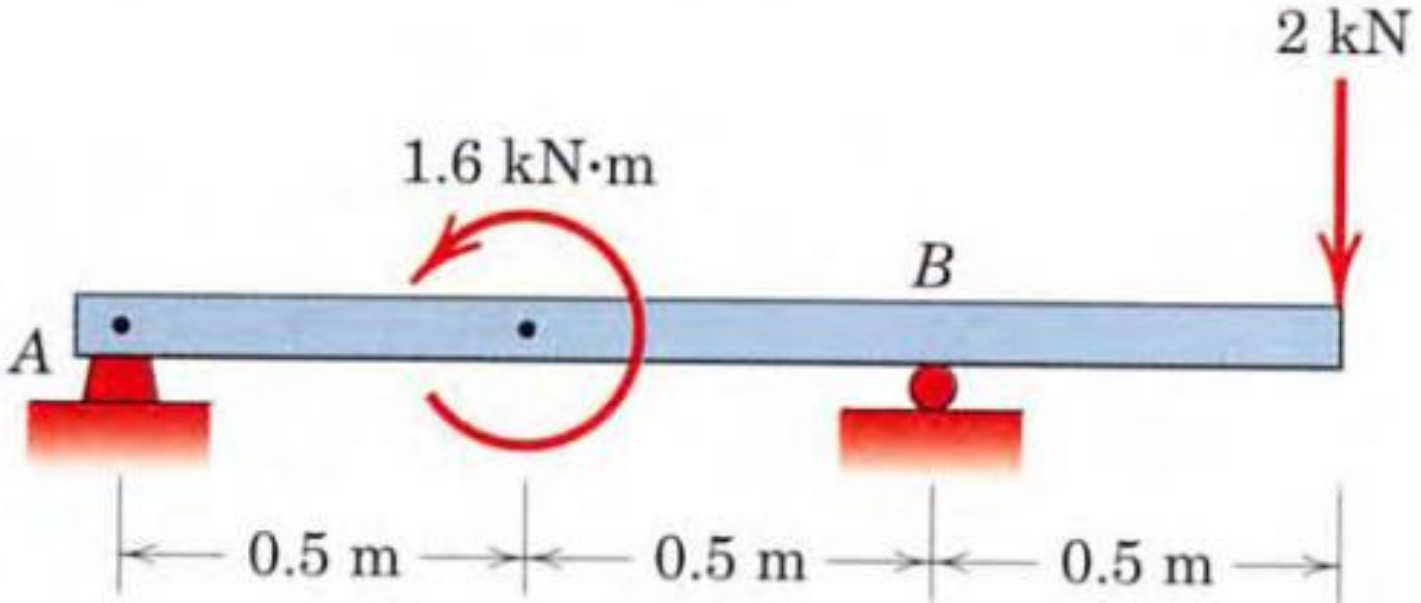


5/113 Determine the shear-force and bending-moment distributions produced in the beam by the concentrated load. What are the values of the shear and moment when $x = l/2$?

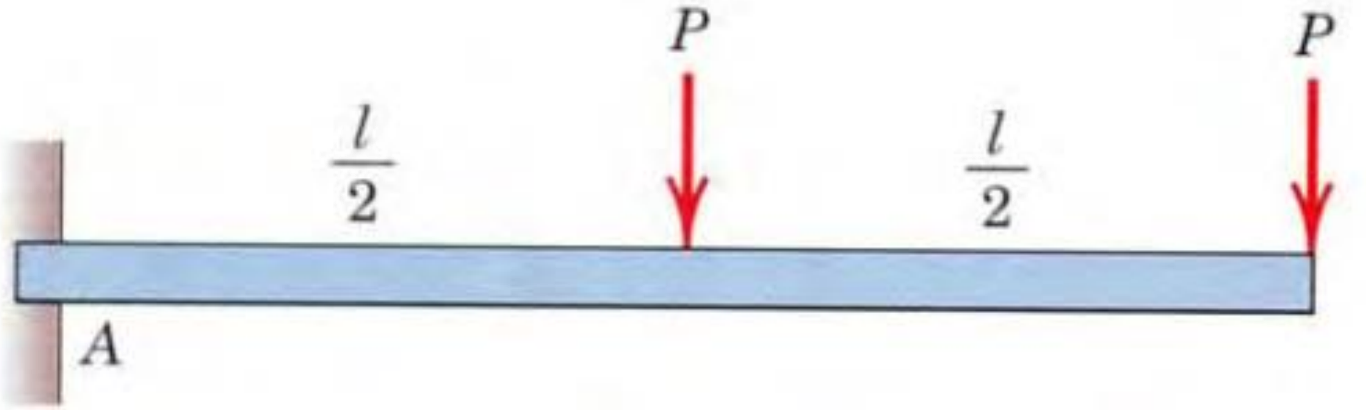
Ans. $V = P/3$, $M = Pl/6$



5/118 Construct the shear and moment diagrams for the beam loaded by the 2-kN force and the 1.6-kN·m couple.



5/114 Draw the shear and moment diagrams for the loaded cantilever beam.



5/117 Draw the shear and moment diagrams for the beam subjected to the two point loads. Determine the maximum bending moment M_{max} and its location.

Ans. $M_{max} = \dfrac{5Pl}{16}$ at $x = \dfrac{3l}{4}$