ME 308
MACHINE ELEMENTS II

CHAPTER 4

JOURNAL BEARINGS_2

(NO ROLLING ELEMENTS,

ONLY A SHAFT AND A HOLE AND SOME LUBRICANT/OIL IN BETWEEN)

The performance parameter temperature increase (*ΔT)* is formulated as

in American units

$$\Delta T_{o_F} = \frac{0.103P}{\left[1-\frac{1}{2}\times\frac{Q_s}{Q}\right]}\times\frac{\frac{r}{c}f}{\frac{Q}{rcNl}}$$

$$P \to \frac{lb}{in^2}, \quad Q \,\&\, Q_s \to \frac{in^3}{\sec}, \quad r,c,l \to in \quad \text{etc.}$$

in SI units

$$\Delta T_{o_C} = \frac{8.30P}{\left[1-\frac{1}{2}\times\frac{Q_s}{Q}\right]}\times\frac{\frac{r}{c}f}{\frac{Q}{rcNl}}$$

$$P \to \frac{MN}{m^2}, \quad Q \,\&\, Q_s \to \frac{m^3}{\sec}, \quad r,c,l \to m \quad \text{etc.}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$$

or

$$\Delta T_{o_C} = T_{\text{var}} \frac{P}{\gamma C_H}$$

*** To be able to determine temp increase *ΔT* we need to know parameters:**

$$\frac{r}{c}f, \quad \frac{Q}{rcNl} \,\&\, \frac{Q_s}{Q} \quad \textit{and} \quad T_{var}$$

and these parameters are dependent on S and hence on *μ* .

***μ* is also dependent on *ΔT again* !!**

If you prefer using

$$\Delta T_{o_C} = T_{\text{var}} \frac{P}{\gamma C_H}$$

where T_{var} is *S* dependent given in fig. 12.12

To be able to use a correct value of *μ* in *S* eqn.

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$$

we need to know *μ* which is dependent on *ΔT again*

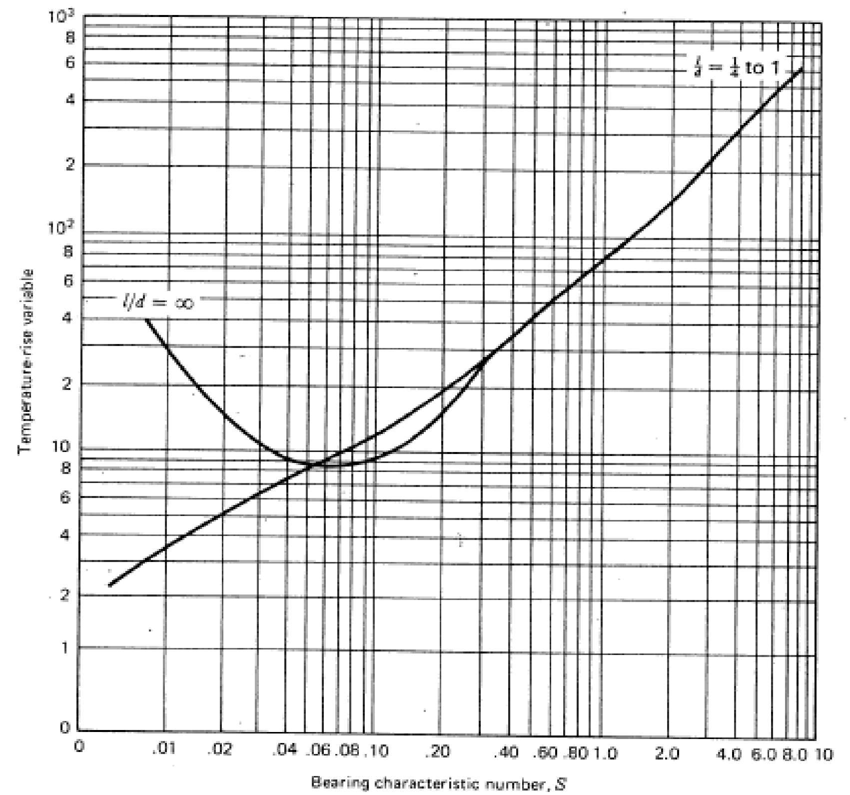


FIGURE 12-12 Chart for temperature-rise variable $T(\text{var}) = \gamma C_H \Delta T/P$. In plotting this chart it was found that the curves for $l/d = \frac{1}{4}, \frac{1}{2}$, and 1 were so close together that they could not be distinguished from a single curve.

Thus there is a situation where we need to start an assumption of either ΔT or μ and check whether the assumption is correct as in the case of tapered roller bearings (K=1.5) of TIMKEN.

There are two trial methods:

First one:

1) a) Assume a trial value for the temperature increase (ΔT = ?)

   b) Calculate T_{ave} and determine μ from chart μ vs T_{ave}

   c) Calculate S and other related parameters

   d) Calculate ΔT_{new} and check if $\Delta T_{new} \cong \Delta T_{old}$; if so OK.

   e) if not re-assume ΔT to be between ΔT_{new} & ΔT_{old} and re-do the calculations from *b* to *d* until $\Delta T_{new} \cong \Delta T_{old}$

Second trial method is:

2) a) Assume a trial value for the viscocity (μ= ?) and
b) then calculate S and ralated parameters such as $\frac{r}{c} f, \frac{Q_s}{Q}$ etc.

c) calculate temperature increase ΔT and average temperature $T_{ave} = T_{in} + \frac{\Delta T}{2}$

d) determine μ_{new} from μ vs T_{ave} chart.

e) check if $\mu_{new} \cong \mu_{old}$; if so design is OK.

f) if not re-assume μ to be between μ_{new} & μ_{old} and re-do the calculations from b to e until $\mu_{new} \cong \mu_{old}$

EXAMPLE 4.1

A sleeve bearing is 10 mm in diameter and 10 mm long. SAE 10 lubricating oil at an inlet temperature of 50° C is used to lubricate the bearing. The bearing is copper-lead alloy and the journal rotates at 3600 rpm. If the radial load on the bearing is 68 N. Find:

a) temperature rise of the lubricant,
b) the minimum oil-film thickness,
c) the power loss in the bearing.
d) oil flowrate-oil consumption

Given:
l = 10 mm
d = 10 mm } $l/d = 1$
SAE 10 oil
T_i = 50° C
T_o = ? } ΔT = ?
T_{ave} = ?
bearing copper-lead
N = 3600 rev/min = 60 rev/sec
W = 68 N

Required:
ΔT = ? ° C
h_o = ?
P_{loss} = ?

S dependent

$$\Delta T_{o_C} = \frac{8.30P}{\left[1 - \frac{1}{2} \times \frac{Q_s}{Q}\right]} \times \frac{\frac{r}{c} f}{\frac{Q}{rcNl}}$$

or

$$\Delta T_{o_C} = T_{\mathrm{var}} \frac{P}{\gamma C_H}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$$

S is μ dependent

$$P = \frac{W}{ld}$$

μ is T_{ave} dependent hence ΔT dependent.

a) So,

$$\Delta T \rightarrow S \rightarrow \mu \rightarrow T_{ave} \rightarrow \Delta T$$

There is a vicious circle of which the starting point can not be found.
Now, we have to use trail and error method of assuming ΔT or μ

Let $\Delta T = 6\ ^{o}C$

$$\Delta T_{^oC} = T_{\text{var}} \frac{P}{\gamma C_H}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$$

Where c is unknown → use Table 12.4

Table 12-4 SOME CHARACTERISTICS OF BEARING ALLOYS

Alloy name	Thickness, mm	Clearance ratio r/c	Load capacity	Corrosion resistance
Tin-base babbit	0.559	600–1000	1.0	Excellent
Lead-base babbit	0.559	600–1000	1.2	Very good
Tin-base babbit	0.102	600–1000	1.5	Excellent
Leaded bronze	Solid	500–1000	3.3	Very good
Copper-lead	0.559	500–1000	1.9	Good
Aluminum alloy	Solid	400–500	3.0	Excellent
Silver plus overlay	0.330	600–1000	4.1	Excellent
Cadmium (1.5% Ni)	0.559	400–500	1.3	Good

For Copper-lead
r/c= 500-1000
let r/c= 600;

N= 60 rev/sec given
P=W/(l*d)=68N/(10mm*10mm)
P=0.68MPa=680 000Pa
μ=? At ave temp
Tvar=?

$$T_{ave} = T_{in} + \frac{\Delta T}{2} = 50 + 3 = 53\ ^{o}C \rightarrow SAE \quad 10 \rightarrow \mu = 17\ mPa - \sec$$

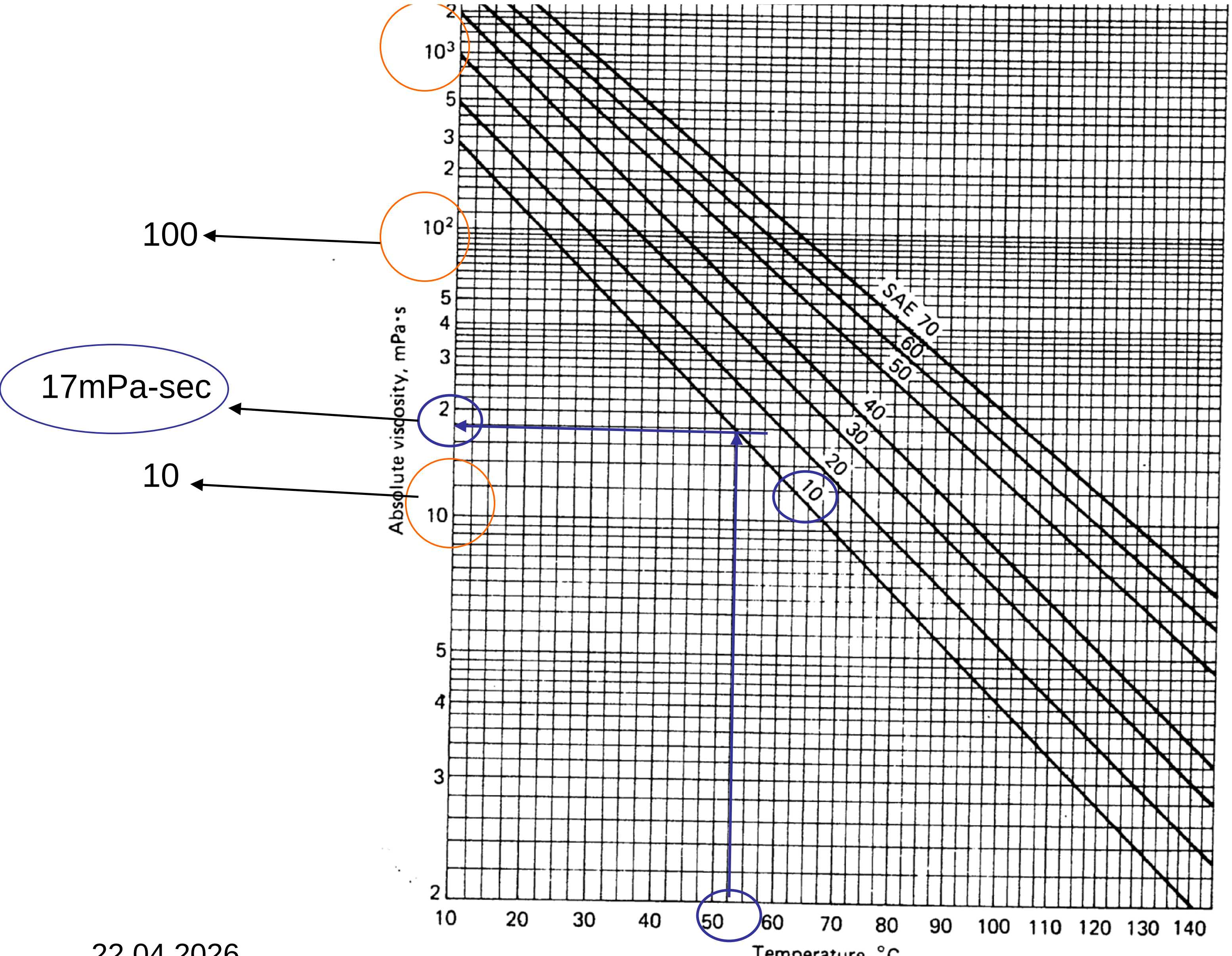


FIGURE 12-10 Viscosity-temperature chart.

$$S=(600)^2\,\frac{\left(17\times10^{-3}\right)\times60}{\left[68/(0.01\times0.01)\right]}=(600)^2\,\frac{0.017\times60}{680\,000\;Pa}=0.54$$

For S=0.54 & l/d=1 Then Tvar=47

$T_{var}=47$

$$\Delta T_{o_C}=T_{\text{var}}\,\frac{P}{\gamma\;C_H}$$

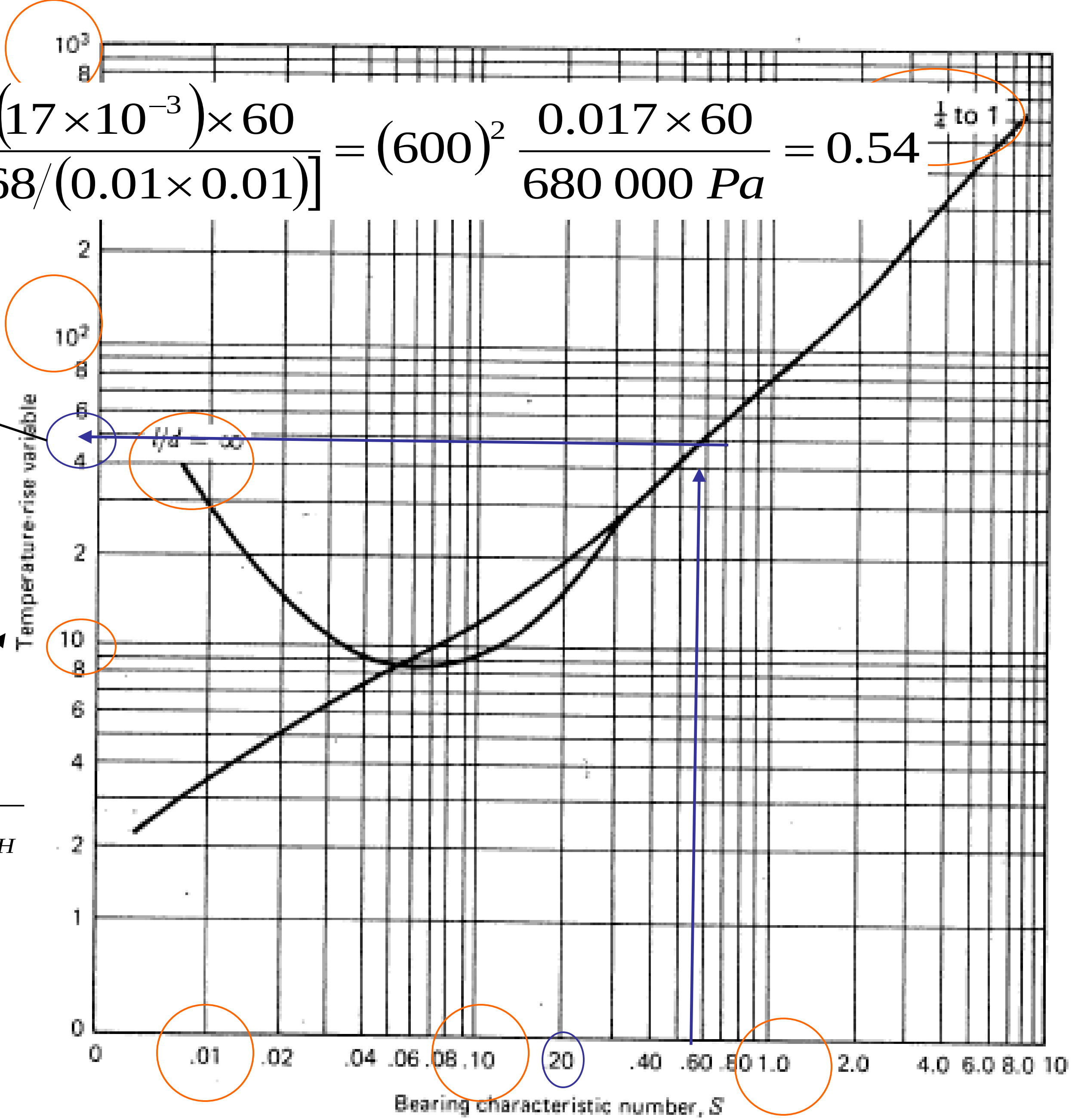


FIGURE 12-12 Chart for temperature-rise variable $T(\text{var})=\gamma C_H\,\Delta T/P$. In plotting this chart it was found that the curves for $l/d=\frac{1}{4}$, $\frac{1}{2}$, and 1 were so close together that they could not be distinguished from a single curve.

NOTE: both axes are <u>logarithmic</u>

And note different curves for different <u>l/d values</u>

So,

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} \qquad P \rightarrow \frac{W}{ld}$$

$$S = (600)^2 \frac{\left(17\times10^{-3}\right)\times 60}{\left[68/(0.01\times 0.01)\right]} = (600)^2 \frac{0.017\times 60}{680\,000\ Pa} = 0.54$$

Use S-T_{var} chart ⟶ for S=0.54 and l/d=1; T_{var} = 47

$$\Delta T = T_{\text{var}} \frac{P}{\gamma C_H} = 47 \frac{680\,000}{861\times 1760} = 21\ ^oC$$

$$\Delta T_{ass} = 6\ ^oC \quad < \quad \Delta T_{calc} = 21\ ^oC$$

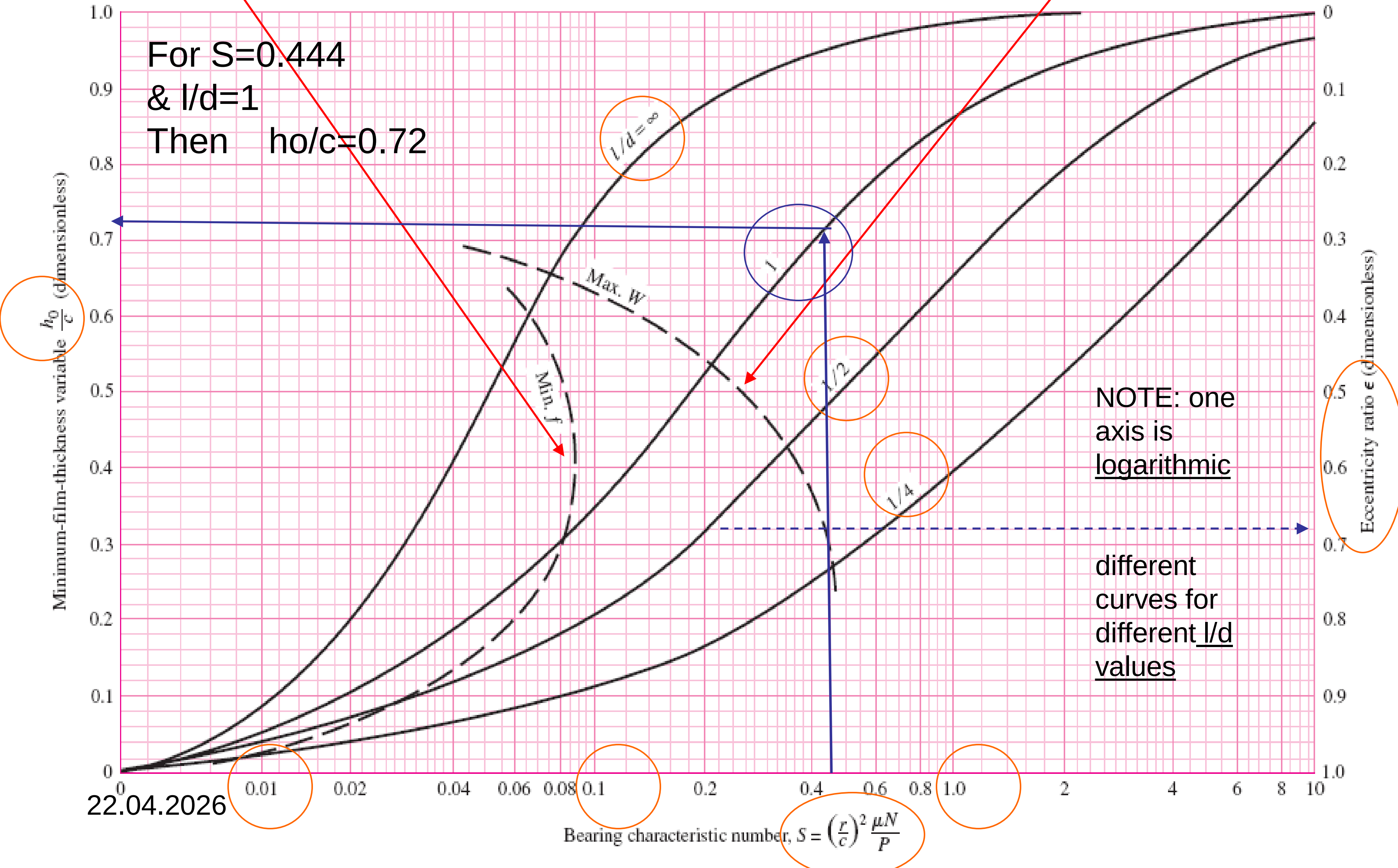
Let ΔT = 16 °C

$$T_{ave} = T_{in} + \frac{\Delta T}{2} = 50 + 8 = 58\ ^oC \rightarrow SAE \quad 10 \rightarrow \mu = 14\ mPa - \sec$$

$$S = (600)^2 \frac{0.014\times 60}{680\,000\ Pa} = 0.444$$ ⟶ T_{var} = 38 and *ΔT = 17* °C

$$\Delta T_{ass} \cong \Delta T_{calc}\ ; \quad \Delta T = 16\ ^oC$$

This is the chart for <u>minimum film-thickness variable</u> and <u>eccentricity ratio</u>. The left boundary(hidden line) of the zone defines the optimal h_0 *for* ***minimum friction**;* *t*he right boundary is optimum h_0 *for* ***max. Load capacity***.



b) h_o= ? h_o /c = ?

$$S = 0.444 \rightarrow \frac{h_o}{c} = 0.72 \quad h_o = 0.72c$$

$$\frac{r}{c} = 600 \rightarrow c = \frac{r}{600} = \frac{5}{600} = 0.0083 \qquad h_o = 0.72 \times 0.0083 = \underline{\underline{6\ \mu m}}$$

c)

$$P_{loss} = T_{loss} \times w \frac{rad}{\sec} \qquad T_{loss} = F_{fric} \times r \qquad F_{fric} = W \times f$$

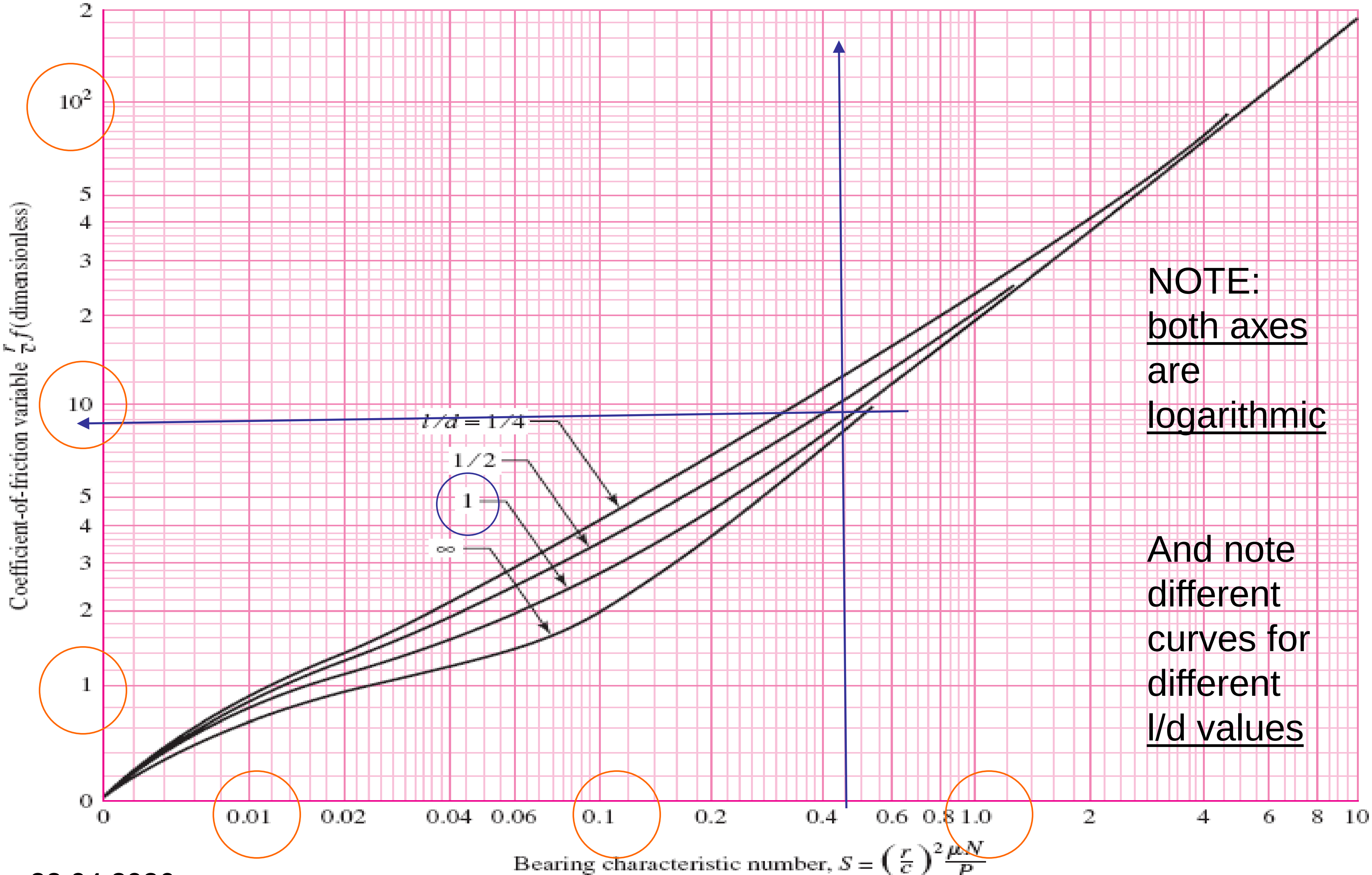
$$S = 0.444; \quad \frac{r}{c} f = 9.3 \rightarrow \quad f = 0.0155$$

$$F_{fric} = 68 \times 0.0155 = \quad 1.054\ N$$

$$T_{loss} = 1.054 \times 0.005 = 5.27 \times 10^{-3}\ Nm$$

$$P_{loss} = 5.27 \times 10^{-3} \times 3600 \frac{rev}{\min} \times \frac{2\pi rad}{rev} \times \frac{\min}{60 \sec} = 1.986\ watt$$

This is the chart for coefficient-of-friction variable;



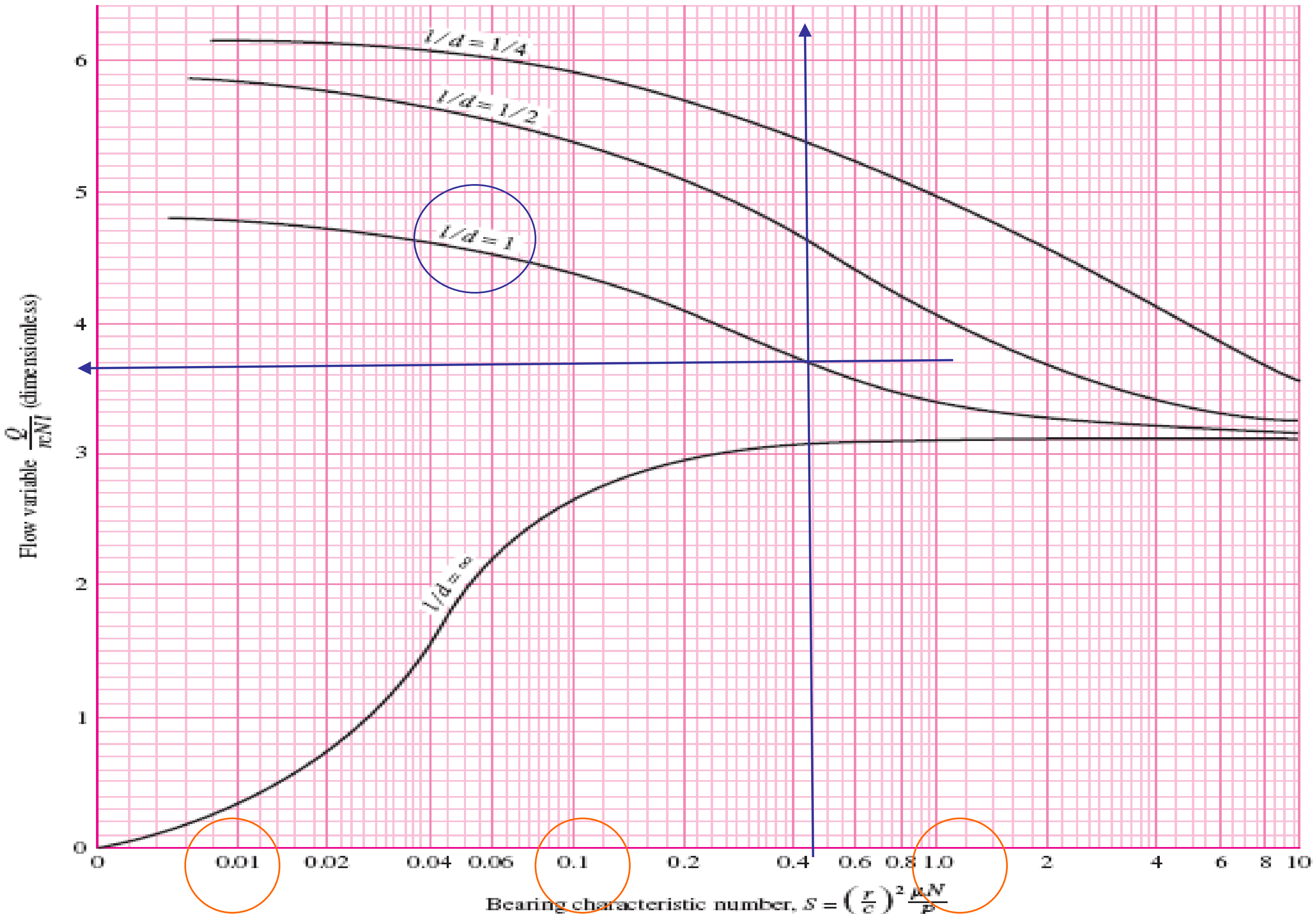
NOTE:
<u>both axes</u>
are
<u>logarithmic</u>

And note
different
curves for
different
<u>l/d values</u>

- Oil flowrate Q is determined from charts again:
- Q/rcNl vs S graph will help to determine Q
- From Chart Q/rcNl=??
- Q=??*rcNl= ……mm3/sec =…lt/min =…kg/hr

This is the chart for flow variable.

Note: Not to be used for pressure-fed bearings**.**



EXAMPLE 4.2

An 80 mm diameter full bearing on an air compressor is to be designed for a load of 6,6 kN, N= 300 rpm. It is desired that the bearing operates at a reasonable steady-state temperature (average temperature) of 50 ^oC. The journal is ground and operates in reamed-cast-bronze bearing.

a)Compute all parameters of the bearing (*d,l,c*),h_0 , *f*, P_{max} , *Q*, Q_S , *ΔT*

b)Specify the oil to be used (SAE ??)

c)Compute the amount of power loss in the bearing

SOLUTION:

GIVEN:

D=80mm

W= 6,6 kN

N= 600 rpm = 300/60= 5 rps

T_{ave}= 50 ^oC

Ground journal

Reamed-cast-bronze bearing

Air compressor

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$$

S is the main parameter to determine all other parameters.

S requires

r, N, P, c and μ

r=d/2=40mm

N=5rps

Others (P, c and μ) are not given or known

1) Since application type is known then P can be limited in table 12-3
P= 1-2 MPa for air compressor main bearings

Table 12-3 RECOMMENDED UNIT LOADS FOR SLEEVE BEARINGS

Application	Unit load, kPa	Application	Unit load, kPa
Air compressors:		Diesel engines:	
Main bearings	1 000– 2 000	Main bearings	6 000–12 000
Crankpin	2 000– 4 000	Crankpin	8 000–15 000
Automotive engines:		Wristpin	14 000–15 000
Main bearings	4 000– 5 000	Electric motors	800– 1 500
Crankpin	10 000–15 000	Gear reducers	800– 1 500
Centrifugal pumps	600– 1 200	Steam turbines	800– 1 500

$$P = \frac{W}{l \times d}$$

$$l = \frac{W}{P \times d} = \frac{6600}{1 \times 10^6 \times 0.08} = 0.0825$$

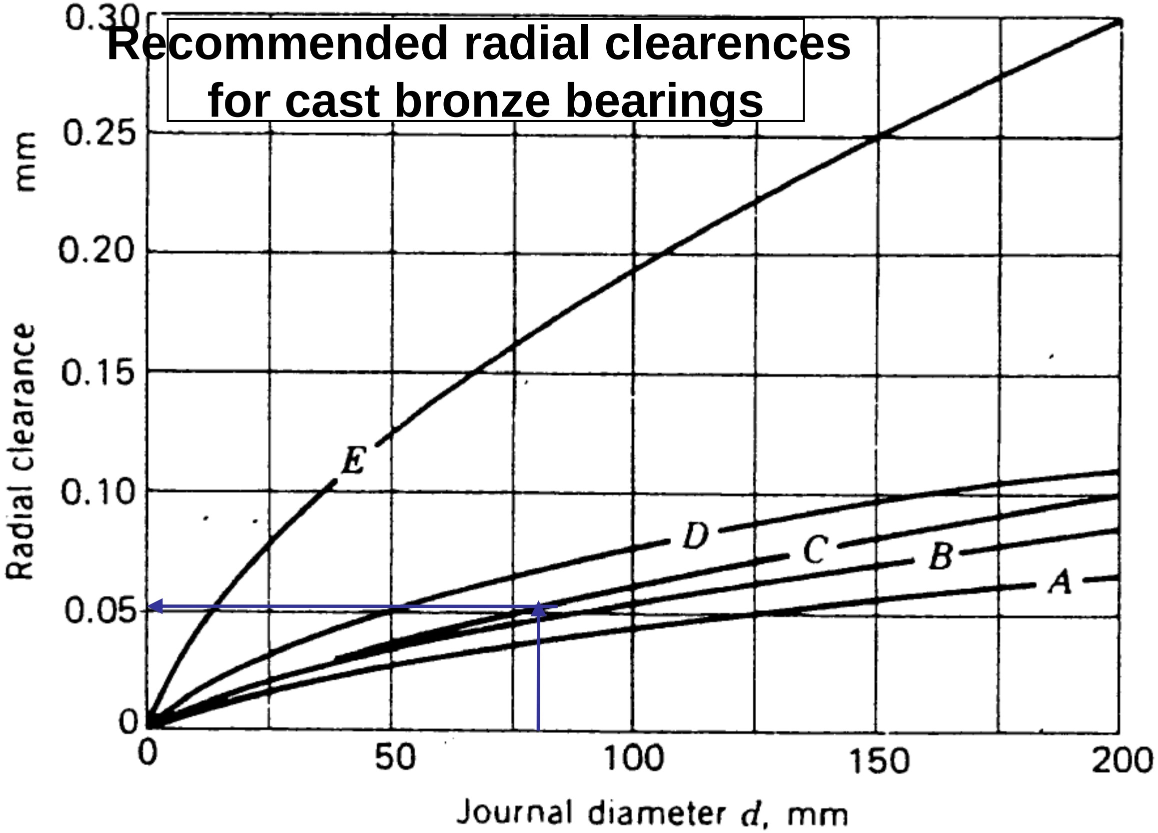
$$l = 82.5\, mm$$

For d=80 mm l=80 mm $P = \frac{W}{ld}$

$$P = \frac{6600}{80 \times 80} = 1.031\, MPa \qquad \text{(in the range OK)}$$

Thus $\frac{l}{d} = \frac{80}{80} = 1$

2) Since bearing type is known the clearance value can be determined from Fig.12.29 pp.559 **curve C**. (Fig.12.27 pp.458 curve C.) **for ground journal and reamed cast-bronze bearing**



- All parameters of the bearing h_0 , f, P_{max} , Q, Q_S , ΔT *depends on S value*
- S is the main parameter to determine all other parameters.

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$$

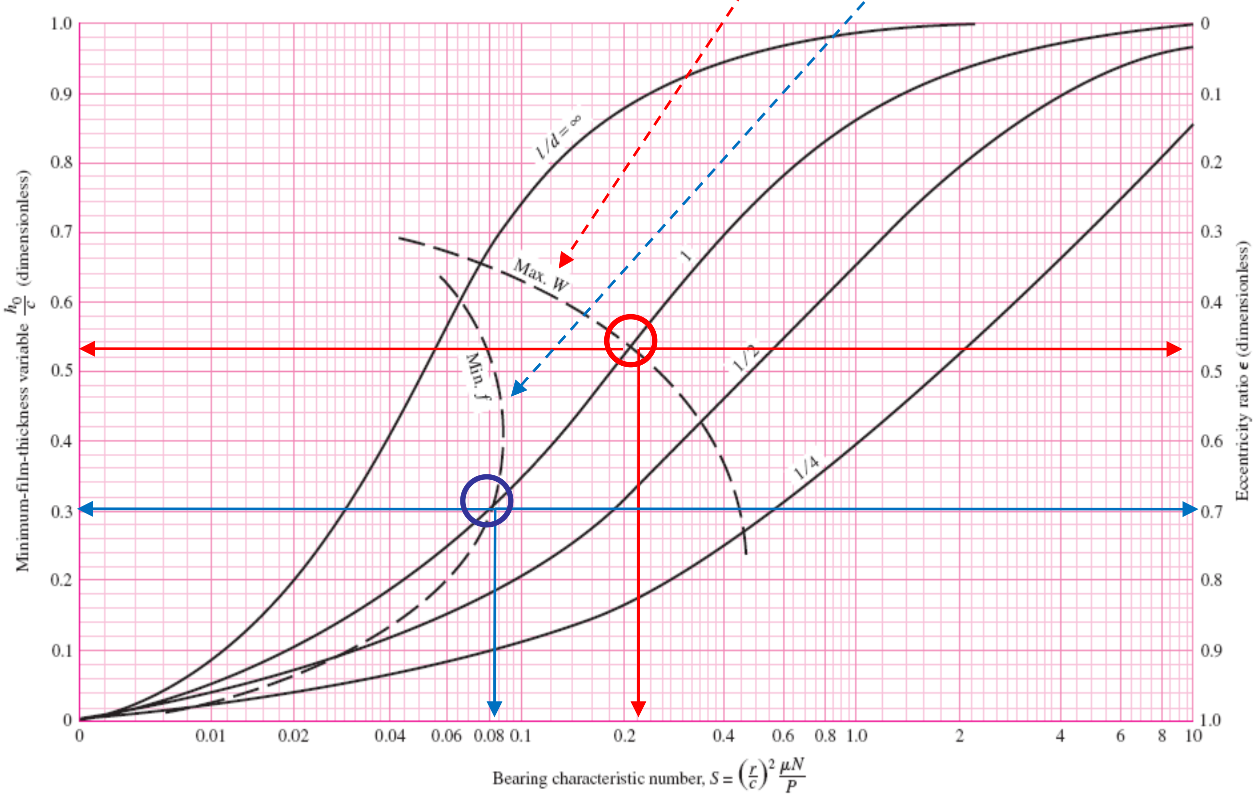
S requires viscosity μ
Viscosity μ requires oil type
But oil type is unkown
That means S can not be determined from the currently given parameters

Now we have the option of designing bearing based on two criteria

- Either for max load carrying capacity
- Or for minimum friction hence minimum power loss condition

3--) Now since $\frac{l}{d} = 1$ and $c = 0.05\,mm$

And none of the parameters are known we will make use of chart (f.12.14) of $\frac{h_0}{c}$ vs S for maximum load carrying capacity cond. and for min friction cond.



For max. W

$\frac{h_0}{c} = 0.53$

$S = 0.21$

$\varepsilon = 0.47$

For min. f

$\frac{h_0}{c} = 0.3$

$S = 0.08$

$\varepsilon = 0.7$

4) Since

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = 0.21$$

Thus for max load capacity condition

$$\mu = \frac{S \times P}{N}\left(\frac{c}{r}\right)^2 = \frac{0.21 \times 1.031 \times 10^6}{5}\left(\frac{0.05}{40}\right)^2 = 0.0676 \;\; Pa - \mathrm{sec}$$

$$\mu = 67.6 \;\; mPa - \mathrm{sec}$$

again for min friction condition

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = 0.08$$

$$\mu = \frac{S \times P}{N}\left(\frac{c}{r}\right)^2 = \frac{0.08 \times 1.031 \times 10^6}{5}\left(\frac{0.05}{40}\right)^2$$

$$\mu = 25.75 \;\; mPa - \mathrm{sec}$$

5) Now from chart Fig. 12.12 pp. 534 (Fig. 12.10 pp. 435)

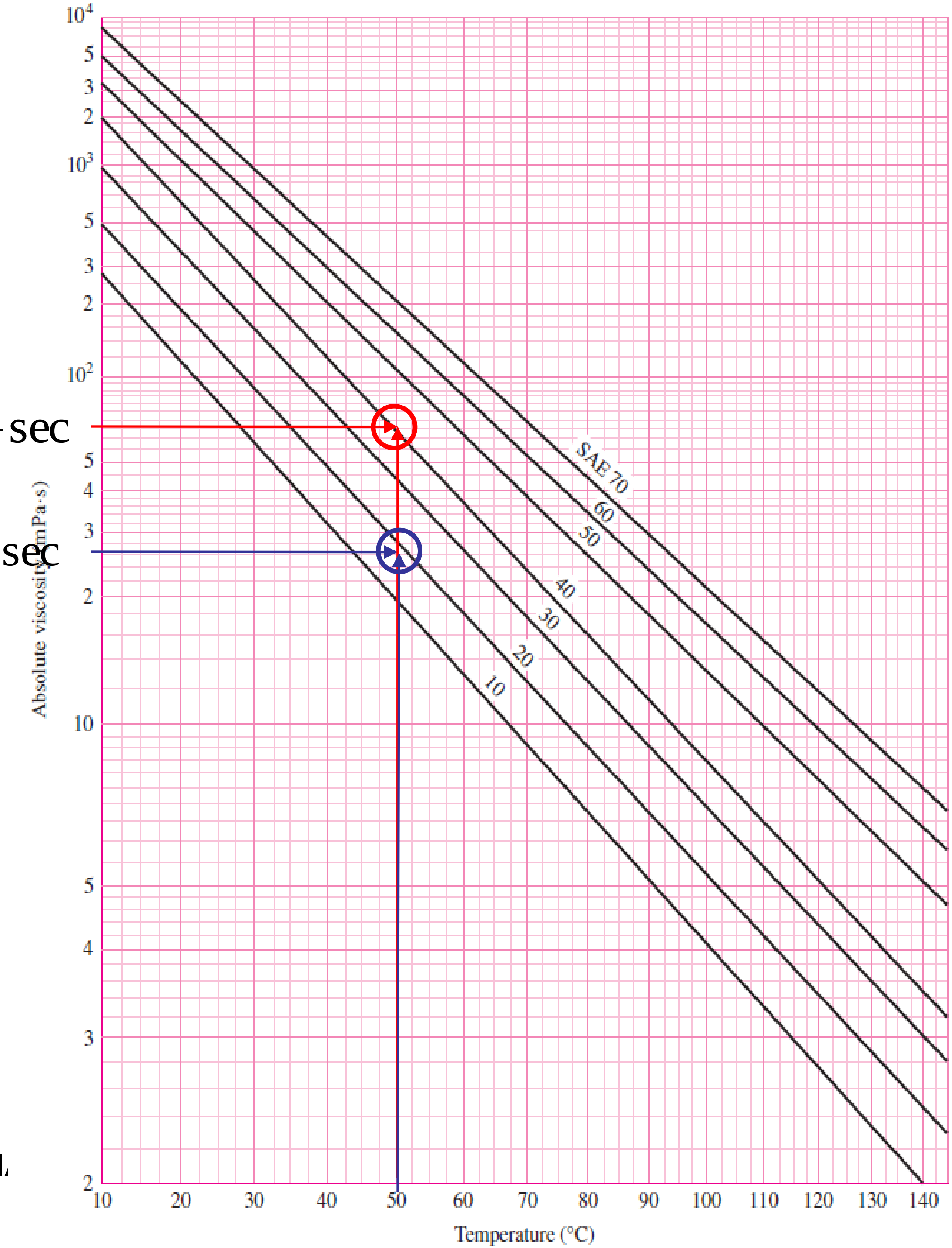
SAE 40 is the most suitable one for max. W

$$\mu = 67.6 \ \ mPa - \mathrm{sec}$$

$$\mu = 25.75 \ \ mPa - \mathrm{sec}$$

and SAE 20 is suitable for f_{min}

T_{ave}= 50 °C for both cases



6) Now because S and l/d are known most other parameters can be determined from Boyd chart (S=0.21 and l/d=1):

$$\left(\frac{r}{c}\right) f = 4.8 \quad (\text{p}540)(\text{p}444) \rightarrow\rightarrow \; f = 4.8\left(\frac{c}{r}\right) = 4.8\frac{0.05}{40} = 0.006$$

$$\frac{Q}{rcNl} = 4.1 \quad (\text{p}541)(\text{p}445) \;\rightarrow\rightarrow\; Q = 4.1 rcNl = 3280 \quad mm^3/\text{sec}$$

$$\frac{Q_s}{Q} = 0.55 \quad (\text{p}542)(\text{p}446) \;\rightarrow\rightarrow\; Q_s = 0.55 \times 3280 = 1804 \quad mm^3/\text{sec}$$

$$\frac{P}{P_{\max}} = 0.46 \quad (\text{p}543)(\text{p}446) \;\rightarrow\rightarrow\; P_{\max} = \frac{P}{0.46} = \frac{1.031 MPa}{0.46} = 2.241 \quad MPa$$

$$\frac{h_0}{c} = 0.53 \quad (\text{p}543)(\text{p}446) \;\rightarrow\rightarrow\; h_0 = 0.53 \times c = 0.0256\, mm = 25.6\, \mu m$$

7)

$$\Delta T_{^oC} = \frac{8.30P}{\left[1-\frac{1}{2}\times\frac{Q_s}{Q}\right]}\times\frac{\frac{r}{c}f}{\frac{Q}{rcNl}} = \frac{8.3\times1.031MPa}{1-\frac{1}{2}\times0.05}\times\frac{4.8}{4.1} = 13.8\ ^oC$$

$$\Delta T_{^oC} = T_{\text{var}}\frac{P}{\gamma C_H} = ??\ ^oC \quad T_v = ?$$

$$\text{Fig.12.12 (S = 0.21)},\ \frac{l}{d} = 1 \ \rightarrow\rightarrow\ T_v = 20$$

$$\Delta T_{^oC} = 20\times\frac{1.031\times10^6}{861\times1760} = 13.7\ ^oC$$

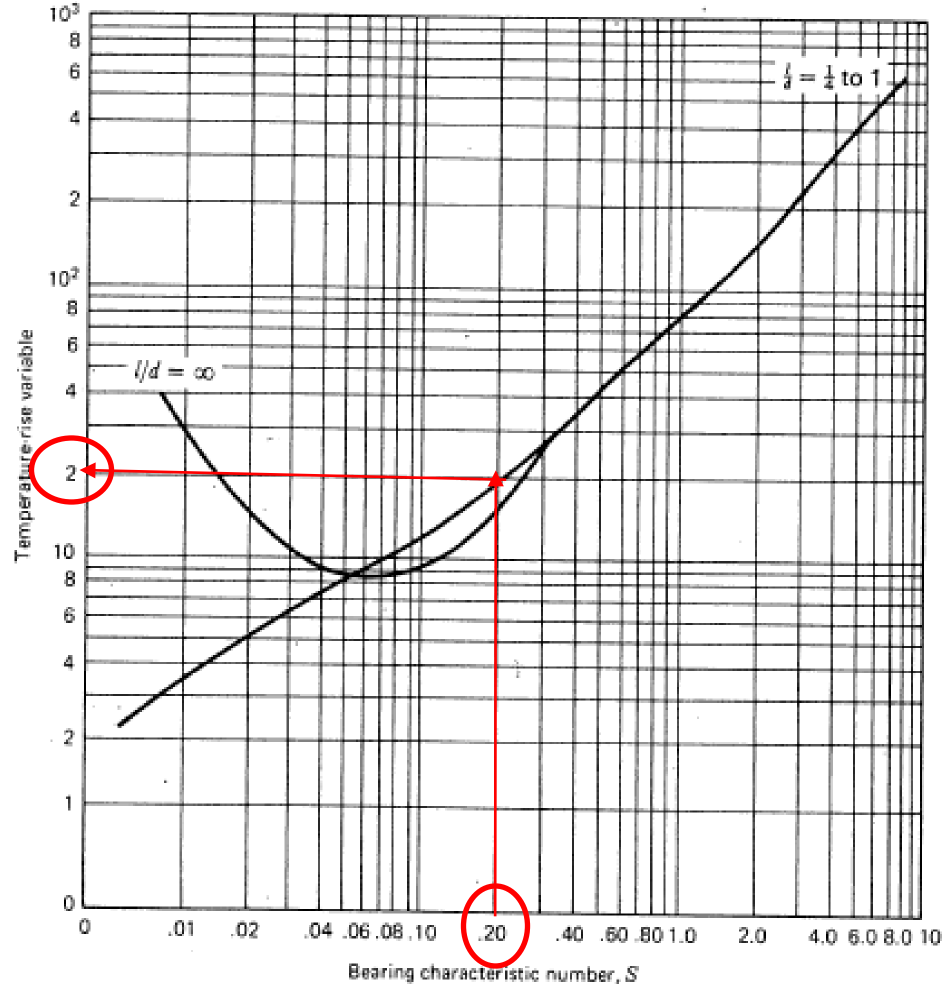


FIGURE 12-12 Chart for temperature-rise variable $T(\text{var}) = \gamma C_H \Delta T/P$. In plotting this chart it was found that the curves for $l/d = \frac{1}{4}, \frac{1}{2}$, and 1 were so close together that they could not be distinguished from a single curve.

8) The torque required to overcome the friction is

$$T_f = f \times W \times r = 0.006 \times 6600 \times 0.04 = 1.584\ Nm$$ For max W cond S=0.21 and l/d=1

$$T_f = f \times W \times r = 0.003 \times 6600 \times 0.04 = 0.792\ Nm$$ For min friction cond S=0.08 and l/d=1

The power loss is

$$P_{loss} = T_f \times \omega_{\frac{rad}{\sec}} = 1.585 \times 300 \frac{rev}{\min} \times \frac{\min}{60 \sec} \times \frac{2\pi rad}{rev} = 49.76\ Nm/\sec$$

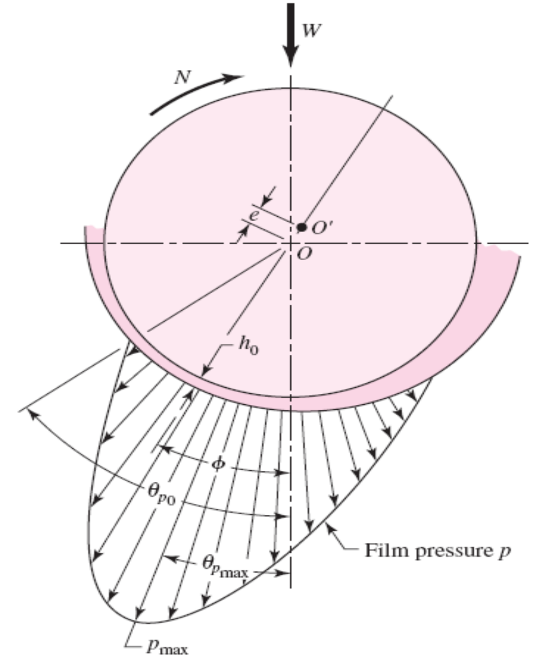
$$P_{loss} = 49.76\ watt$$ For max W cond S=0.21 and l/d=1

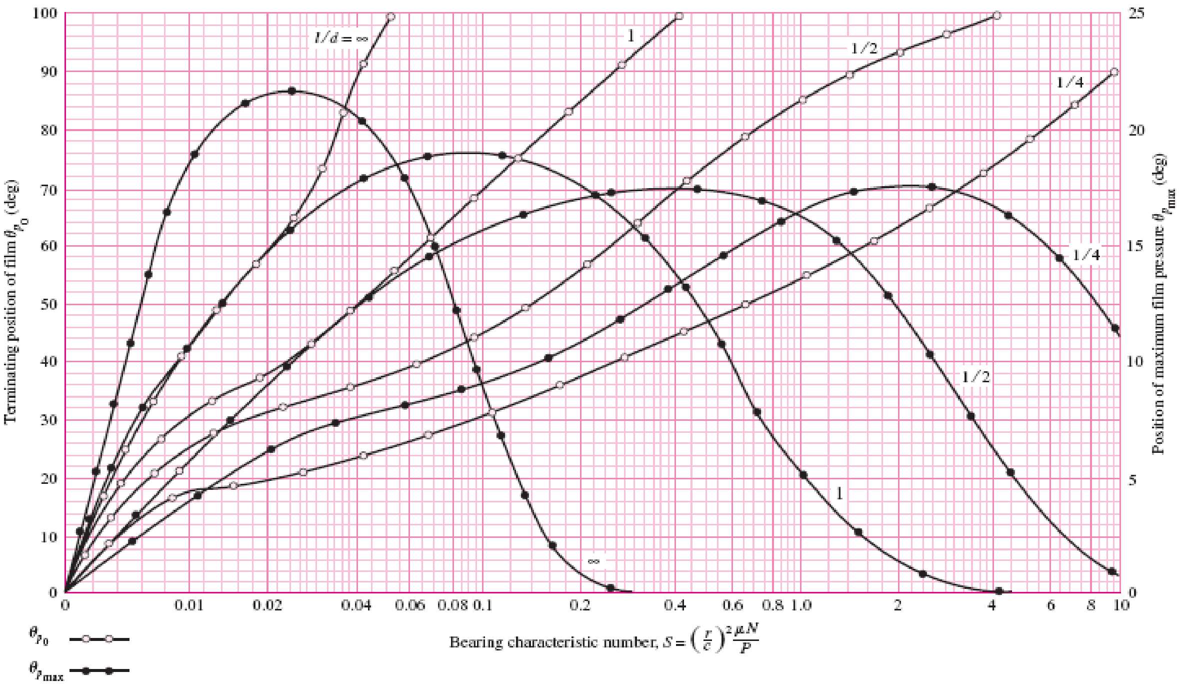
$$P_{loss} = 24.88\ watt$$ For min friction cond S=0.08 and l/d=1

if required $\left(\theta_{P_0} \ \& \ \theta_{Pmax}\right)$ for $S = 0.21, \dfrac{l}{d} = 1$

from Fig. 12.21 p554

$$\theta_{P_0} = 85^o \ \& \ \theta_{Pmax} = 21.25^o$$





OPTIMIZATION OF J.B's

In designing a journal bearing for thick-film lubrication,
the engineer must :

- select the grade of oil to be used and
- determine suitable values for *P*, *N*, *r*, *c*, and *l*.

A poor selection of these parameters or inadequate control of them during manufacture or in use may result in a film that is too thin, so that the oil flow is insufficient, causing the bearing to overheat and, eventually, fail.

Furthermore, the radial clearance *c* is difficult to hold accurate during manufacture, and it may increase because of wear.

What is the effect of an entire range of radial clearances, expected in manufacture, and what will happen to the bearing performance if *c* increases because of wear?

Most of these questions can be answered and the design optimized by plotting curves of the performance as functions of the quantities over which the designer has control.

Figure shows the results obtained when the performance of a particular bearing is calculated for a whole range of radial clearances and is plotted with clearance as the independent variable.

The graph shows that if the clearance is too tight; the outlet temperature T_2 will be too high and the minimum film thickness (h_o) will be too low.

High temperatures may cause the bearing to fail by fatigue. If the oil film is too thin, dirt particles may be unable to pass without scoring or may embed themselves in the bearing.

In either event, there will be excessive wear and friction, resulting in high temperatures and possible seizing.

H represents the power loss in the bearing due to friction

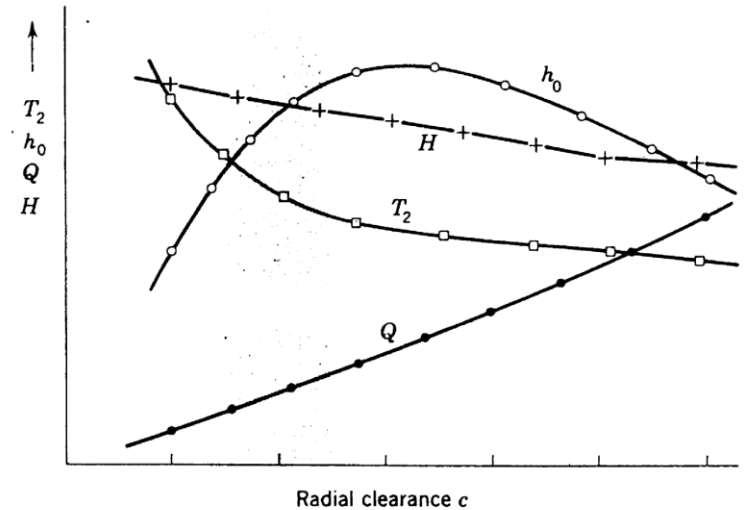


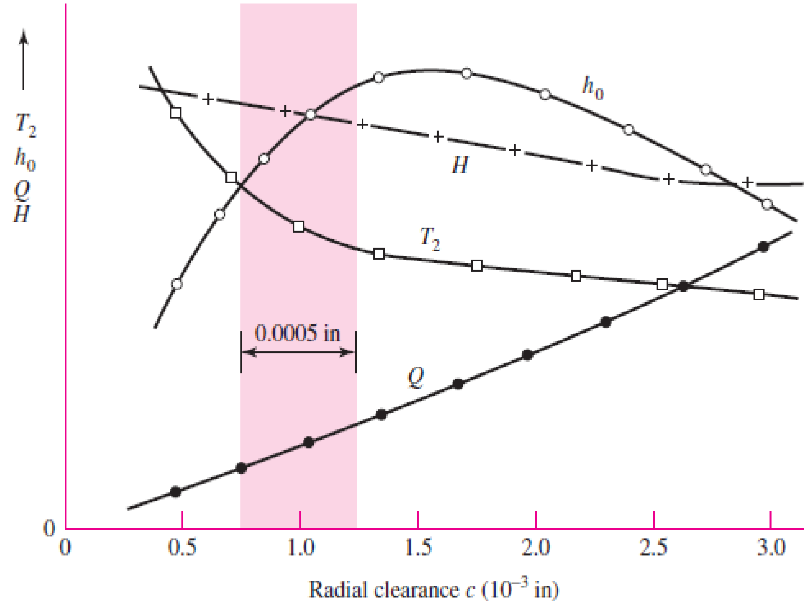
FIGURE 12-21 A plot of some performance characteristics. The bearing outlet temperature is T_2. New bearings should be designed for the shaded zone because wear will move the operating point to the right.

A tight clearance results in a high temperature. It would seem that a large clearance will permit the dirt particles to pass through and also will permit a large flow of oil. This lowers the temperature and increases the life of the bearing. However, if the clearance becomes too large, the bearing becomes noisy and the minimum film thickness begins to decrease again.

In between these two limitations there exists a rather large range of clearances that will result in satisfactory bearing performance.

When both the production tolerance and the future wear on the bearing are considered, it is seen, from Fig, that the best compromise is a clearance range slightly to the left of the top of the minimum-film-thickness curve.

In this way, future wear will move the operating point to the right and increase the film thickness and decrease the operating temperature



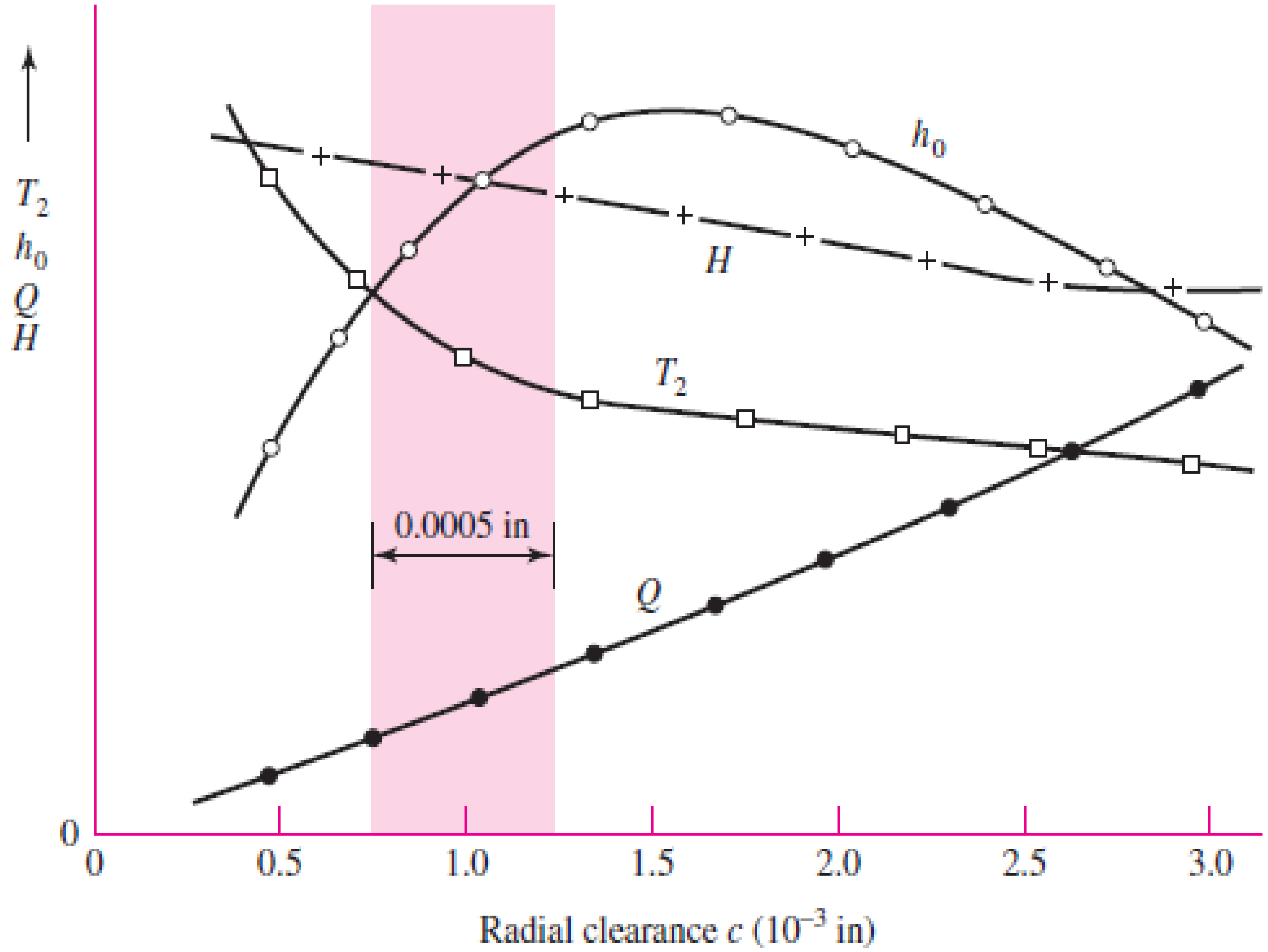


Fig 12.25 A plot of some performance characteristics of the bearing of Exs. 12–1 to 12–4 for radial clearances of 0.0005 to 0.003 in. The bearing outlet temperature is designated T2. <u>New bearings should be designed for the shaded zone, because wear will move the operating point to the right.</u>

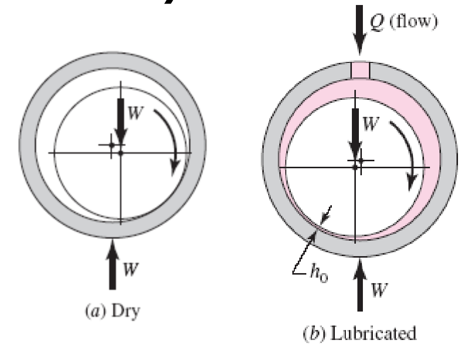
PRESSURE - FED BEARINGS (HYDRO-STATIC JOURNAL BEARING)

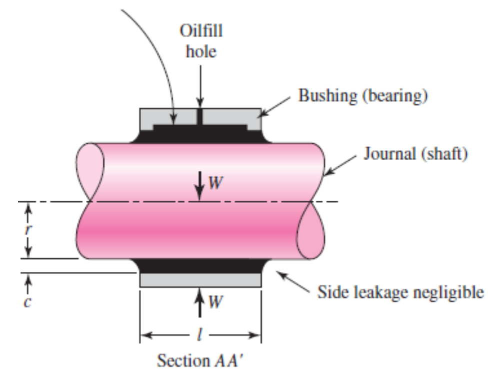
The load-carrying capacity of self-contained natural-circulating (hydro-dynamic) journal bearings is quite restricted.

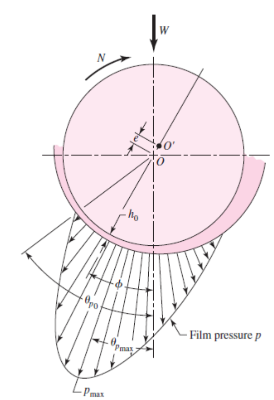
The factor limiting better performance is usually the heat-dissipation capability of the bearing.

A first thought of a way to increase heat dissipation is to cool the sump with an external fluid such as water. The high-temperature problem is in the film where the heat is generated but cooling is not possible in the film until later. This does not protect against exceeding the maximum allowable temperature of the lubricant.

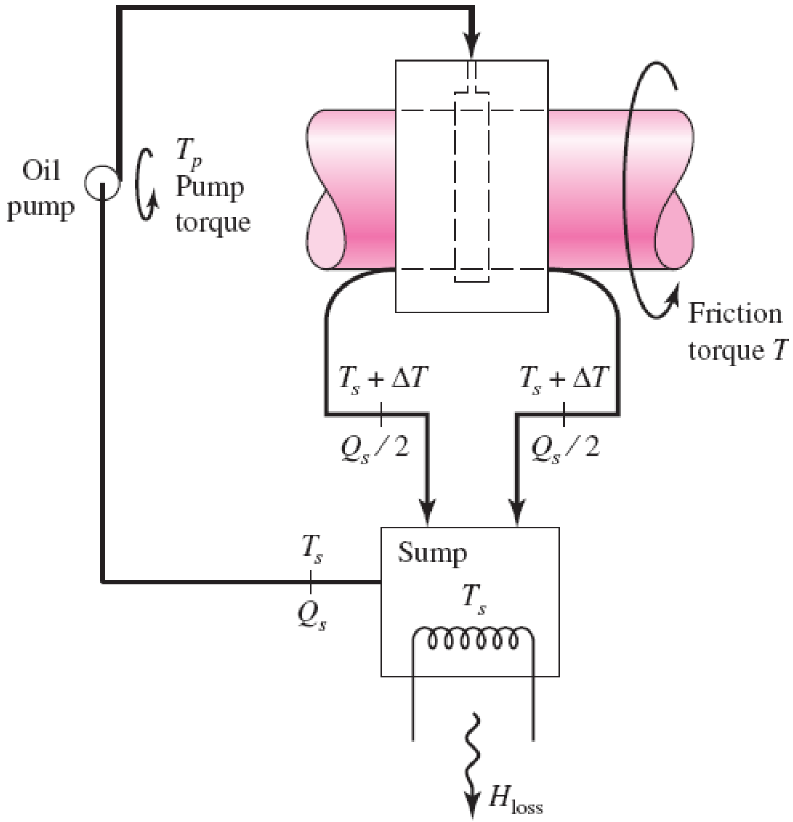
A second alternative is to reduce the *temperature rise* in the film by dramatically increasing the rate of lubricant flow.



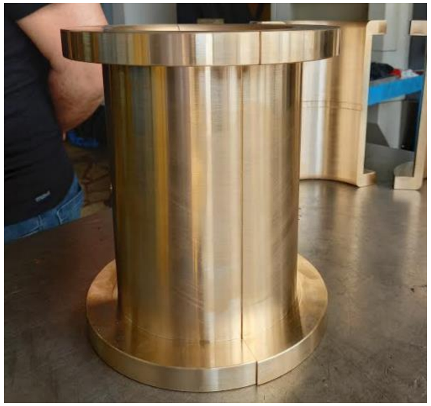




To increase lubricant flow, an external pump must be used with lubricant supplied at pressures higher than atmospheric pressure.



Because the lubricant is supplied to the bearing under pressure, such bearings are **called** ***pressure-fed bearings (or hydrostatic J barings)***.


To force a greater flow through the bearing and thus obtain an increased cooling effect, a common practice is to use a circumferential groove at the center of the bearing, with an oil-supply hole located opposite the load-bearing zone.

Such a bearing is shown in Fig. 12–27. The effect of the groove is to create two half-bearings, each having a smaller *l/d* ratio than the original.

The groove divides the pressure-distribution curve into two lobes and reduces the minimum film thickness, but it has wide acceptance among lubrication engineers because such bearings carry more load without overheating.

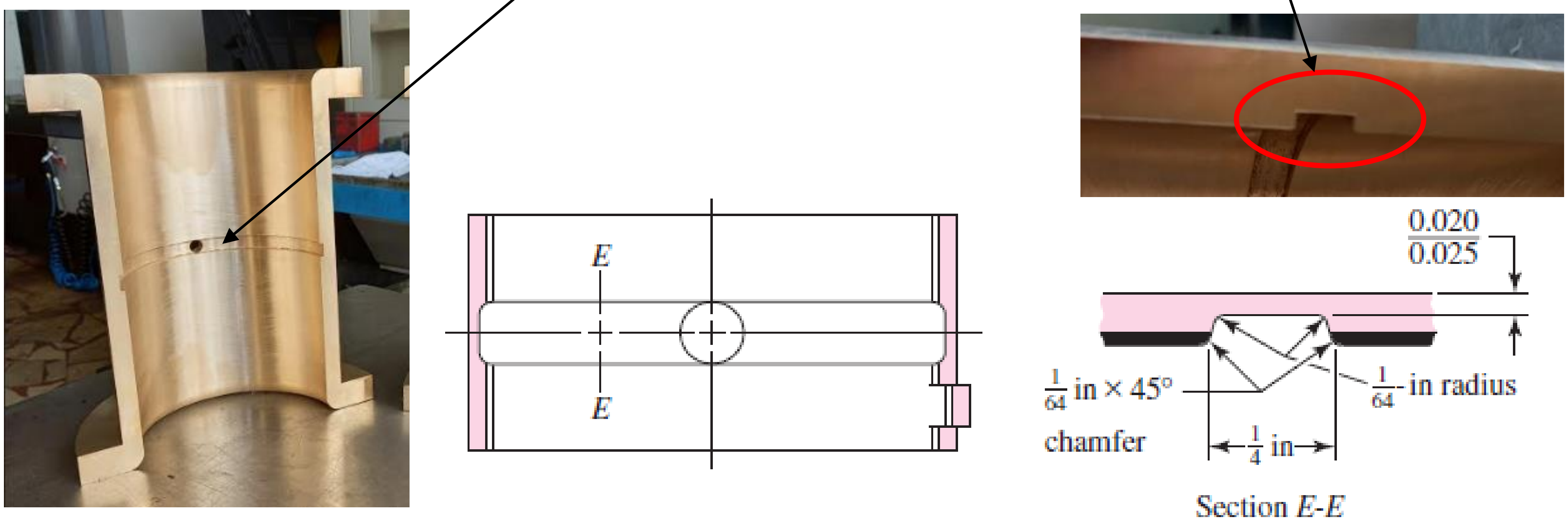


Fig. 12.27 Centrally located full annular groove.

$l' = (l - l_g) / 2$

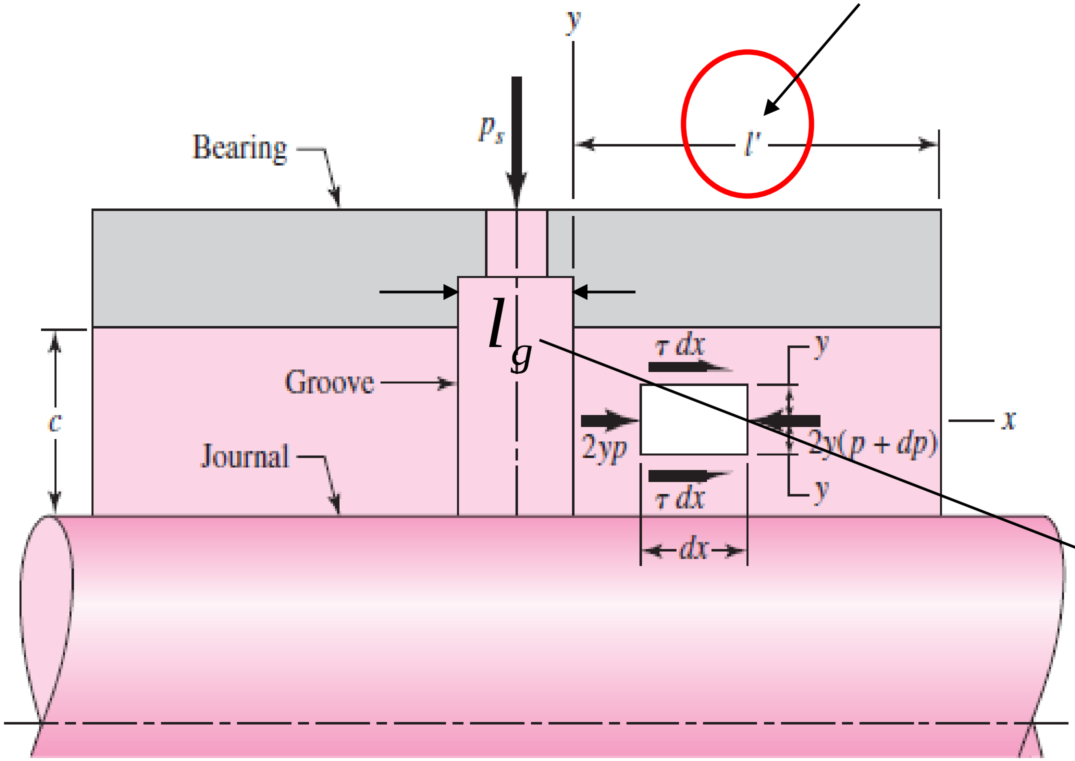


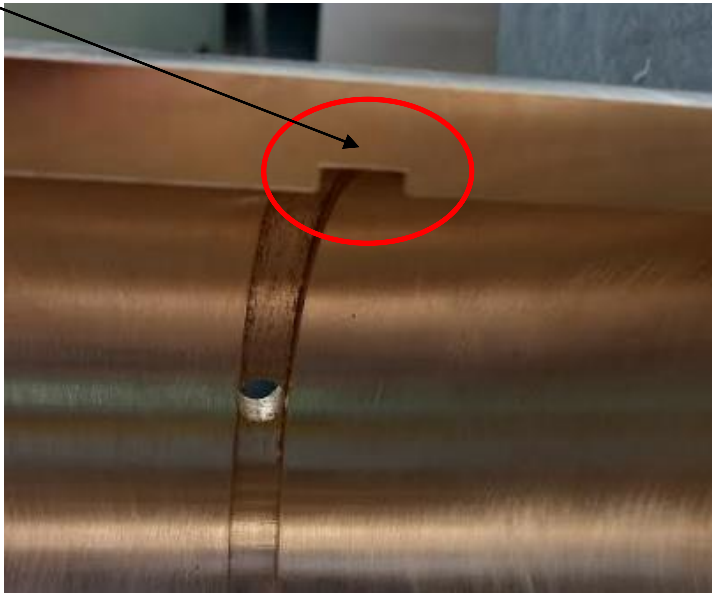
Fig. 12.28 Flow of lubricant from a pressure-fed bearing having a central annular groove.

Here we assume that the bearing is two fold each having properties:

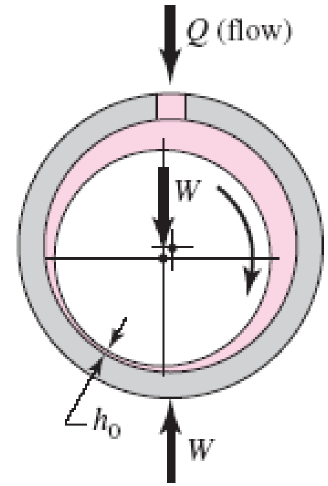
l'/d (instead of l/d) and

P=(W/2)/(l'd)

$$P = \frac{W/2}{l' * d}$$

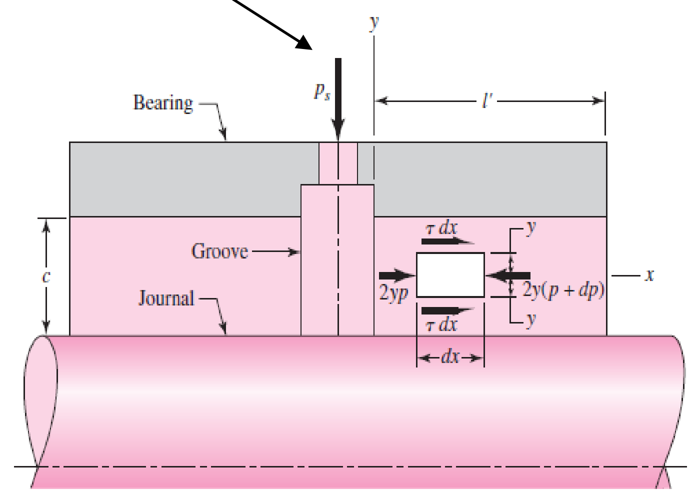


These pressure-fed (hydro-static) bearings do not rely on wedging action of the lubricant by the rotation of journal only but lubricant is forced into the bearing clearance under a pressure p_s



(b) Lubricated

This is the method used in application where the normal lubricant flow Q is not sufficient to carry away the heat generated by the hydrodynamic action within the bearing clearance for the load carried is high to be hold up.



Therefore “a special oil flowrate formula Q” will be developed for pressure-fed (hydro-static) bearings. <u>**The Qs/Q and Q/rcNl figures will NOT be used.**</u>

To set up a method of solution for oil flow rate Q, we shall assume a groove ample enough that the pressure drop in the groove itself is small.

Initially we will neglect eccentricity and then apply a correction factor for this condition.

The oil flow, then, is the amount that flows out of the two halves of the bearing in the direction of the concentric shaft.

If we neglect the rotation of the shaft, the flow of the lubricant is caused mainly by the supply pressure *ps*.

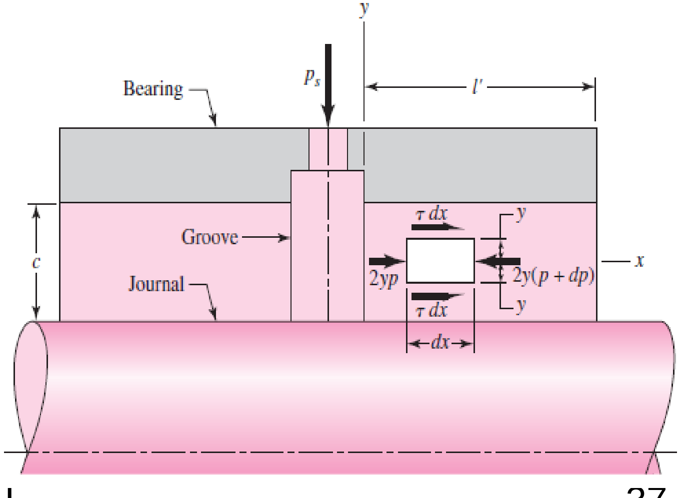


Figure 12–29 shows a graph of this relation fitted into the clearance space *c* so that you can see how the velocity of the lubricant varies from the journal surface to the bearing surface.

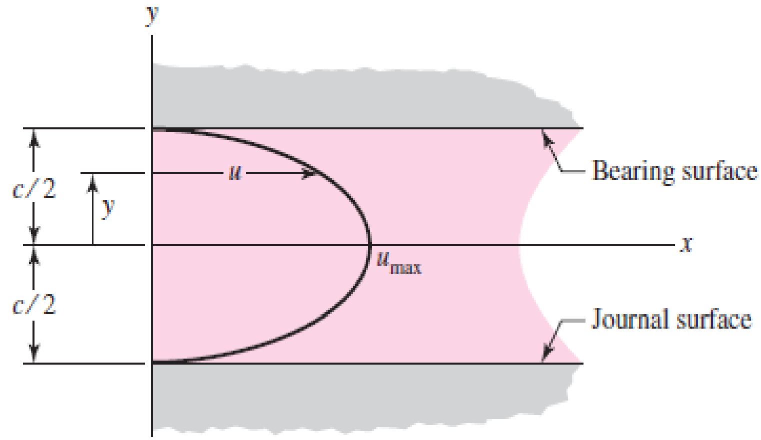


Fig.12.29 Parabolic distribution of the lubricant velocity.

The distribution is parabolic, as shown, with the maximum velocity occurring at the center, where *y* = 0. The magnitude is, from Eq. (12–21),

$$u_{\max} = \frac{p_s c^2}{8\mu l'} \qquad \text{(i)}$$

To consider eccentricity, as shown in Fig. 12–30, the film thickness is *h = c − e cos θ.* Substituting *h* for *c* in Eq. (*i*), with the average ordinate of a parabola being two-thirds the maximum, the average velocity at any angular position *θ* is

$$u_{ave} = \frac{2}{3}\frac{p_s h^2}{8\mu l'} = \frac{p_s}{12\mu l'}(c - e\cos\theta)^2 \qquad \text{(j)}$$

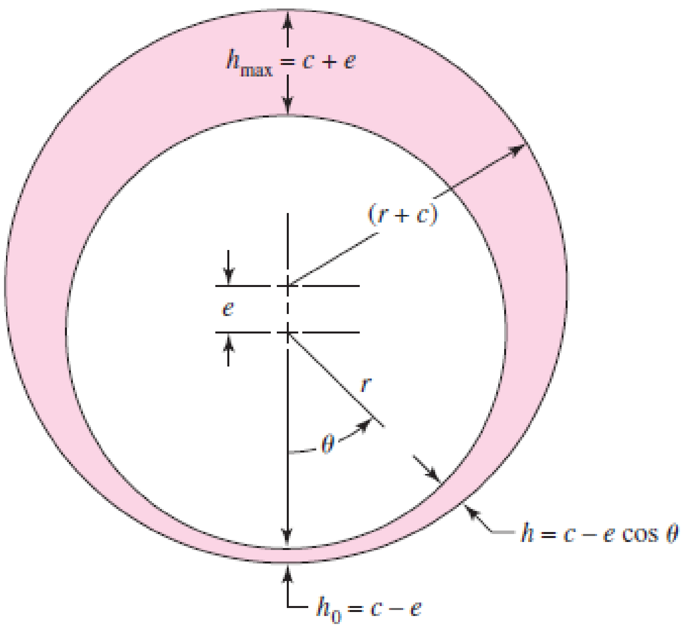


Fig. 12.30

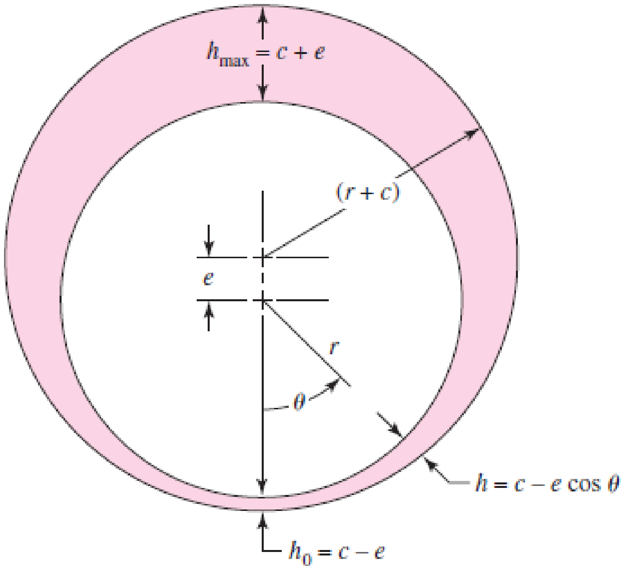


Fig. 12.30

$$h_o = c(1-\varepsilon)$$

$$\varepsilon = \frac{e}{c}$$

We can compute the amount of lubricant that flows out both ends;

$$dQ_s = u_{ave} \times dA \qquad dA = h \times rd\theta$$

Total side flow is twice that value and *dA* is the elemental area;

$$dQ_s = 2 \times u_{ave} \times hrd\theta$$

$$u_{ave} = \frac{p_s}{12\mu l'}(c - e\cos\theta)^2$$

$$dQ_s = \frac{p_s r}{6\mu l'}(c - e\cos\theta)^3 d\theta$$

Integrating around the bearing gives the total side flow as

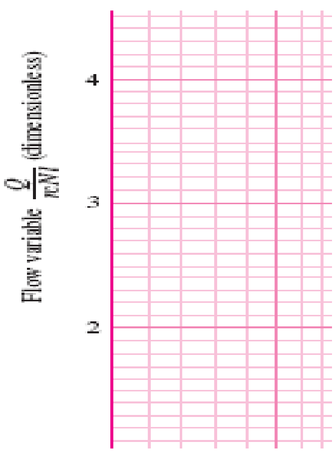
$$Q_s = \int dQ_s = \frac{p_s r}{6\mu l'} \int_0^{2\pi} (c - e\cos\theta)^3 \, d\theta = \frac{p_s r}{6\mu l'} (2\pi c^3 - 3\pi c^3)$$

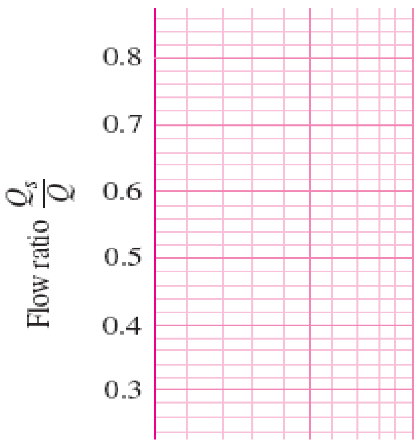
Rearranging, with $\varepsilon = e/c$, gives

$$Q_s = \frac{\pi p_s r c^3}{3\mu l'} (1 + 1.5\varepsilon^2)$$

Flow rate eqn for "pressure-fed bearings"

The charts for
flow variable (Q/rcNl) and
flow ratio (Qs/Q) (Figs. 12–19 and 12–20)
DO NOT apply to pressure-fed bearings.





In analyzing the performance of pressure-fed bearings, **the bearing length should be taken as *l ′*.**

The characteristic pressure in each of the two bearings that constitute the pressure-fed bearing assembly *P* is given by

$$P = \frac{W/2}{2rl'} = \frac{W}{4rl'}$$

Also, the maximum film pressure Pmax given by Fig. 12–21 must be increased by the oil supply pressure p_s to obtain the total film pressure.

Pmax = Pmax +Ps

Since the oil flow has been increased by forced feed, Eq. (12–14) will not give a correct temperature rise (will give too high temp rise) because the side flow carries away all the heat generated. A new temp. rise eqn (ΔT) is derived for pressure fed bearings.

To calculate heat gain- temperature rise (ΔT)- of the fluid passing through the bearing the following steps are followed:

1) $$\frac{l'}{d} = \frac{\left(l - l_g\right)/2}{d}$$

2) $$P = \frac{W/2}{2rl'} = \frac{W}{4rl'}$$

The Sommerfeld number may be expressed as

3) $$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = \left(\frac{r}{c}\right)^2 \frac{\mu N}{\frac{W}{4rl'}} = \left(\frac{r}{c}\right)^2 \frac{4rl'\mu N}{W}$$

4) The corresponding temperature rise (ΔT) equation in SI units uses the bearing load *W* in kN, lubricant supply pressure p_s in *kPa*, and the journal radius *r* in mm:

$$\Delta T_{^oC} = \frac{978\times 10^6}{1+1.5\varepsilon^2}\times\frac{\left(\frac{r}{c}f\right)SW^2}{p_s r^4}$$

Where ΔT in ^{o}C (temperature increase)
W= bearing load in kN
p_s= supply pressure in kPa
r= journal radius in mm

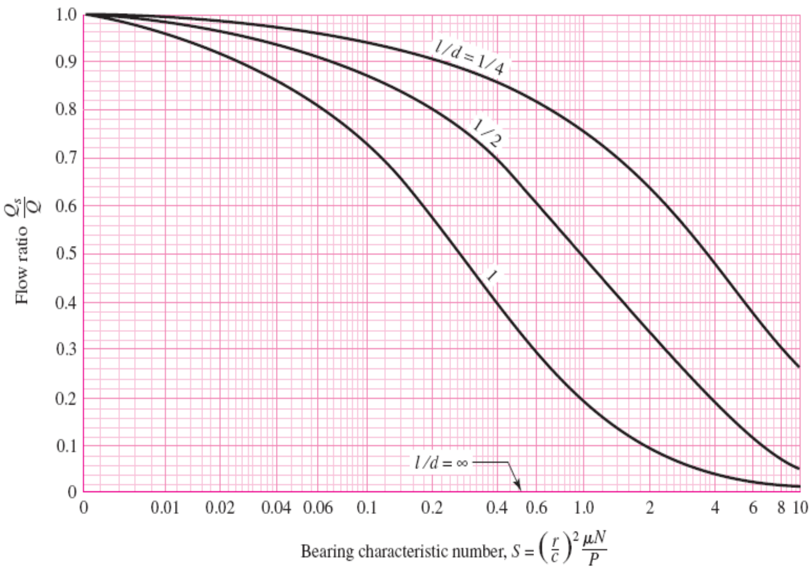
OR YOU CAN USE

$$\Delta T_{^oC} = \frac{1.5\left(\frac{r}{c}f\right)}{\gamma C_H\left(1+1.5\varepsilon^2\right)}\times\frac{SW^2}{p_s r^4}$$

Where γ= 861 kg/m^3
C_H= 1760 J/kg ^{o}C
W= bearing load in N
p_s= supply pressure in Pa
r= journal radius in m

5) **DO NOT** use figures 12.18 (17) of flow rate variable

and 12.19 (18) flow rate



Instead, use the equation

$$Q_s = \frac{\pi p_s r c^3}{3\mu l'}(1 + 1.5\varepsilon^2)$$

6) Use figures 12.20 (40) but to find maximum pressure add two values P_{max} & p_s .

$$P_{\max\, act} = P_{\max} + p_s$$

Example 3: **(12-20) (prob 12.16)**

An 8- cylinder diesel engine has a front main bearing 90 mm in diameter and 51 mm long. The bearing has a central annular oil groove 6 mm wide.

It is pressure lubricated with SAE 30 oil at on inlet temperature of 82 ^oC and at a supply pressure of 345 kPa.

Corresponding to a radial clearance of 0.064 mm, a speed of 2800 rpm and a radial load of 20500 N, find;

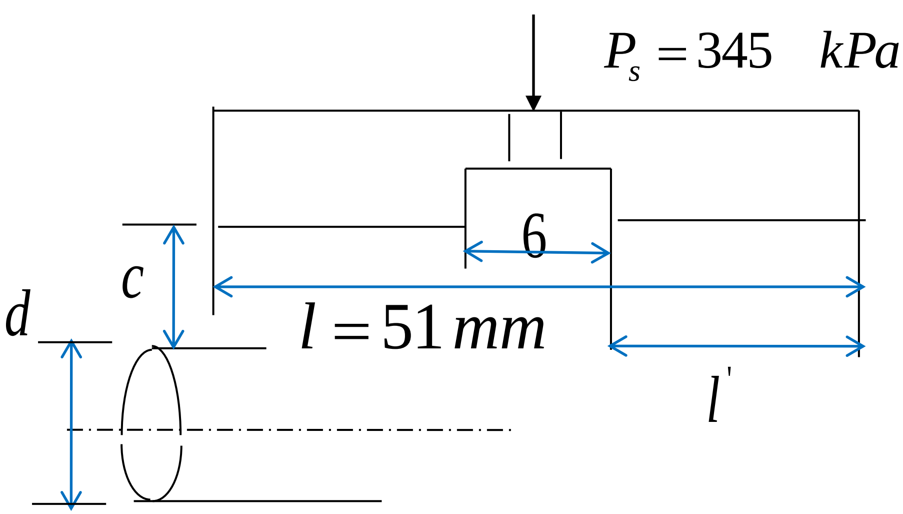
a) The temperature rise

b) Minimum oil film thickness

c) maximum oil film pressure

d) oil flow

e) θ_{h_0} & $\theta_{P_{max}}$



$$l = 51\ mm$$

$$d = 90\ mm \rightarrow \quad r = 45\ mm$$

$$l_g = 6\ mm \rightarrow l' = \frac{1}{2}\left(l - l_g\right) = 22.5\ mm$$

$$\frac{l'}{d} = \frac{22.5}{90} = \frac{1}{4}$$

$T_i = 82\ ^oC$

$T_o = ?\quad \mapsto\ \mu$ at T_{ave} Since ΔT & μ are unknowns we need to use trail and error method

$P_s = 345\ kPa$

$c = 0.064\ mm$

$r = 45\ mm$

$n = 2800\ rpm \rightarrow N = 46.7\ rev/\sec$

$W = 20500\ N$

$$P = \frac{W}{4rl} = \frac{20500}{4 \times 45 \times 22.5} = 5.06\ \ MPa$$

Table 12-3 RECOMMENDED UNIT LOADS FOR SLEEVE BEARINGS

Application	Unit load, kPa	Application	Unit load, kPa
Air compressors:		Diesel engines:	
Main bearings	1 000– 2 000	Main bearings	6 000–12 000
Crankpin	2 000– 4 000	Crankpin	8 000–15 000
Automotive engines:		Wristpin	14 000–15 000
Main bearings	4 000– 5 000	Electric motors	800– 1 500
Crankpin	10 000–15 000	Gear reducers	800– 1 500
Centrifugal pumps	600– 1 200	Steam turbines	800– 1 500

(see pp.558 Table 12.2, 6-12 MPa) diesel engine front main bearing a)

$$\Delta T_{^oC} = \frac{978\times 10^6}{1+1.5\varepsilon^2}\times\frac{\left(\frac{r}{c}f\right)SW^2}{p_s r^4} \qquad \text{where } \varepsilon \ \& \ \frac{r}{c}f \quad \text{depends on } S \ \& \ \frac{l}{d}$$

$$\text{but } S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} \quad \text{depends on } \mu$$

$$\text{also } \mu \quad \text{depends on } T_i + \frac{\Delta T}{2}$$

Start by assuming ΔT=20 ^{o}C

$$T_{ave} = T_i + \frac{\Delta T}{2} = 82 + 10 = 92 \ {}^oC \rightarrow \mu = 8.5\times 10^{-3} \quad Pa-\sec$$

$$\text{Then } S = \left(\frac{45}{0.064}\right)^2 \frac{8.5\times 10^{-3}\times 46.7}{5.06\times 10^6} = 0.038$$

For $S = 0.038$ $\quad \frac{h_0}{c} = 0.075 \quad \& \quad \varepsilon = 0.925 \quad$ (Fig.12.14 or 12.16)

$\frac{l'}{d} = \frac{1}{4}$ $\quad \frac{r}{c} f = 2.1 \quad$ (Fig.12.17 or 12.18)

$$\Delta T_{^oC} = \frac{978\times10^6}{1+1.5\times(0.925)^2}\times\frac{2.1\times0.038\times(20.5)^2}{345\times(45)^4} = 10.15\ ^oC < 20\ ^oC$$

Re-try $\Delta T = 10^oC$

$$T_{ave} = T_i + \frac{\Delta T}{2} = 82 + 5 = 87\ ^oC \rightarrow \mu = 10\times10^{-3} \quad Pa - \sec$$

$$\text{Then } S = \left(\frac{45}{0.064}\right)^2 \frac{10\times10^{-3}\times46.7}{5.06\times10^6} = 0.045$$

$$\text{For } S = 0.045 \qquad \frac{h_0}{c} = 0.08 \quad \& \quad \varepsilon = 0.92 \quad (\text{Fig.}12.16)$$

$$\frac{l'}{d} = \frac{1}{4} \qquad \frac{r}{c} f = 2.3 \quad (\text{Fig.}12.18)$$

$$\Delta T_{{}^oC} = \frac{978\times10^6}{1+1.5\times(0.92)^2} \times \frac{2.3\times0.045\times(20.5)^2}{345\times(45)^4} = 13.2(12.6)\ {}^oC > 10\ {}^oC$$

Re-try ΔT= 11°C;

$$T_{ave} = T_i + \frac{\Delta T}{2} = 82 + 5.5 = 87.5\ {}^oC \rightarrow \mu = 9.5\times10^{-3} \quad Pa-\text{sec}$$

$$\text{Then } S = \left(\frac{45}{0.064}\right)^2 \frac{9.5\times10^{-3}\times46.7}{5.06\times10^6} = 0.042$$

For $S = 0.042$ $\qquad \dfrac{h_0}{c} = 0.075 \quad \& \quad \varepsilon = 0.925 \quad (\text{Fig.}12.16)$

$\dfrac{l'}{d} = \dfrac{1}{4}$ $\qquad \dfrac{r}{c} f = 2.2 \quad (\text{Fig.}12.18)$

$$\Delta T_{o_C} = \frac{978\times10^6}{1+1.5\times(0.925)^2}\times\frac{2.2\times0.042\times(20.5)^2}{345\times(45)^4} = 11.75(11.25)\ {}^oC \cong 11\ {}^oC$$

$\Delta T_{o_C} = 11.25\ {}^oC \cong 11\ {}^oC$ OK…

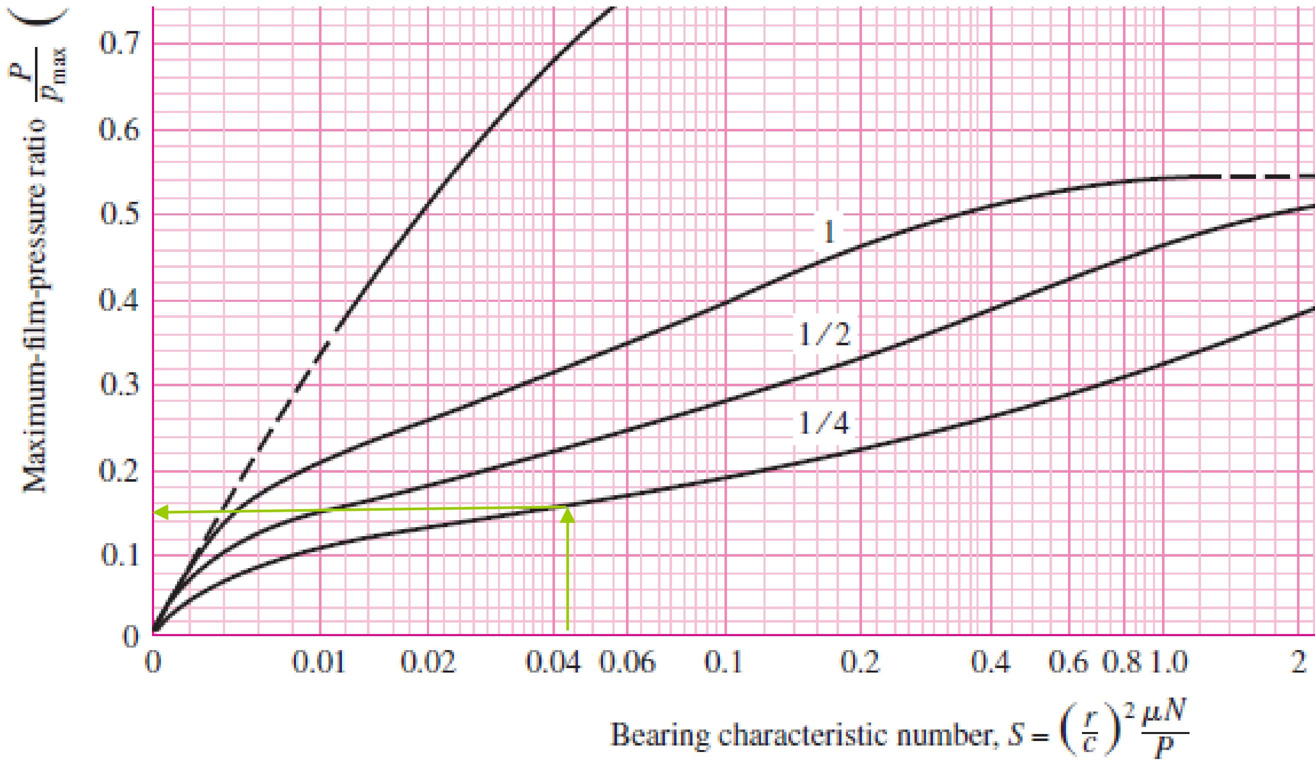
$b)\quad h_0 = 0.075 \times c = 0.075 \times 0.64$

$$h_0 = 4.8 \times 10^{-3}\ mm = 4.8\, micron$$

c) $P_{max} = ? \quad P_{max} + P_s \quad \text{from Fig. 12.21 for S} = 0.042 \rightarrow \dfrac{P}{P_{max}} = 0.15$

$$So, \quad P_{max} = \frac{P}{0.15} = \frac{5.06}{0.15} = 33.73\ \ MPa$$

$$33.73 \times 10^6 + 345 \times 10^3 = 34078\ \ kPa = 34.078\, MPa$$



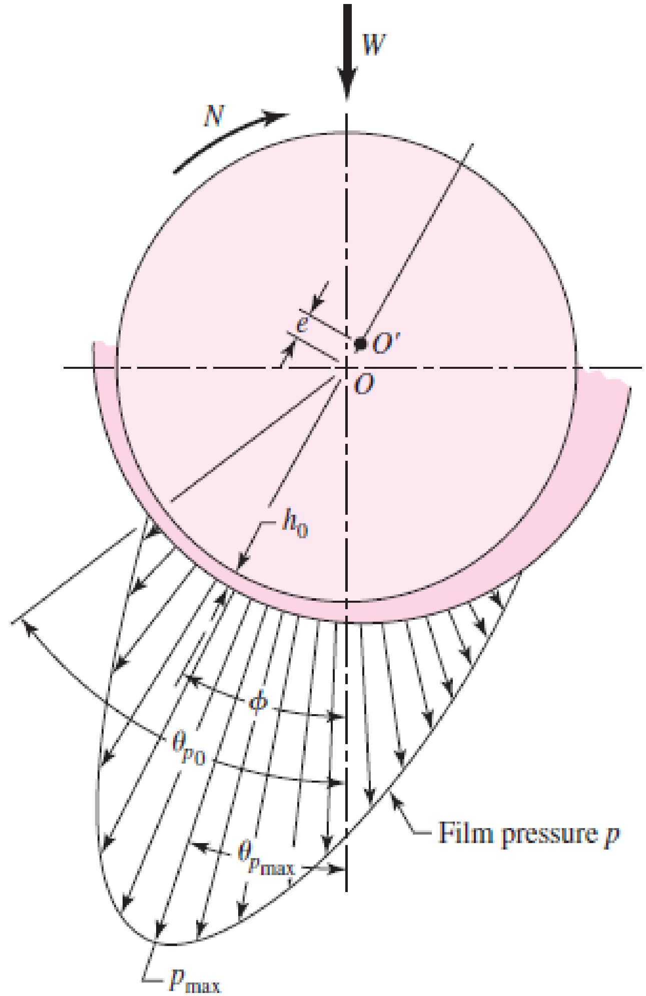
d) Q_s =?

$$Q_s = \frac{\pi P_s r c^3}{3\mu l^{'}}(1+1.5\varepsilon^2) = \frac{\pi \times 345\times 10^3 \times 45 \times (0.064)^3}{3\times 9.5\times 10^{-3}\times 22.5}\left(1+1.5\times(0.925)^2\right)$$

$$Q_s = 45528\, mm^3/\sec$$

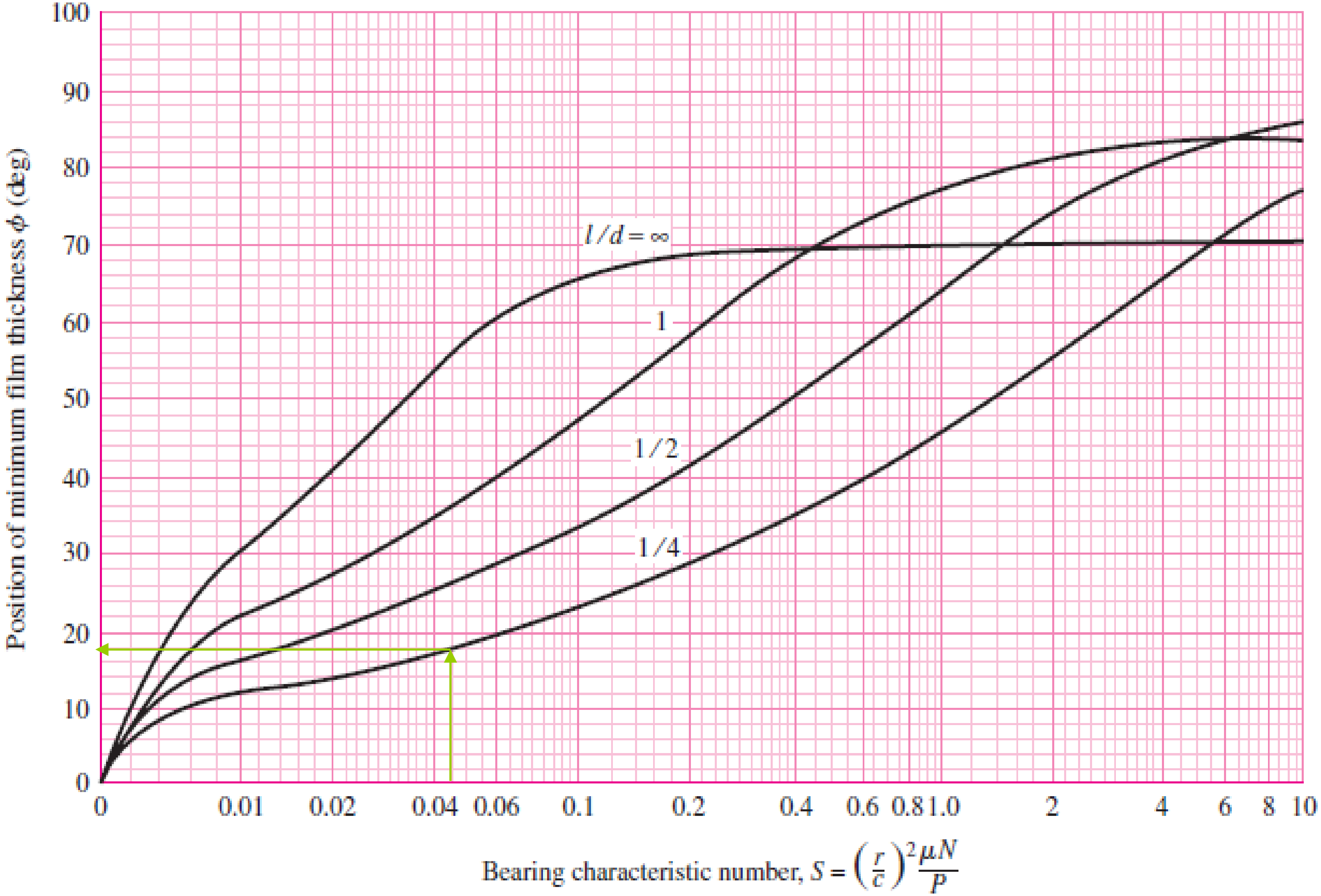
e)

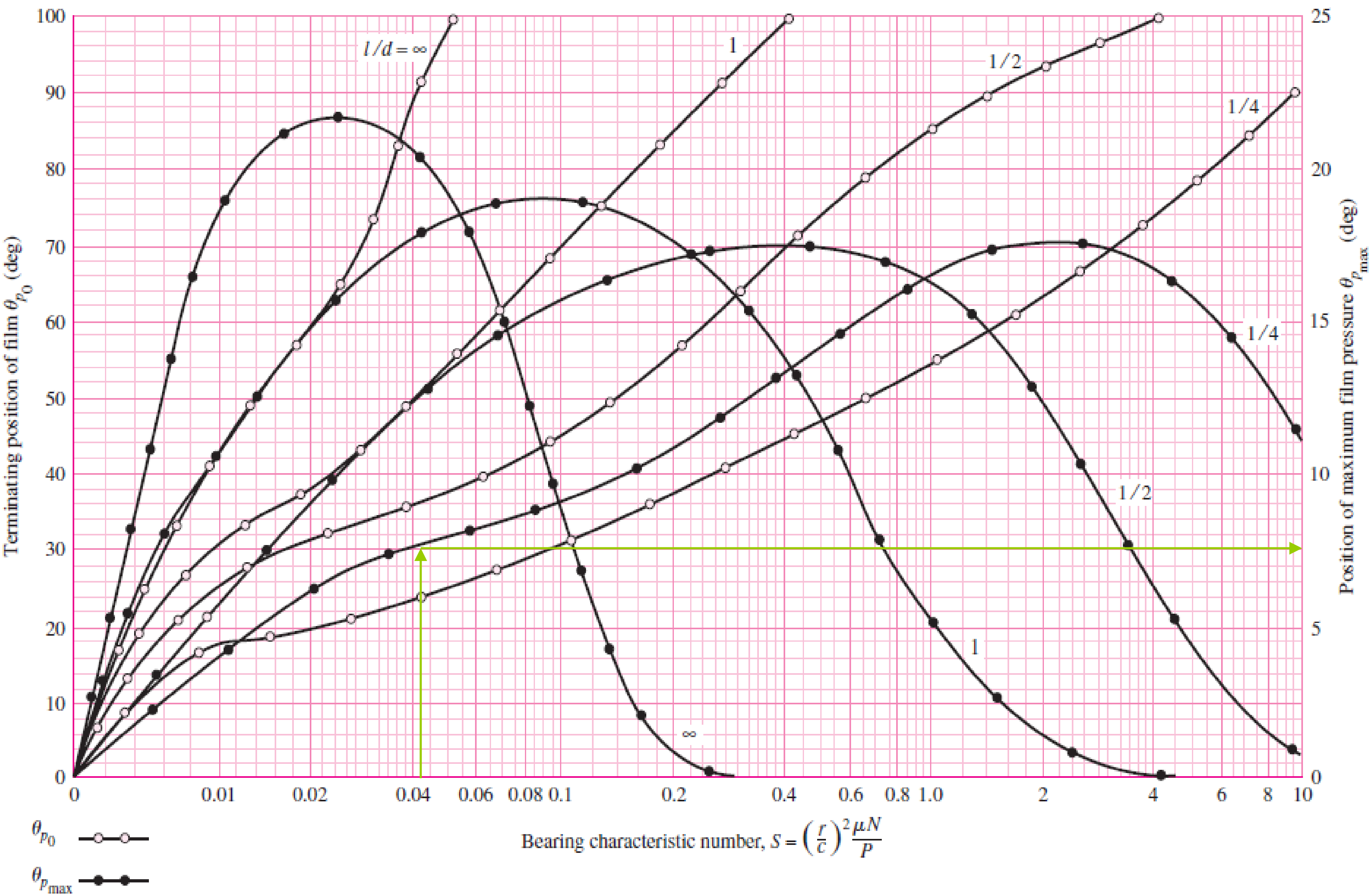
For θ_{h0} & $\theta_{P_{\max}}$ use related figures



e)

For θ_{h0} & $\theta_{P_{max}}$ use the Fig. 12.21





Terminating position of film θ_{p_0} (deg)
Position of maximum film pressure $\theta_{p_{max}}$ (deg)
$l/d = \infty$
1
1/2
1/4
1/4
1/2
1
∞
0 10 20 30 40 50 60 70 80 90 100
0 5 10 15 20 25
0 0.01 0.02 0.04 0.06 0.08 0.1 0.2 0.4 0.6 0.8 1.0 2 4 6 8 10
Bearing characteristic number, $S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$
θ_{p_0}
$\theta_{p_{max}}$

Design Suggestions

1. Start design (also analysis) by listing the given parameters

2. List the required parameters

3. Write down the relation between required parameters & the known parameters of $\Delta T= ?$ or $S= ?$

4. Use some recommended (suggested) values given in the tables & figures of P values & c values & PV_{max} values if not given in the problem or Assume them.

5. Start with an l/d= unity if geometry of bearing is not fixed then increase this ratio if thin film lubrication is likely to occur decrease this ratio if thick film lubrication or high temperatures are likely to occur.

6. A long bearing (large *l/d* ratio) reduces the coefficient of friction & side flow and therefore is desirable where thin-film lubrication is present.
   On the other hand, a short bearing results in a greater flow of oil out of the end and thus keeping the bearing cooler.
   Where force – feed or positive lubrication ($P_s>0$) is present. The *l/d* ratio should be relatively small.

7. If shaft deflection is likely to be severe, a short bearing should be used to prevent metal-to-metal contact at the ends of the bearing.

8. Always consider using of a partial bearing if high temperatures are a problem, become relieving the non-load bearing area of a bearing can very substantially reduce the heat generated.

9. Choose bearing material with satisfactory compressive ad fatigue strength while having low melting point and low modulus of elasticity.(table 12-3) It should have resistance to wear and corrosion. It should have low friction coefficient (can be reduced by some design means

Example 4:

A journal bearing has a diameter of 64 mm and length of 32 mm. The journal is to operate at a speed of 1800 rpm and carry a load of 3500 N. If SAE 20 oil at an inlet temperature of 44 ºC is to be used. What should be the radial clearance for:

a) Optimum load carrying capacity

b) Minimum friction or power loss condition.

d= 64 mm → r= 32 mm

l= 32 mm → l/d= 32/64= 0,5

N= 1800 rpm = 30 rps

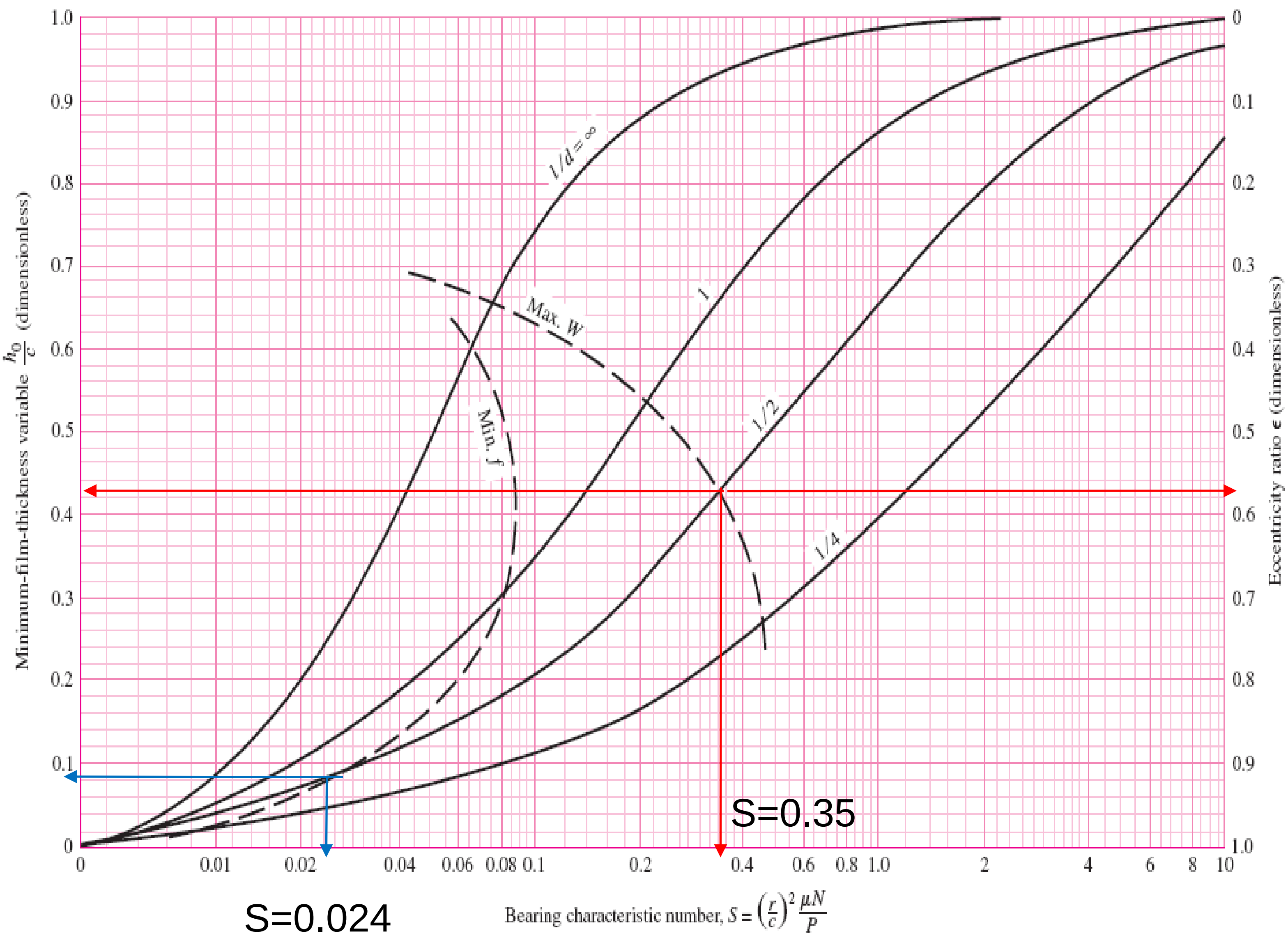
W= 3500 N

T_i = 44ºC

SAE 20 oil

c= ?

a) For optimum load carrying capacity with *l/d* = 1/2



$\Delta T = T_{var}\frac{P}{\gamma C_H} = 34 = {}^oC$

For $S = 0.35$

$$\frac{h_0}{c} = 0.425$$

$$\varepsilon = 0.575$$

$$P = \frac{W}{l*d} = \frac{3500}{32\times 64} = 1.709 \ MPa$$

For $S = 0.35$

$$\frac{l}{d} = \frac{1}{2}$$

$$\frac{r}{c} f = 9.0 \quad \& \quad T_{\text{var}} \cong 30$$

$$\frac{Q}{rcNl} = 4.8 \quad \& \quad \frac{Q_s}{Q} = 0.72$$

$$\Delta T_{o_C} = \frac{8.30P}{\left[1 - \frac{1}{2}\times\frac{Q_s}{Q}\right]} \times \frac{\frac{r}{c} f}{\frac{Q}{rcNl}} = 41.55 \ {}^oC$$

$$\Delta T_{o_C} = 30 * \frac{1.71*E6}{\gamma * C_H} = 34C$$

$$T_{ave} = T_{in} + \frac{\Delta T}{2} = 44 + \frac{41.5}{2} = 64.75 \ {}^oC \quad \rightarrow \mu = 15\times 10^{-3} \ Pa - \text{sec}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = 0.35$$

$$c = \left[\frac{r^2}{S}\frac{\mu N}{P}\right]^{1/2} = 0.0277 \ mm \cong 28 \ \mu m$$

b) For minimum friction or power loss:

$$S = 0.024 \quad \& \frac{h_0}{c} = 0.11$$

For $S = 0.024$

$$\frac{l}{d} = \frac{1}{2}$$

$$\frac{r}{c} f = 1.7 \quad \& \quad T_{\text{var}} \cong 7$$

$$\Delta T_{^oC} = 7 * \frac{1.71 * E6}{\gamma * C_H} = 8C$$

$$\frac{Q}{rcNl} = 5.65 \quad \& \quad \frac{Q_s}{Q} = 0.94$$

$$\Delta T_{^oC} = 8.05\ ^oC \quad \left(< \Delta T = 41.5\ ^oC \quad \text{based on max load} \right)$$

$$T_{ave} = T_{in} + \frac{\Delta T}{2} = 44 + \frac{8.05}{2} = 48\ ^oC \quad \rightarrow \mu = 32 \times 10^{-3}\ Pa - \sec$$

$$c = \left[\frac{r^2}{S} \frac{\mu N}{P} \right]^{1/2} = \left[\frac{32^2}{0.034} \frac{32 \times 10^{-3} \times 30}{1.709 \times 10^6} \right]^{1/2} = 0.13\, mm \cong 130\ \mu m$$

$$130\ \mu m > 28 \mu m$$

$$c_{fr} > c_{load}$$

Example 5:

A sleeve bearing is 100 mm diameter and has *l/d* ratio of 1/2. It has a frictional power loss of 100 watts when the load is 3000 N and the journal speed is 900 rpm for c/r=0.002.

a)Compute the minimum film thickness (h_0=?)

b)What is the viscosity of the oil and proper grade for an operating temperature of 71 ºC (T_{ave}=?).

c)What is the temperature increase? Θ_{Pmax} =? P_{max} =?

Solution:

Given:

$$\frac{l}{d} = \frac{1}{2} \qquad d = 100\ mm \qquad W = 3000\ N \qquad P_{loss} = 100\ watts$$

$$N = 900\ rpm = 15\ rps \qquad \frac{c}{r} = 0.002 \qquad c = 0.002 \times 50 = 0.1\ mm$$

a) The frictional power loss is given by;

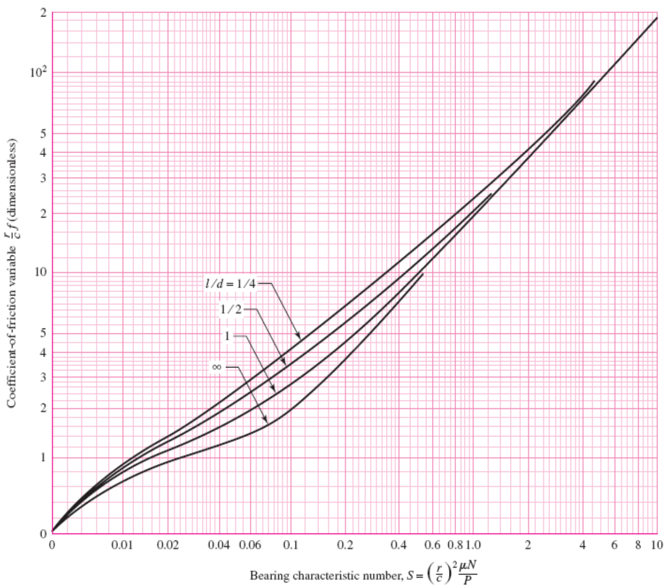
$$P_{loss} = T_{loss} \times w \frac{rad}{\sec} = \left(f \times W \times r \right) \times \left(2\pi N \right)$$

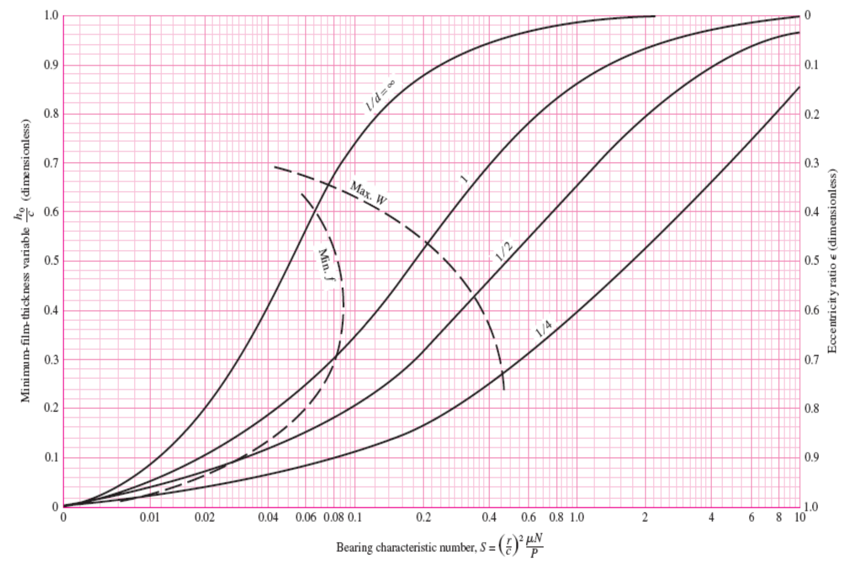
$$T_{loss} = F_{fric} \times r$$

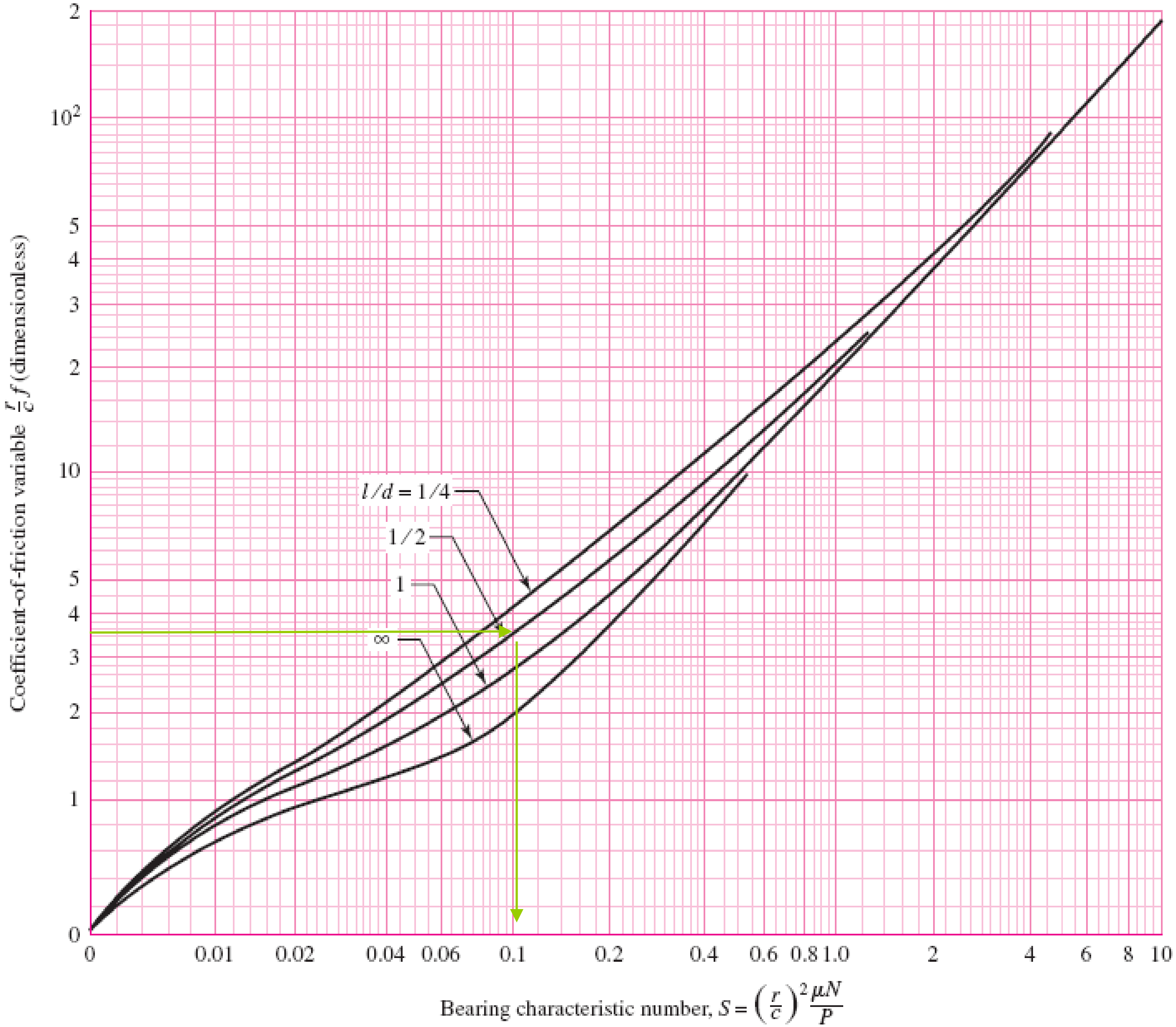
$$F_{fric} = W \times f$$

where only f is unknown

$$f = \frac{P_{loss}}{w \times r \times 2\pi N} = \frac{100}{3000 \times 0.05 \times 2\pi \times 15} = 0.00707$$







2
10^2
5
4
3
2
10
5
4
3
2
1
0
Coefficient-of-friction variable $\frac{r}{c}f$ (dimensionless)
$l/d = 1/4$
1/2
1
∞
0
0.01
0.02
0.04
0.06
0.1
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
Bearing characteristic number, $S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$

from this figure S can be determined.

$$\frac{r}{c} = \frac{1}{0.002} = 500 \rightarrow \frac{r}{c} f = 3.535 \quad \& \quad \frac{l}{d} = \frac{1}{2}; \quad \underline{\underline{S = 0.1}}$$

$$\frac{h_0}{c} = 0.21 \rightarrow h_0 = 0.21 \times c = 0.021\, mm \qquad \underline{\underline{h_0 = 21\, \mu m}}$$

b)

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} \qquad P = \frac{W}{ld} = \frac{3000}{50 \times 100} = 0.6 \;\; MPa$$

$$\mu = \frac{SP}{N\left(\frac{r}{c}\right)^2} = \frac{0.1 \times 0.6 \times 10^6}{15(500)^2} = 0.016\, Pa - \sec = \quad \underline{\underline{16\, mPa - \sec}}$$

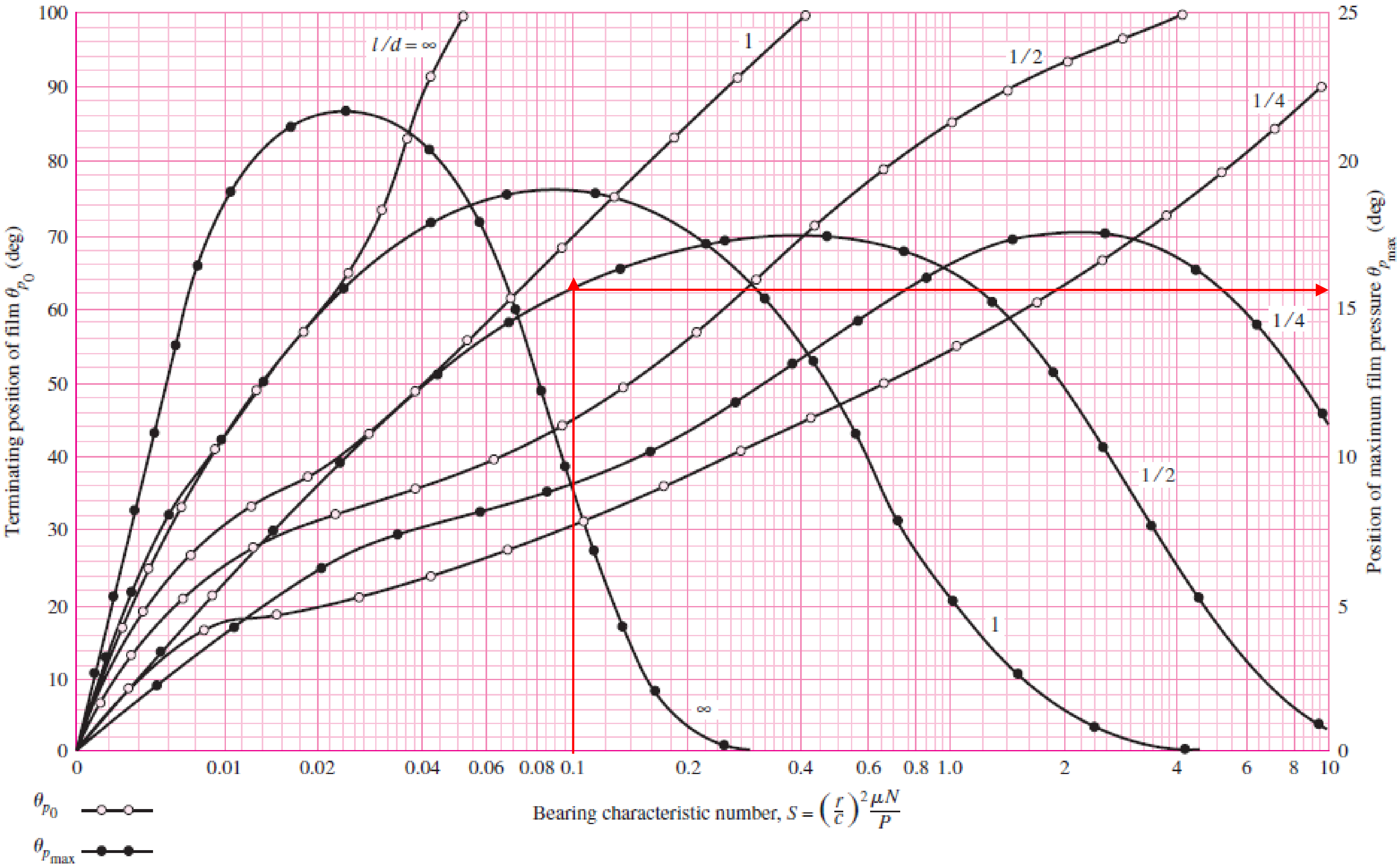
For operating temperature of 71 oC (*Tave*=71).
<u>SAE 30 is the suitable one to be used</u>.

SAE 30 is the surtable oil to be used.

c)

$$\Delta T_{^oC} = \frac{8.30 \times P}{\left[1 - \frac{1}{2} \times \frac{Q_s}{Q}\right]} \times \frac{\frac{r}{c} f}{\frac{Q}{rcNl}} = \frac{8.30 \times 0.6}{1 - \frac{1}{2} \times 0.87} \times \frac{3.535}{5.4} = 5.77 \quad {}^oC$$

$$\theta_{P_{\max}} = 16.5^o \qquad \frac{P}{P_{\max}} = 0.27 \qquad P_{\max} = \frac{0.6}{0.27} = 2.22 \quad MPa$$



100
90
80
70
60
50
40
30
20
10
0
25
20
15
10
5
0
Terminating position of film θ_{p_0} (deg)
Position of maximum film pressure $\theta_{p_{max}}$ (deg)
$l/d = \infty$
1
1/2
1/4
1/4
1/2
1
∞
0
0.01
0.02
0.04
0.06
0.08
0.1
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
θ_{p_0}
$\theta_{p_{max}}$
Bearing characteristic number, $S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$

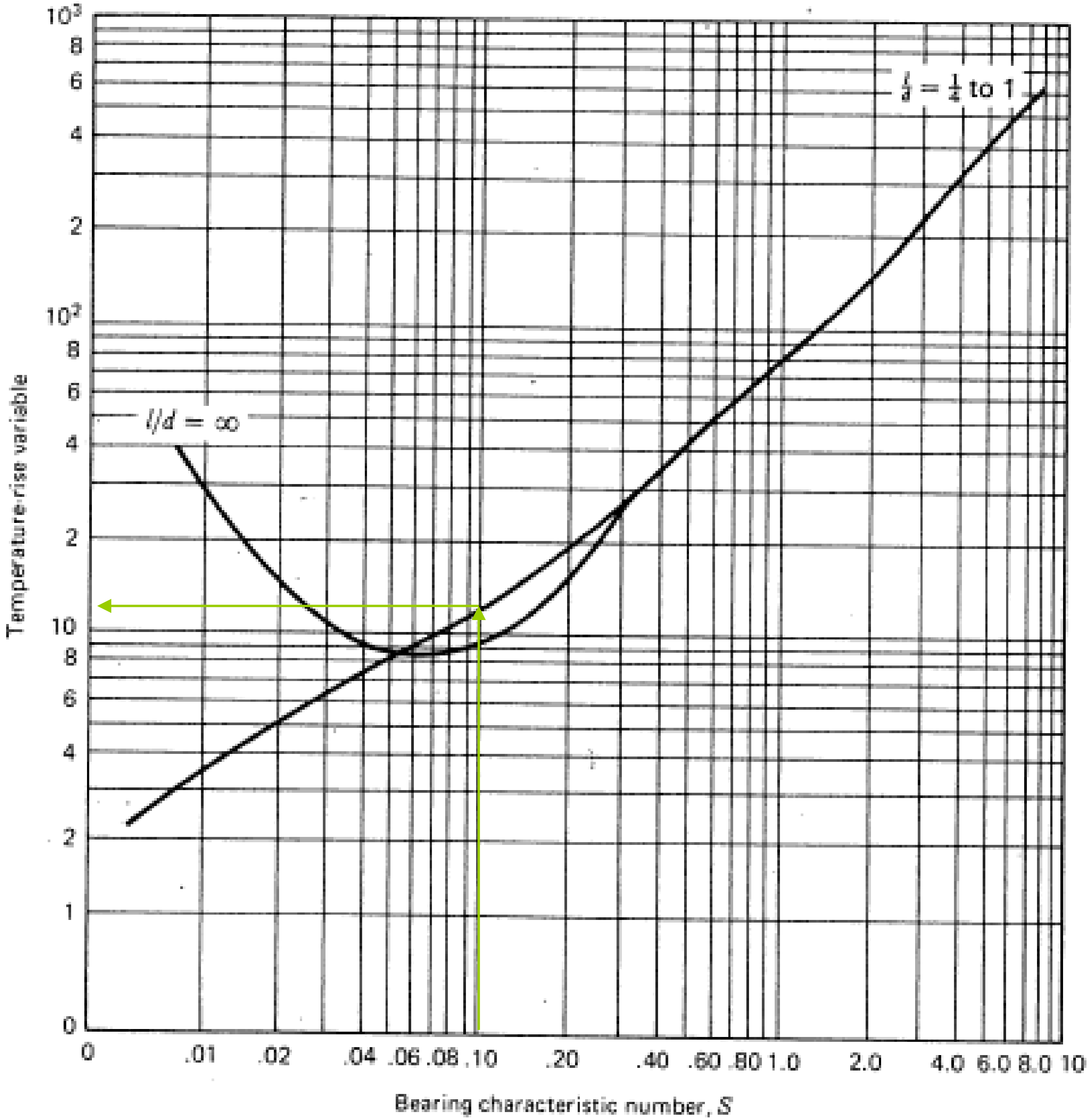


FIGURE 12-12 Chart for temperature-rise variable $T(\text{var}) = \gamma C_H \Delta T/P$. In plotting this chart it was found that the curves for $l/d = \frac{1}{4}, \frac{1}{2}$, and 1 were so close together that they could not be distinguished from a single curve.

Based on eqn.

$$\Delta T_{o_C} = T_{\text{var}} \frac{P}{\gamma\ C_H}$$

S=0.1, l/d=1/2

Tvar=14

P=0.6 MPa

861kg/m3

1760J/kg-C

$$\Delta T_{o_C} = 14 * \frac{P}{\gamma\ C_H} = 5.5$$

Example 6:

A journal which is turned and operates in bored and reamed cast bronze bearing has a diameter of 50 mm and a length of 25 mm.
The speed of journal is 1200 rpm. If SAE 20 oil of an inlet temperature of 45 ^{o}C is used for optimum (minimum) power loss.
Determine:
a)The temperature rise of the oil
b)The maximum load that can be supported by this bearing.
c)Total oil flow and the side flow.
d)The coefficient of friction.
e)The maximum oil pressure and its angular location.
f)The terminating position of the oil film.

Solution:
Turned journal
Bored and reamed cost bronze bearing
d=50 mm l=25 mm → l/d= 1/2
N= 1200 rpm = 20 rps
SAE 20 oil T_i =45 ^{o} C

T=?

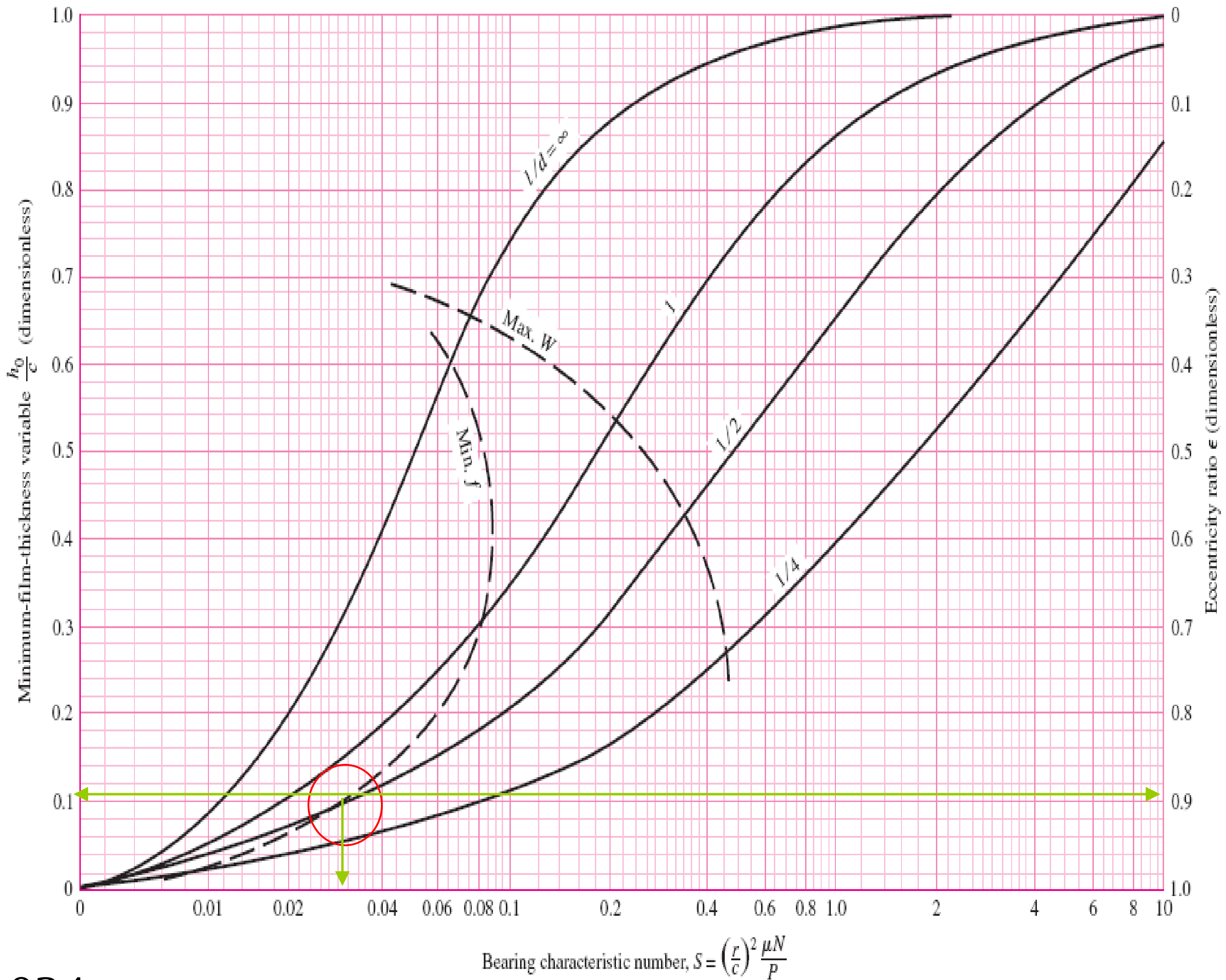
W=?

Q?,Qs?

f=?

etc

From the graph for minimum power loss



For $\frac{l}{d} = \frac{1}{2}$

$$\frac{h_0}{c} = 0.11 \quad \text{and} \quad S = 0.034$$

a) From the graph for minimum power loss

For $\frac{l}{d} = \frac{1}{2}$

$\frac{h_0}{c} = 0.11$ and $S = 0.034$

P=W/(ld)=?,
W=?

$$\Delta T_{o_C} = \frac{8.30 \times P}{\left[1 - \frac{1}{2} \times \frac{Q_s}{Q}\right]} \times \frac{\frac{r}{c} f}{\frac{Q}{rcNl}}$$

or

$$\Delta T = T_{\text{var}} \frac{P}{\gamma C_H}$$

For $S = 0.034$

$$\frac{l}{d} = \frac{1}{2} \qquad \frac{r}{c} f = 1.7 \quad \& \quad \frac{Q}{rcNl} = 5.65 \quad \& \quad \frac{Q_s}{Q} = 0.94$$

T_v =7.

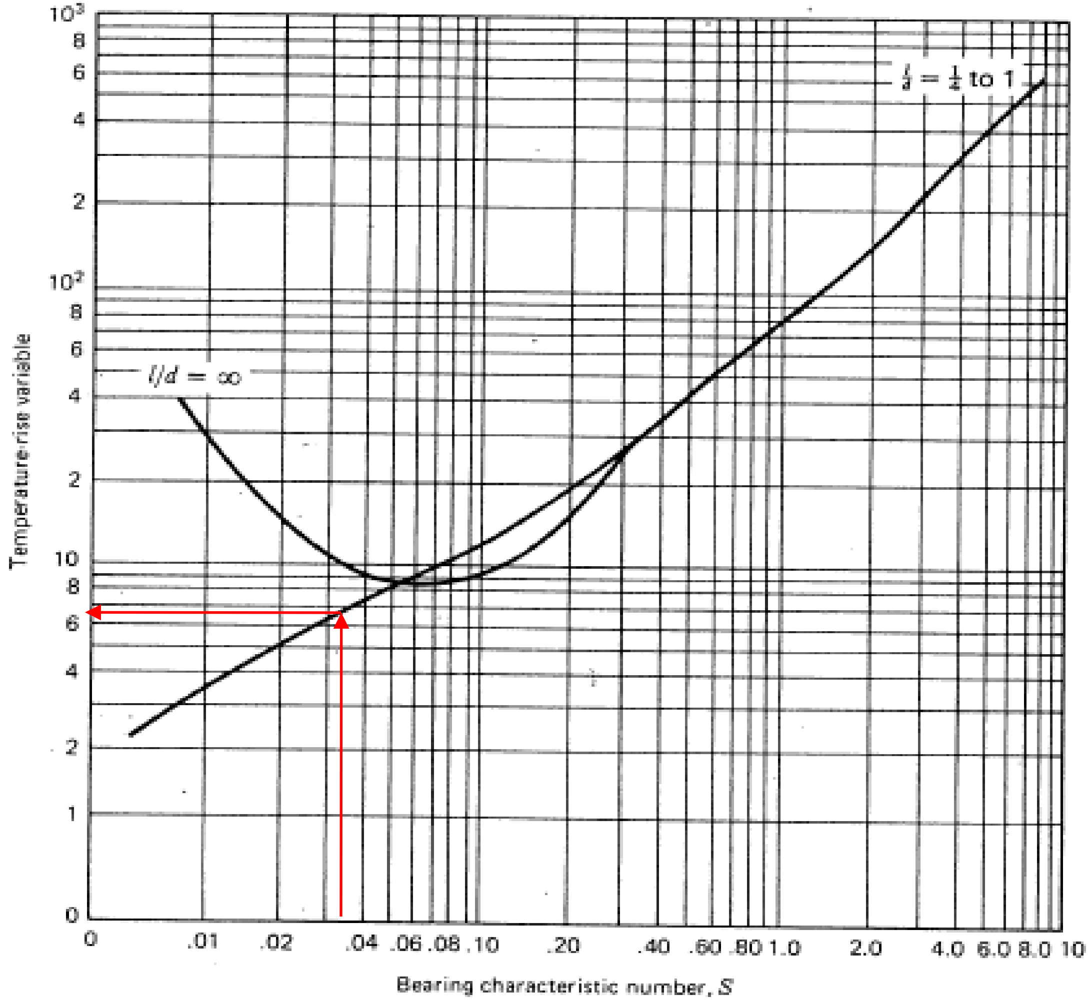
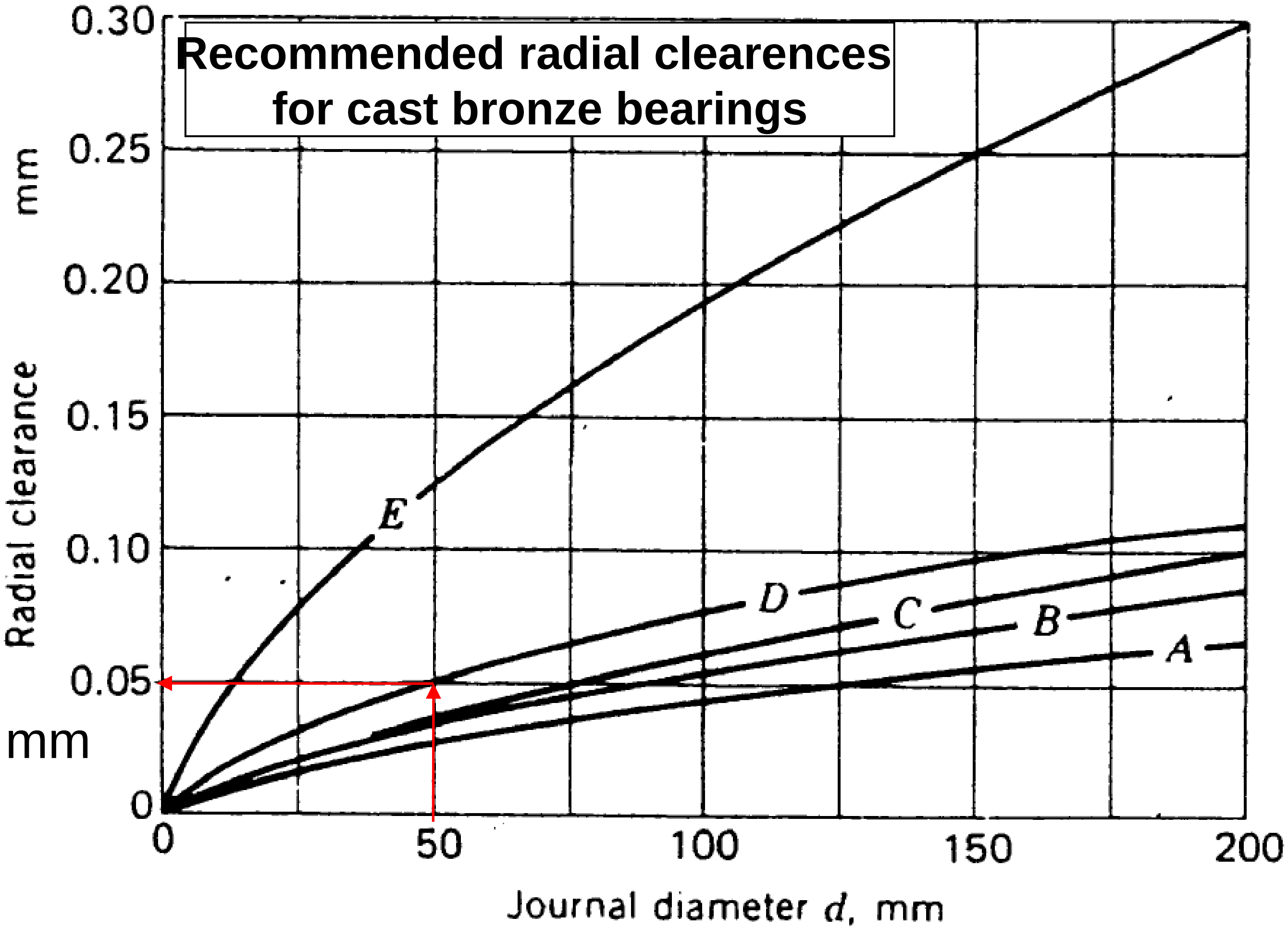


FIGURE 12-12 Chart for temperature-rise variable $T(\text{var}) = \gamma C_H \Delta T/P$. In plotting this chart it was found that the curves for $l/d = \frac{1}{4}, \frac{1}{2}$, and 1 were so close together that they could not be distinguished from a single curve.


Recommended radial clearences
for cast bronze bearings
0.30
0.25
0.20
0.15
0.10
0.05
0
Radial clearance mm
E
D
C
B
A
0
50
100
150
200
Journal diameter d, mm

C_{min} =0.05 mm

Since ΔT depends on P ;

$$\Delta T = T_{\text{var}} \frac{P}{\gamma C_H}$$

and also P depends on μ

$$P = \left(\frac{r}{c}\right)^2 \frac{\mu N}{S}$$

and μ depends on ΔT

$$T_{ave} = T_{in} + \frac{\Delta T}{2}$$

we will use trial & error method for ΔT ^oC

let ΔT =16 ^oC →

$$T_{ave} = 45 + \frac{16}{2} = 53\ ^oC \rightarrow$$

$$\mu = 24 \times 10^{-3}\ Pa - \sec \quad \text{for SEA 20 oil}$$

Then

$$P = \left(\frac{50/2}{0.05}\right)^2 \frac{24 \times 10^{-3} \times 20}{0.034} = 3.525\ MPa$$

$$\Delta T_{^oC} = T_{\text{var}} \frac{P}{\gamma C_H} = 7 \times \frac{3.525 \times 10^6}{861 \times 1760} = 16.6\ ^oC \cong \text{assumed}\ \ \Delta\text{T}\ \ \text{OK}$$

b) $W = ?, = P \times (l \times d) = 3.525 \times 10^6 \times (0.025 \times 0.05)$

$W = 4406\ N$

c) Q=?

Q_s=?

For $S = 0.34$

$\frac{l'}{d} = \frac{1}{2}$

$$\frac{Q}{rcNl} = 5.65 \rightarrow Q = 5.65 \times 25 \times 0.05 \times 20 \times 50 = 3528\ mm^3/\sec$$

$$\frac{Q_s}{Q} = 0.94 \quad \rightarrow Q_s = 0.94 \times 3528 = 3317\ mm^3/\sec$$

d) $\frac{r}{c} f = 1.7 \quad \rightarrow \quad f = 1.7 \frac{0.05}{25} = 0.0034$

e) $\frac{P}{P_{\max}} = 0.27 \quad \rightarrow P_{\max} = \frac{P}{0.2} = \frac{3.525}{0.2} = 17.62\ \mu Pa \qquad \theta_{P_{\max}} \cong 12^o$

f) $\theta_{P_0} \cong 34^o$

g) $h_0 = ? \quad \frac{h_0}{c} = 0.11 \rightarrow h_0 = 0.11 \times 0.05 = 0.0055\ mm \qquad \underline{\underline{h_0 = 5.5\ \mu m}}$