ME 308
MACHINE ELEMENTS II

<u>CHAPTER 7</u>

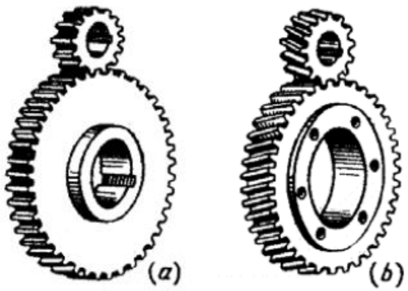
BEVEL GEARS

This chapter is about those gears which transmit power between <u>non-parallel shafts</u> of both

- intersecting and
- non-intersecting

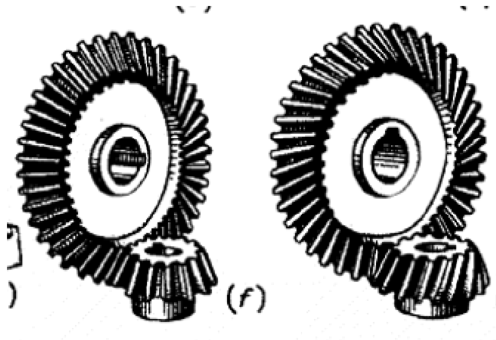
types

DIFFERENT TYPES OF GEARS AND SHAFT CONFIGURATIONS



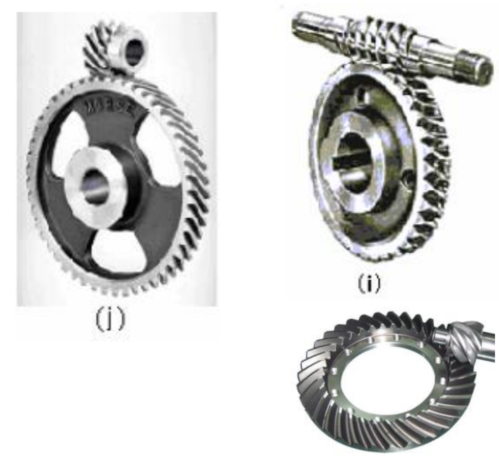
- spur gears and
- helical gears

are used in <u>parallel shafts</u> configuration to transmit power (motion and torque)



- Straight bevel gears and
- Spiral bevel gears

are used in <u>non-parallel intersecting shafts</u> configuration with 90^o shaft angle usually



- Crossed helical gears
- worm gears and
- hypoid bevel gears

are used in <u>non-parallel and non-intersecting shafts</u> configuration

7. BEVEL GEARS

Bevel gears, shown in Fig. 7.1, have teeth formed on conical surfaces and are used mostly for transmitting power and motion between intersecting shafts.

The figure actually illustrates *straight-tooth (spur) bevel gears.*

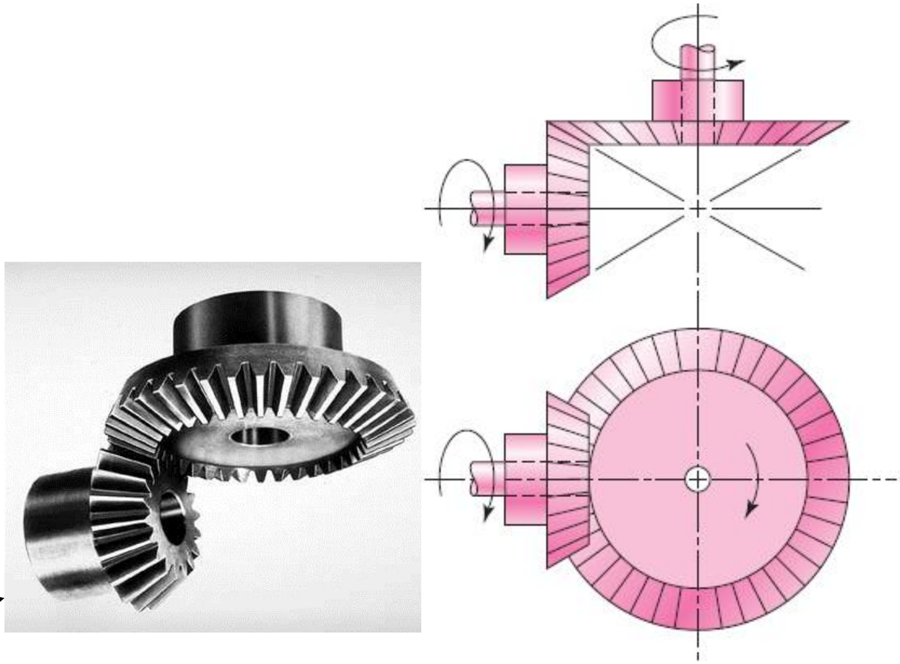
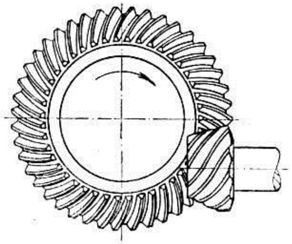


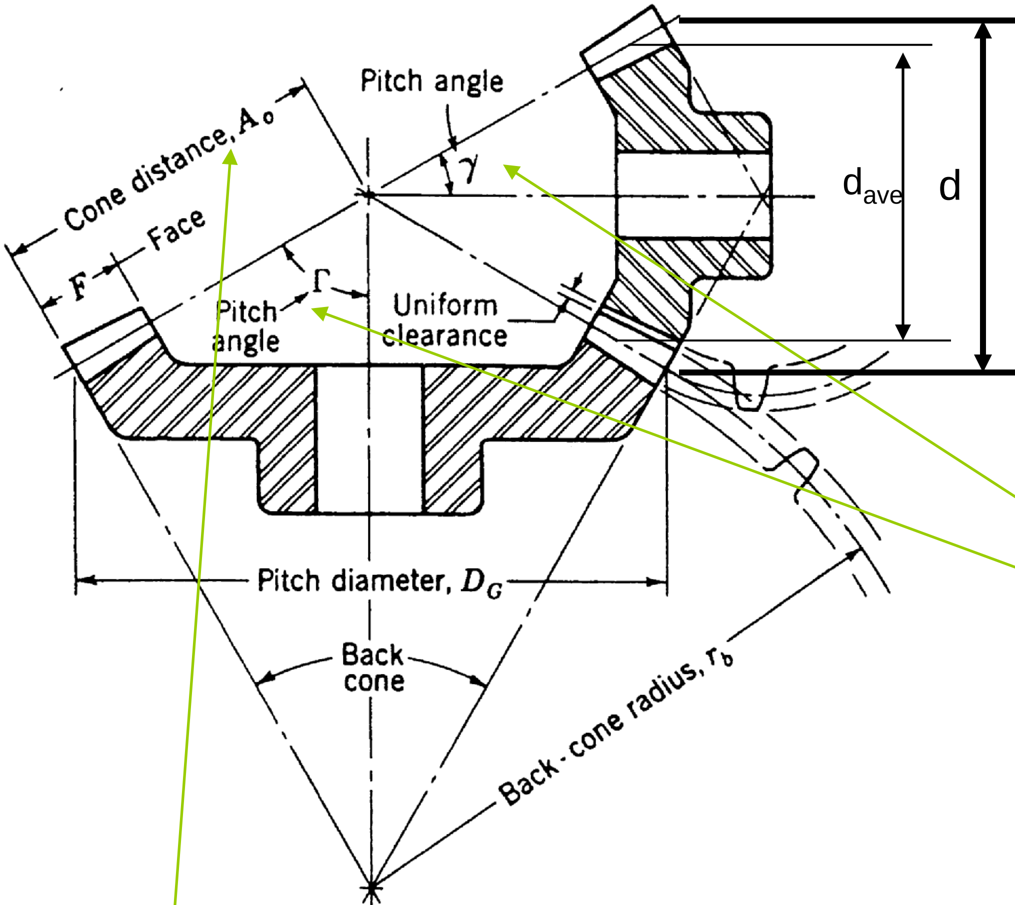
Fig.7.1 Bevel gears are used to transmit rotary motion between intersecting shafts.

Spiral bevel gears are cut so that the tooth is no longer straight, but forms a circular arc.

Hypoid gears are quite similar to spiral bevel gears except that the shafts are offset hence nonintersecting.



STRAIGHT BEVEL GEAR KINEMATICS



$$r_{ave} = d_{ave} / 2$$

$$d_{ave} = d - F * Sin(pitchangle)$$

γ is pitch angle of pinion

Γ is pitch angle of gear

$$\tan\gamma = \frac{N_P}{N_G}\left(= \frac{d_P}{d_G} = \frac{N_P}{N_G} \right)$$

$$\tan\Gamma = \frac{N_G}{N_P}$$

$$m_G = \frac{\mathrm{N_G}}{\mathrm{N_P}} \quad \text{gear ratio}$$

$$\text{Face width} = \frac{A_0}{3} \quad \text{or} \quad 10m$$

whichever is smaller

(usually used in design work)

(not used in analysis work)

$$A_0 = \sqrt{\left(\frac{d_P}{2}\right)^2 + \left(\frac{d_g}{2}\right)^2} = \frac{m}{2}\sqrt{N_p^2 + N_g^2}$$

$$m_G = \frac{N_G}{N_P}$$

STRAIGHT BEVEL GEAR KINEMATICS

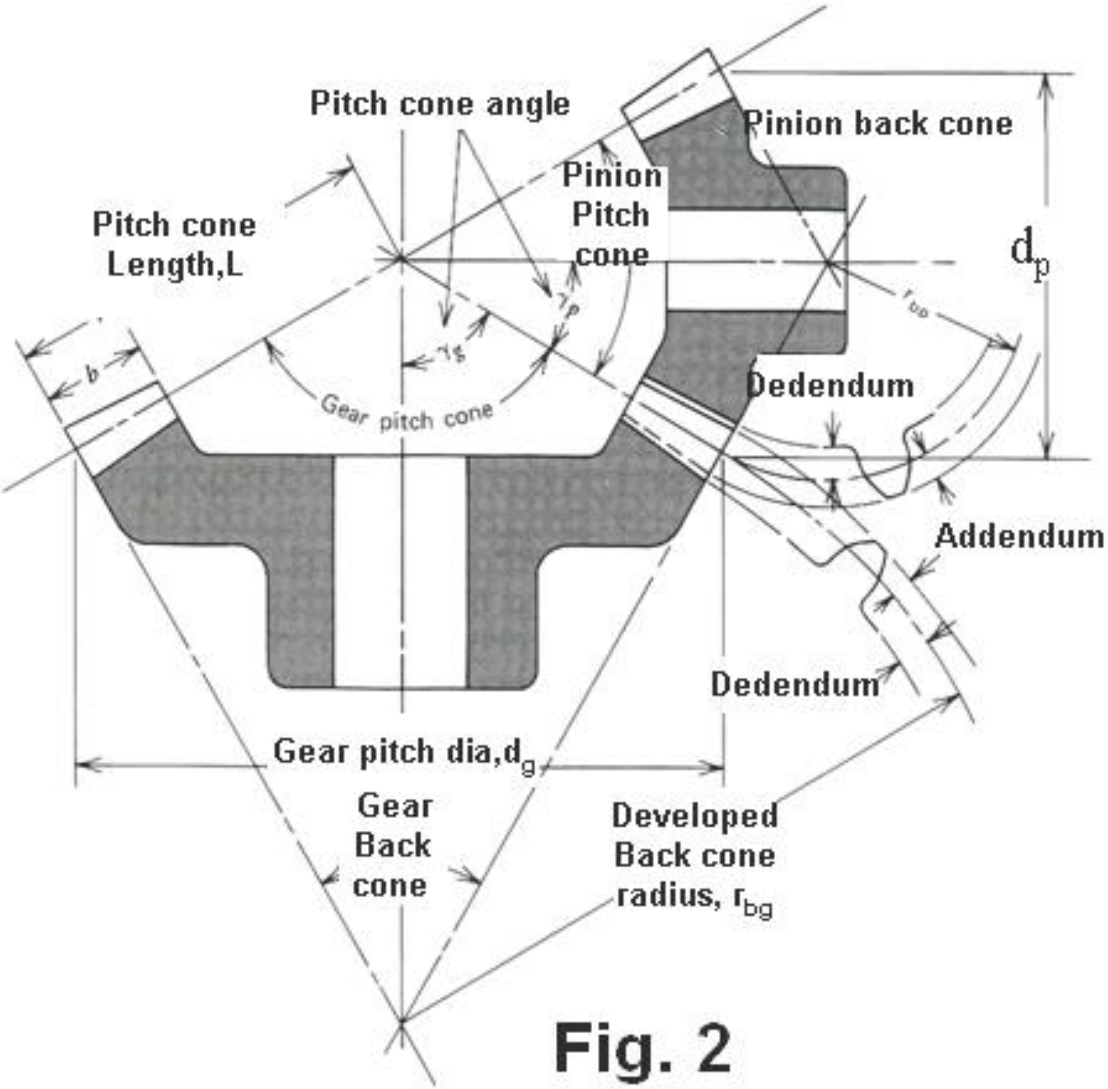
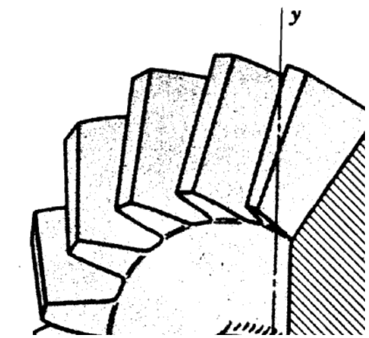


Fig. 2



m is the module at largest end of the tooth

d_p and d_g are the pitch diameters of pinion and gear at large end.

$$d_{ave} = d - F * Sin\left(pitchangle\right)$$

$$W_t = \frac{\text{Power}}{\text{pitchline velocity}}$$

$$W_t = \frac{P\left(watt\right)}{V_{ave}\left(m/s\right)} = \frac{60 \times P}{\pi d_{ave} n}$$

BEVEL GEAR FORCE ANALYSIS

Φ Pressure angle of bevel pinion-gear

γ (γ *or* Γ) is the pitch angle of the bevel on which force analysis done.

$$W_t = W\cos\phi \qquad W = \frac{W_t}{\cos\phi}$$

$$W_t = \frac{P(watt)}{V_{ave}(m/s)} = \frac{60\times P}{\pi d_{ave} n}$$

$$W_r = W\sin\phi\cos\gamma$$

$$= \frac{W_t}{\cos\phi}\sin\phi\cos\gamma$$

$$= W_t\tan\phi\cos\gamma$$

$$W_r = W_t\tan\phi\cos\gamma$$

$$W_a = W_t\tan\phi\sin\gamma$$

(Γ if gear forces are analysed)

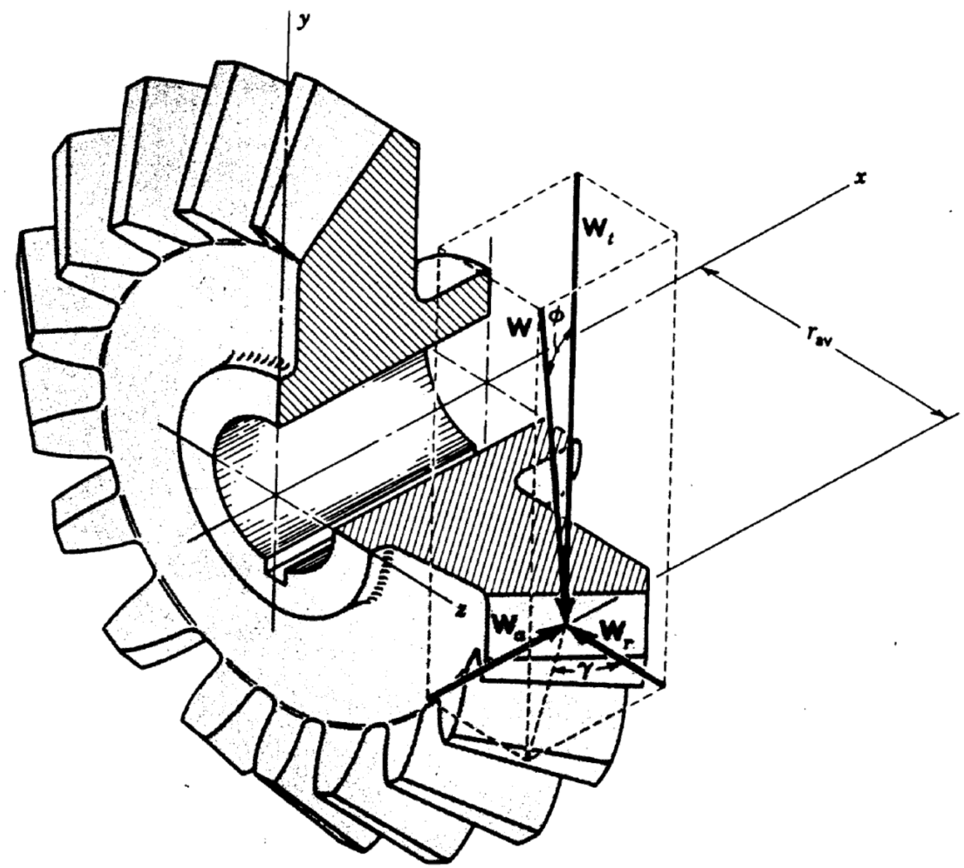


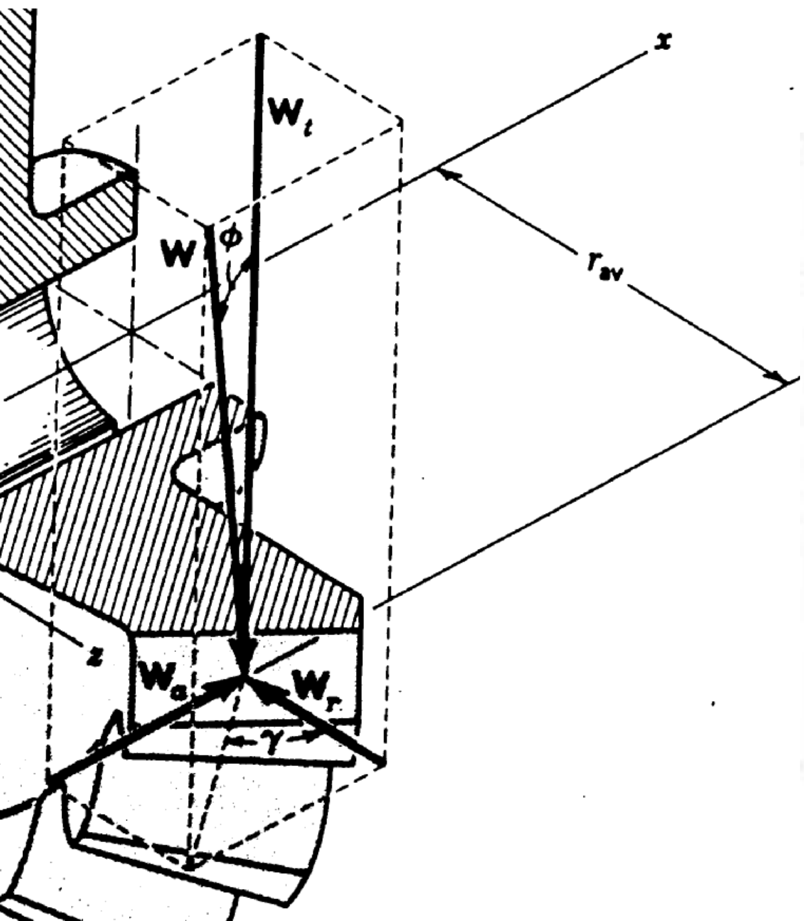
FIGURE 14-22 Bevel-gear tooth forces.

W_t is common to both pinion and gear

$$W_{t_{pin}} = W_{t_{gear}}$$
$$W_{r_{pin}} = W_{a_{gear}}$$
$$W_{a_{pin}} = W_{r_{gear}}$$

$$W_{t_{pin}} = \frac{Torque_{pin}}{r_{ave_{pin}}}$$

$$Torque_{pin} = \frac{Power\ Input}{n_{pin}(rad/\sec)}$$



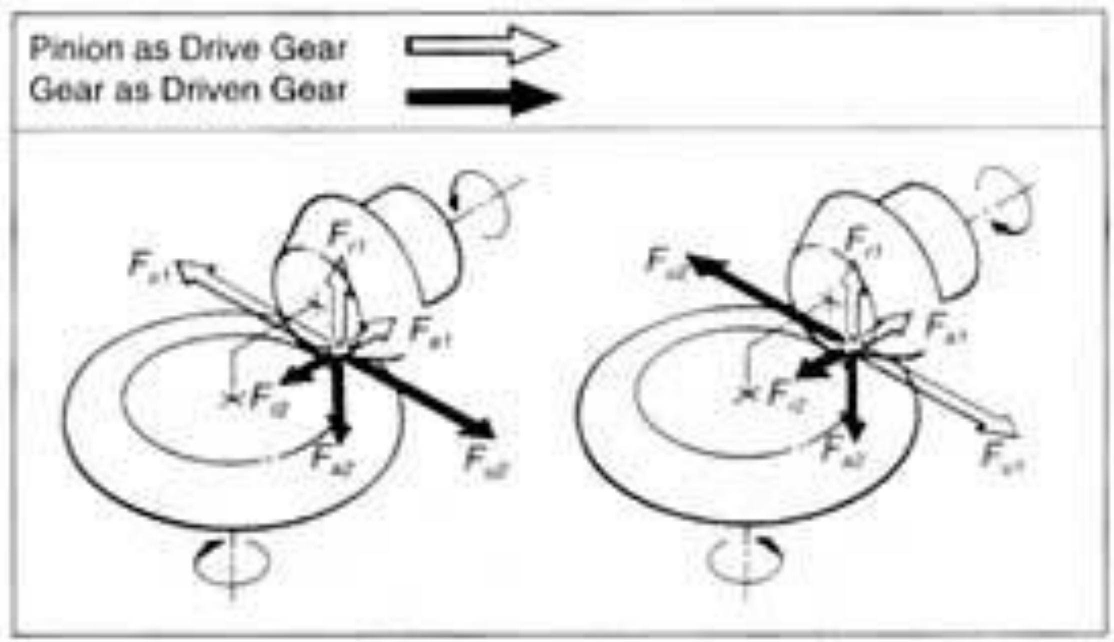
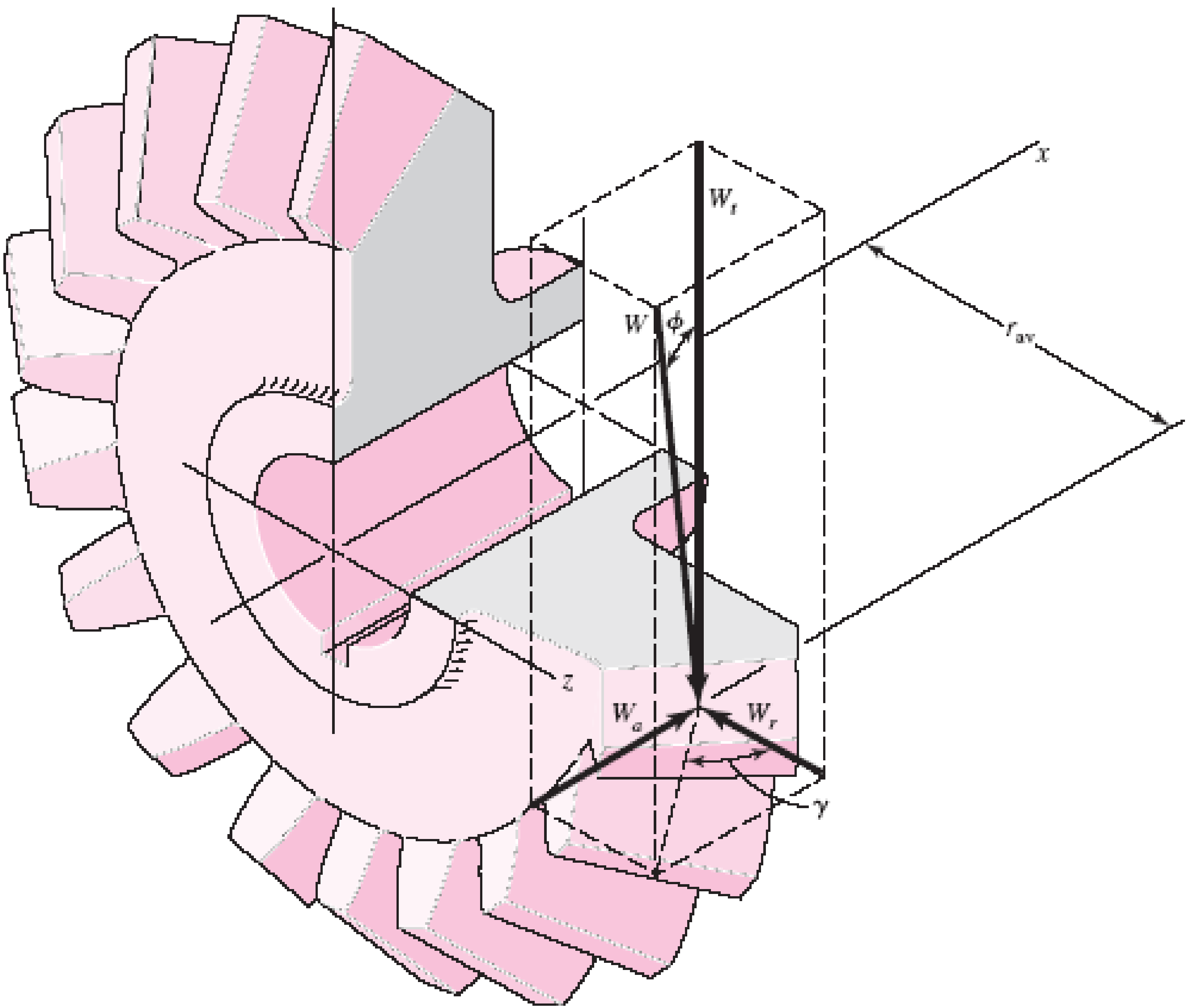


Fig. 16-6 Directions of Forces Acting on a Straight Bevel Gear Mesh



x
W_t
W
ϕ
r_{av}
z
W_a
W_r
γ

Bevel gear bending stress & strength

$$\sigma = \frac{W_t . P}{K_v \times F \times J}$$

spur gear formula is used for bevel gears too

P diametral pitch
m module

$$\sigma = \frac{W_t}{K_v \times F \times J \times m} \quad (\text{in SI})$$

W_t has to be calculated by using power/velocity equation

$$V = \frac{\pi d_{ave} n}{12} \ \text{or} = \frac{\pi d_{ave} n}{60}$$

$$(d_{ave} = d - F \sin \gamma(\Gamma))$$

Geometry factor J is given in fig. 14.25.

K_v as for spur gear $\frac{a}{a + \sqrt{V}}$ etc.

F face width of bevel gear (or pinion)

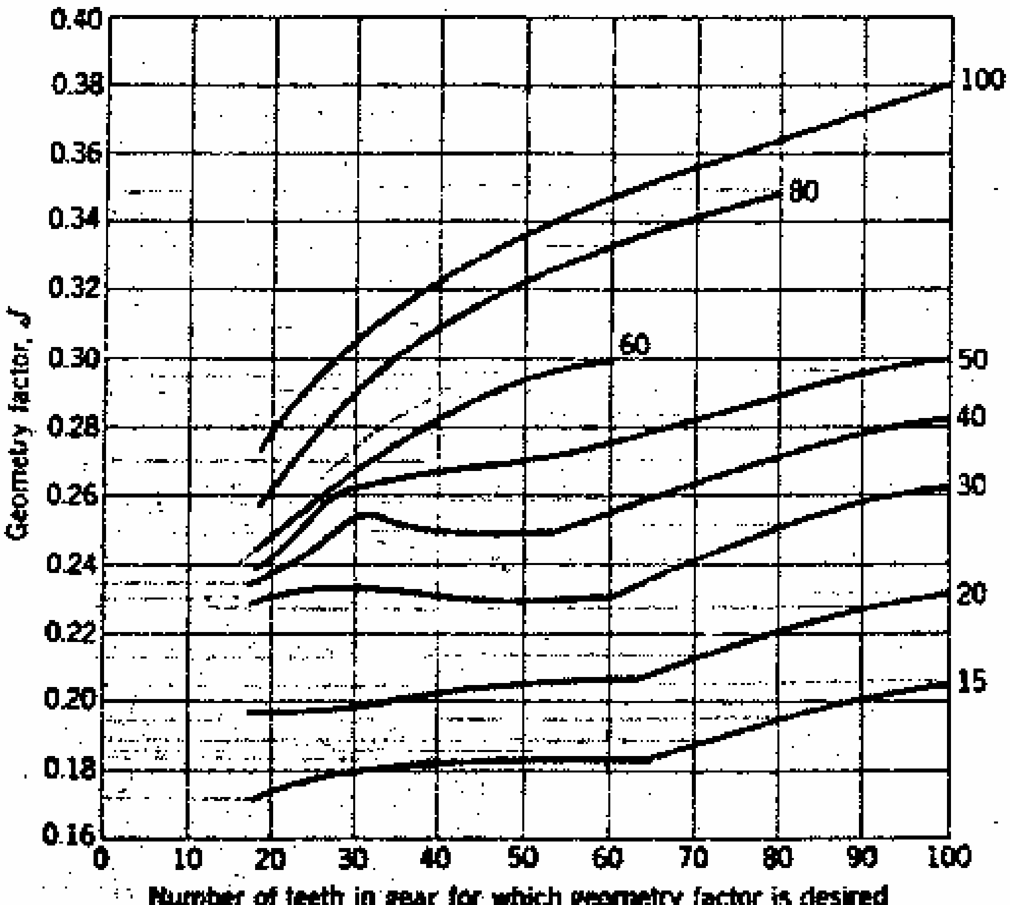


FIGURE 14-25 Geometry factors J for straight bevel gears; these are for a 90° shaft angle, 20° pressure angle, and a clearance of $c = 0.240/P$ in. (*AGMA Information Sheet 225.01.*)

Table 14-9 APPROXIMATE BEVEL-GEAR LOAD DISTRIBUTION FACTORS K_m AND C_m

Application	Both gears inboard	One gear outboard	Both gears outboard
General industrial	1.00–1.10	1.10–1.25	1.25–1.40
Automotive	1.00–1.10	1.10–1.25	
Aircraft	1.00–1.25	1.10–1.40	1.25–1.50

Source: *AGMA Information Sheet* 225.01, 1967, table 4.

Bevel gear bending stress & strength

Similarly $$n_G = \frac{S_e}{\sigma}$$

$$n_G = nK_oK_m$$

$$S_e = k_a \cdot k_b \cdot \ldots S_e^{'}$$

K_m given in table 14-9 for bevels.

K_o as for spur gears.

$$S_e^{'} = 0.5S_{ut}$$

(Sut) Up to 1400 MPa (200 kpsi)

$$S_e^{'} = 700\ MPa \quad (\text{or}\ 100\ kpsi)$$

Over 1400MPa (200kpsi)

for steels

Bevel Gear Surface Durability

$$\sigma_H = -C_P \sqrt{\frac{W_{tp}}{C_v F d_p I}}$$

Similar to spur gear equation

C_v at average diametral speed

$$V_{ave} = \frac{\pi d_{ave} n}{12} \qquad V_{ave} = \frac{\pi d_{ave} n}{60}$$

$$(d_{ave} = d - F \sin\gamma(\Gamma))$$

Elastic coefficient C_p is given in Table. 14.10 (for bevels).

Geometry factor *I* is given in fig. 14.26 (for bevels).

All other parameters as in spur gears of Chapter 13.

$$S_H = \frac{C_L C_H}{C_T C_R} S_C \quad \text{or} \quad \mathrm{S_H} = \sigma_H = C_P \sqrt{\frac{W_{tp}}{C_v F d_p I}}$$

$$S_C = 2.76 \times HB - 70 \; MPa$$

$$\& \quad n_G = \frac{W_{tp}}{W_t} \qquad n_G = n C_m C_o$$

Cm=Km is given Table. 14.9 for bevels.

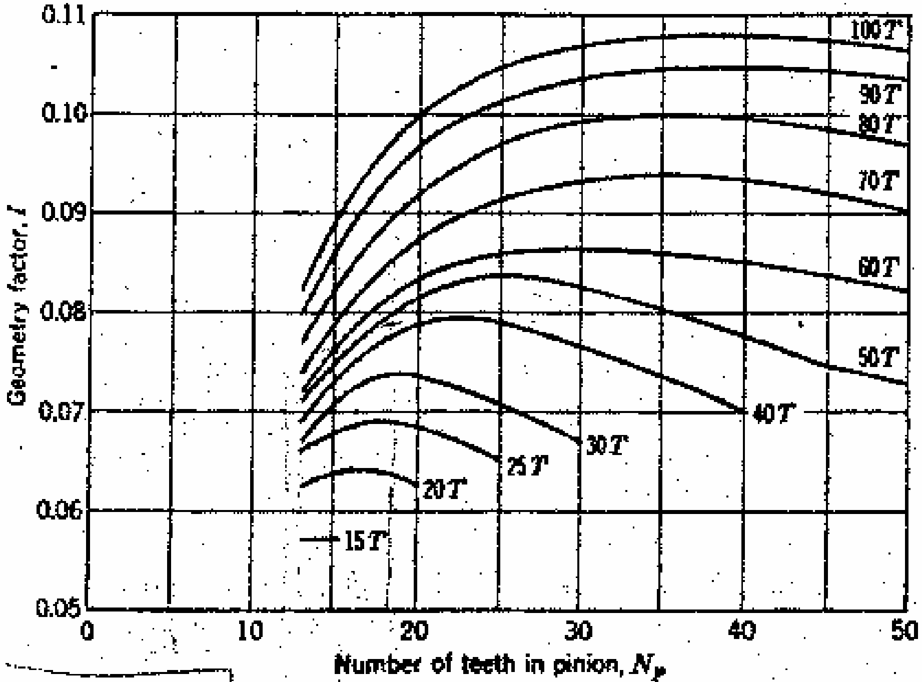


FIGURE 14-26 Geometry factors I for straight bevel gears of 20° pressure angle mounted at a 90° shaft angle. (*AGMA Information Sheet 212.02.*)

Table 14-10 VALUES OF THE ELASTIC COEFFICIENT C_p FOR BEVEL GEARS AND OTHERS WITH LOCALIZED CONTACT*

	Gear			
Pinion	**Steel**	**Cast iron**	**Aluminum bronze**	**Tin bronze**
Steel, $E = 30$ Mpsi	2800	2450	2400	2350
Cast iron, $E = 19$	2450	2250	2200	2150
Aluminum bronze, $E = 17.5$	2400	2200	2150	2100
Tin bronze, $E = 16$	2350	2150	2100	2050

Source: *AGMA Information Sheet* 212.02

* In each case the modulus of elasticity is in Mpsi.

Steel on Steel

$C_p = 2800 \sqrt{Mpsi}$

$C_p = 0.083 \times 2800$

$C_p = 232.5 \sqrt{MPa}$

Conversion factor = 0.083

Example: 1

$N_p = 15T$

$P = 6 teeth / inch \quad (m = 4.233\, mm)$

cold-drawn UNSG10180 steel without heat treatment.

teeth are hobed.

$n_{pm} = 900\ rpm$

pinion mounted outboard of its bearings.

$N_g = 60T$

material grade 30 CI

$F = 31.75\, mm\, (1.25'')$

$\phi = 20^0$

gear is outboard mounted,

shaft angle is 90^0 between pinion and gear.

Based on; <u>bending strength,</u> 50% reliability, n=4, general moderate use,
Determine the maximum safe hp capacity of this bevel gear set.

$$Power = W_t \times V_{P_{ave}}$$

$$W_t = ? \rightarrow \sigma = \frac{W_t}{FmJK_v} \rightarrow W_t = \sigma FmJK_v \qquad (1)$$

$$V_{P_{ave}} = ? \quad V_{P_{ave}} = \frac{\pi d_{ave} n}{60} \,.........\, (2)$$

In 1 a) $\sigma = ? \quad = \frac{S_e}{n_G} = \frac{k_a \cdot k_b \cdot ... S_e^{'}}{nK_o K_m}$

$$S_e^{'} = ? \ \text{pinion or gear?}$$

$S_{ep}^{'} = 0.5(S_{ut})?$ For pinion in appendix Sut=440 MPa <1400 MPa

$S_{eg}^{'} = 0.5(S_{ut})?$ For gear in appendix Sut=214 MPa < 1400 MPa

$$S_{ep}^{'} = 0.5(S_{ut}) = 0.5 \times 440 = 220\ MPa$$

$$S_{eg}^{'} = 0.45(S_{ut}) = 0.45 \times 214 = 96.3\ MPa$$

$\text{since} \quad S_{eg}^{'} < S_{ep}^{'} \rightarrow$ Design is based on gear tooth strength.

$k_a ......k_e$ are assumed to not effect Se too much

Thus:

$$k_a = 0.8, \quad k_b = 0.925, \quad k_c = 1.0,$$

$$k_d = 1.0, \quad k_e = 1.0 \quad k_f = 1.33,$$

$$S_e^{'} = 0.45(S_{ut}) = 0.45 \times 214 = 96.3\ MPa$$

$$n = 4, \quad K_o = 1.0, \quad K_m = 1.3,$$

$$\sigma = \frac{96.3}{4 \times 1.0 \times 1.33} \cong 18.5 MPa$$

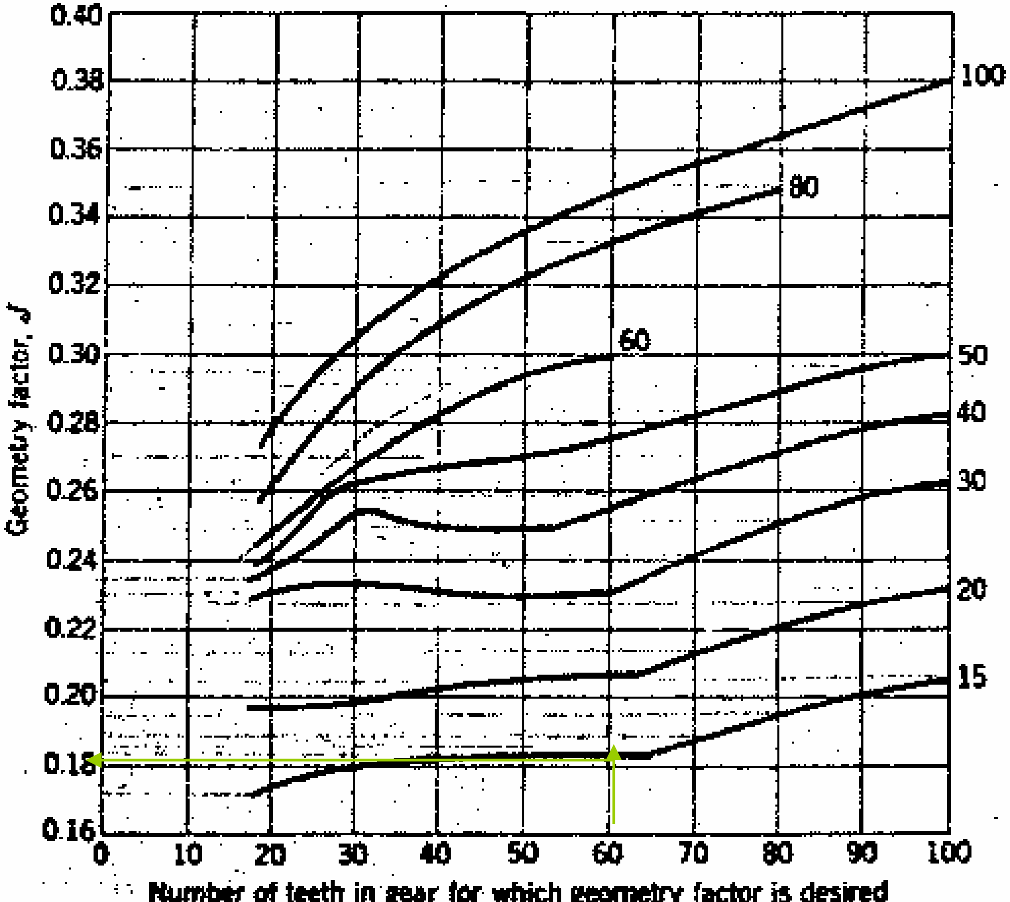


FIGURE 14-25 Geometry factors J for straight bevel gears; these are for a 90° shaft angle, 20° pressure angle, and a clearance of $c = 0.240/P$ in. (*AGMA Information Sheet 225.01.*)

b) *J=?* = 0.185 from Fig. 14.25 (J for bevel gears) 60T vs 15T

Table 14-9 APPROXIMATE BEVEL-GEAR LOAD DISTRIBUTION FACTORS K_m AND C_m

Application	Both gears inboard	One gear outboard	Both gears outboard
General industrial	1.00–1.10	1.10–1.25	1.25–1.40
Automotive	1.00–1.10	1.10–1.25	
Aircraft	1.00–1.25	1.10–1.40	1.25–1.50

Source: *AGMA Information Sheet* 225.01, 1967, table 4.

c) $K_v = \frac{3}{3+\sqrt{V}}$ for CI

$$V_{ave} = \frac{\pi d_{ave} n}{60}$$

$$d_{ave} = d_p - F \sin\gamma$$

$$\tan\gamma = \frac{N_p}{N_g} = \frac{15}{60}, \quad \gamma = 14^0, \quad d_p = m \cdot N_p = 4.233 \times 15 = 63.5\, mm$$

$$d_{ave} = 63.5 - 31.75 \times \sin 14 = 55.82\, mm$$

$$V_{ave} = \frac{\pi \times 55.82 \times 900}{60} = 2.63 \text{ m/sec}$$

$$K_v = 0.532$$

$$Power = W_t \times V_{ave}$$

$$W_t = \sigma \times F \times m \times J \times K_v = 18.5 \times 31.75 \times 4.233 \times 0.185 \times 0.532$$

$$W_t = 244\ N$$

$$\underline{P = 244 \times 2.63 = 643.5\ watt}$$

Example:

Tp=14, Tg=21, m=2.5mm & F=14mm ,

gear material is through hardened steel with HB=550.

determine the factors of safety based on

- a- bending strength
- b- surface durability

If 6 hp is transmitted through pinion at a speed of 3500 rpm.

a- bending strength

$$n = n_G / K_o K_m$$

$$n_G = \frac{S_e}{\sigma}$$

b- contact strength

$$n = n_G / C_m C_o$$

$$n_G = \frac{W_{tp}}{W_t}$$

a- bending strength

Assume uniform-to-medium shock ;

And both gears out board

$$K_o = 1.25$$

$$K_m = 1.5 (T14-9)$$

$$n = n_G / K_o K_m$$

$$n_G = \frac{S_e}{\sigma}$$

$$S_e = k_a . k_b . k_c . k_d . k_e . k_f . S_e ' \rightarrow$$

$$S_e ' = 0.5 S_{ut} ? \rightarrow S_{ut} ? \quad 3.45\text{HB}=1897\text{MPa}$$

$$\mathrm{S_e} ' = 700 MPa \text{ since } \mathrm{S_{ut}} > 1400 MPa$$

$$S_e = 617 MPa (\text{for both pinion and gear})$$

$$\sigma = \frac{W_t}{FmJK_v} \rightarrow W_t = ?$$

$$Power = W_t \times V_{ave} \quad \rightarrow W_t = Power / V_{ave}$$

$$Power = 6 * 746 = 4476 Watt$$

J=0.195, Fig. 14-25 based on pinion(14T)

- $V = \dfrac{\pi d_{ave} x n}{60} \quad \mathrm{d}_{\mathrm{ave}} = d_p = F \sin\gamma$

$$d_p = T_p x m = 14x2.5 = 35mm$$

$$\gamma = tg^{-1}\left(\frac{T_p}{T_g}\right) = tg^{-1}(14/21) = 33.7^o$$

$$d_{ave} = 35 - 14x \sin 33.7 = 27.23$$

$$V = \pi x 27.23 x 3500 / 60 = 4.99 m / \sec$$

$$W_t = 4476 / 4.99 = 897 N$$

$$K_V = \frac{50}{50 + \sqrt{200V}} = 0.612$$

$$T = \frac{897}{14x2.5x0.195x0.612} = 214.75 MPa$$

$$n_G = \frac{S_e}{T} = \frac{617}{215} = 2.87 \rightarrow n = \frac{2.87}{1.25x1.5} = 1.53$$

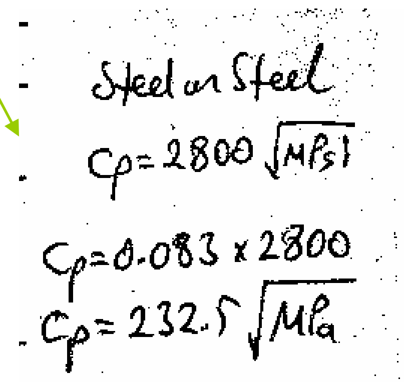
Safe based on bending strength

b- surface (durability) strength

$$n = \frac{n_G}{C_o C_m} \qquad \mathrm{n}_{\mathrm{G}} = \frac{W_{tp}\,?}{W_t \rightarrow 897}$$

$$C_o = K_o = 1.25$$

$$C_m = K_m = 1.5 \;\; \mathrm{W}_{\mathrm{tp}}\,? \rightarrow T_H = C_P \sqrt{\frac{W_{tp}\,?}{F d_p I C_V}} = S_H = \frac{C.C}{C.C} S_C$$



$$S_C = 2.76 HB - 70 = 1448 MPa$$

$$S_H = 1448 MPa$$

$$W_{tp} = \left(\frac{S_H}{C_P} \right)^2 x F d_P I C_V = 736 M$$

$$n_G = \frac{736}{897} = 0.82 < 1.0 \rightarrow \textit{fails} \text{ based on contact strength}$$

$$\mathrm{n} = \frac{\mathrm{n}_{\mathrm{G}}}{C_o C_m} = \frac{0.82}{1.25 x 1.5} = 0.43$$

Example(14-30):

For a pair of bevel gears with

Tp=14, Tg=21, P=10 t/inch, and F=0.55 inch

material is through hardened steel with HB=550.

Determine the factors of safety based on

a)bending strength

b)Surface durability,

if 6 hp is transmitted through pinion at a speed of 3500 rpm.

a)

$$n = n_G / K_o K_m \qquad n_G = \frac{S_e}{\sigma}$$

b)

$$n = n_G / C_m C_o \qquad n_G = \frac{W_{tp}}{W_t}$$

$$S_{ut} = 500 * HB (psi)$$
$$= 500 * 550 = 275000 \text{psi} = 275 \text{kpsi}$$

$$S_c = 0.4 \text{HB} - 10 \, (\text{kpsi})$$
$$= 0.4 * 550 - 10 = 210 \text{kpsi}$$

A)

$$S_e' = 100kpsi \quad \text{since } S_{ut} > 200kpsi$$

$$S_e = k_a k_b \ldots\ldots S_e' = 0.6x0.972x1.0x1.0x1.0x1.5x100$$

$$S_e = 87.5kpsi$$

$$\sigma = \frac{W_t P}{K_V F J}$$

$$W_t = \frac{hpx33000}{V_{ave}}$$

$$V_{ave} = \frac{\pi d_{ave} n_{pin}}{12}$$

$$\gamma = tg^{-1}\left(\frac{N_P}{N_G}\right) = 33.7^o$$

$$V_{ave} = \frac{\pi x1.095x3500}{12}$$

$$d_{ave} = \left(\frac{N_p}{p}\right) - 0.55\sin\gamma$$

$$V_{ave} = 1003rpm$$

$$K_V = \frac{50}{50 + \sqrt{1003}} = 0.612$$

$$d_{ave} = \frac{14}{10} - 0.55x\sin 33.7$$

$$d_{ave} = 1.095''$$

$$W_t = \frac{6x33000}{1003} = 197.5lb$$

$$\sigma = \frac{197.5x10}{0.612x0.55x0.195} = 30090\,psi$$

$$n_G = \frac{87500}{30090} = 2.90$$

$$n = \frac{n_G}{K_o K_m} = \frac{2.90}{1.25x1.5} = 1.55$$

There is a satisfactory tooth bending strength.

b)

$$S_H = C_P \sqrt{\frac{W_{tp}}{C_V F d_p I}}$$

$$S_H \frac{C_L.C_H}{C_R.C_T} S_C = \frac{1.0x1.0}{1.0x1.0} 210 = 210kpsi$$

$$C_p = 2300 \text{ steel-steel}$$

$$\mathrm{C_V} = K_V = 0.612$$

$$d_p = \frac{N_p}{P} = \frac{14}{10} = 1.4$$

$$I = 0.063(\,fig.14-26)$$

$$\text{thus } \mathrm{W_{tp}} = (S_H / C_P)^2 x C_V x F x d_p x I$$

$$W_{tp} = 247.5lb$$

$$n_G = \frac{W_{tp}}{W_t} = \frac{247.5}{197.5} = 1.253$$

$$n = \frac{n_G}{C_o C_m} = \frac{1.253}{1.25x1.5} = 0.668$$

Unsatisfactory tooth surface strength (or less power should be transmitted).

Example:

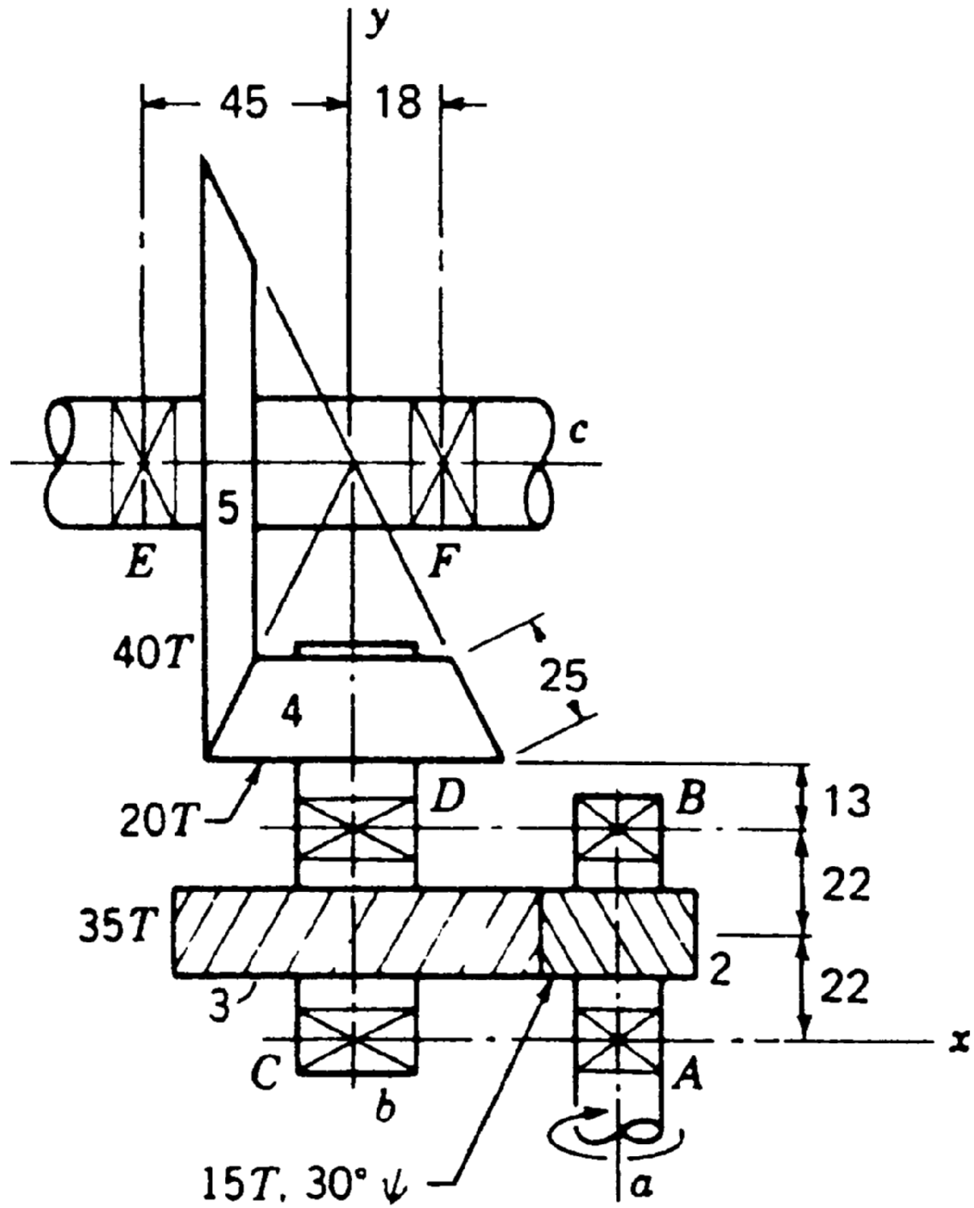
The gear train shown is composed of a pair of helical gears & a pair of straight bevel gears.

Bevel gears have 3 mm module, 20º press. angle and 25 mm face width as shown.

Helical gears have a normal module of 2mm and normal press. Angle of 20⁰.

Shaft C (of the bevel gear) is the output of the train and it delivers 4.5 kw to the load at a speed of 370 rpm.

Input to the train is at shaft 'a' of the helical pinion and in a direction of CW.

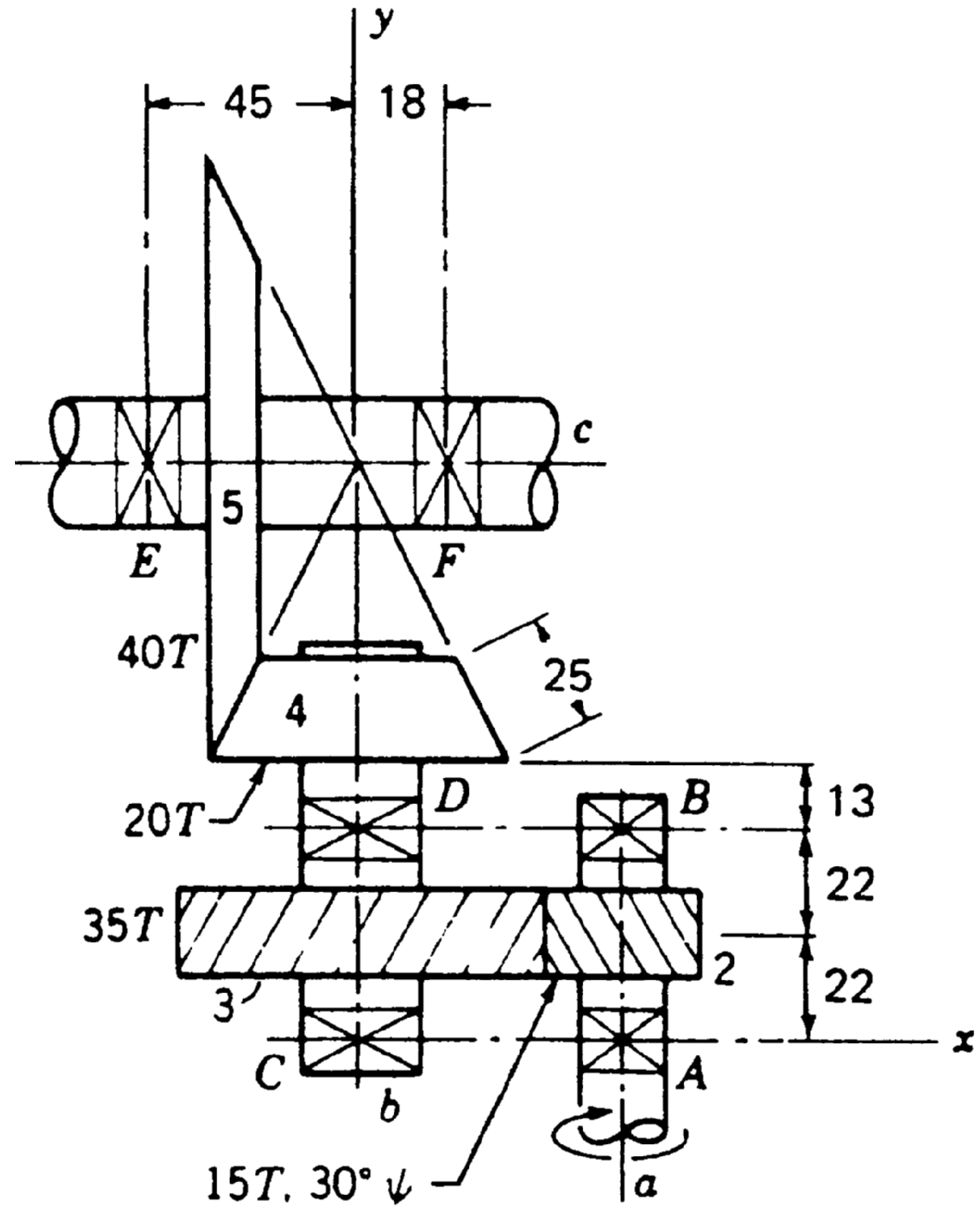


a) If bearing E is to take up both thrust & radial load while bearing F is to take only radial load ,

determine the magnitude & direction of the forces which these bearings exert against the shaft C.

b) Find the forces exerted by bearings C&D on shaft b.

c) Which of these bearings should take the thrust?



$$P_{out} = 4.5\,kw = W_t \times V_{ave}$$
$$= T \times w = (W_t \times r_{ave}) \times w\,(rad/\sec)$$

$$tgr = \frac{N_g}{N_p}$$

$$r = 63.43^o$$

$$W_t = \frac{P_{out}}{V = \frac{\pi d_{ave} n}{60}}$$

$$r_{ave} = \frac{d_{ave}}{2}$$

$$d_{ave} = d_g - F.\sin(r)$$
$$= \mathrm{N_g} xm - F \sin 63.43$$
$$= 40x3-25\sin 63.43$$
$$\mathrm{d_{ave}} = 97.64nn$$
$$r_{ave} = 48.82mm$$

$$W_t = \frac{4500x60}{0.04882x2x\pi x370}$$

$$W_t = 2379N$$

$$W_r = W_t.tg\phi\cos r = 387.3N$$

$$W_a = W_t tg\phi\sin r = 774.4N$$

These are the tooth forces on bevel gear

(Wrp=Wag=774.4N and Wap=Wrg=387.3N)

$$\sum F_x = 0 \qquad \mathrm{E_t} = W_a = 774.4N$$

$$\sum M_E = 0(in\ \text{x-y plane}) \qquad 45\mathrm{xW_r} = (45+18)F_y + r_{ave_g}\, xW_a$$

$$F_y = \frac{45xW_r - r_{ave_g}\, xW_a}{45+18} = \frac{45x387.3 - 48.82x774.4}{63} = -323.45N$$

$$\sum F_y = 0 \qquad \mathrm{E_y} + F_y = W_r$$

$$\mathrm{E_y} - 323.45 = 387.3N$$

$$E_y = 710.75N$$

??

In x-z plane

$$\sum M_E = 0 \qquad 45\mathrm{xW_t} = (45+18)F_z \rightarrow F_z = 1699.3N$$

$$\sum F_z = 0 \qquad \mathrm{W_t} = E_z + F_z \rightarrow E_Z = 679.7N$$

$$\mathrm{E_{thrust}} = W_a = 774.4N(\text{ in +x direction})$$

$$\mathrm{E_{radial}} = \sqrt{E_y{}^2 + E_z{}^2} = 983.44N$$

$$F_{radial} = \sum \sqrt{F_y{}^2 + F_z{}^2} = 1729.8N$$

These are the loads exerted by the bearings E&F On shaft C.

b)

$$W_a = 387.3N$$

$$W_r = 774.4N$$

$$W_t = 2379N$$

These are the tooth forces on bevel pinion

$$r_h = \frac{1}{2}(m_t N_h) = \frac{1}{2} x \frac{2}{\cos 30} x35 = 40.414 \text{ mm}$$

$$m_t = \frac{m_n}{\cos\psi}$$

$$W_t x r_{ave_{pin}} = P_t x r_h$$

$$P_t = W_t \frac{r_{ave}}{r_h} = 2379 \frac{48.817/2}{40.414} = 1437N$$

$$P_r = P_t . tg\phi_t = P_t . tg\left[tg^{-1}\left(\frac{tg\phi_n}{\cos\psi}\right)\right]$$

$$P_r = 1437 xtg\left[tg^{-1}\left(\frac{tg20}{\cos 30}\right)\right] = 604N$$

$$P_a = P_t . tg\psi = 1437 xtg30 = 830N$$

Since
Wa &Pa are opposite &
Pa>Wa bearing can take the thrust load with fillet at D.

If Wa were larger than Pa than bearing C had to be used to take thrust load .