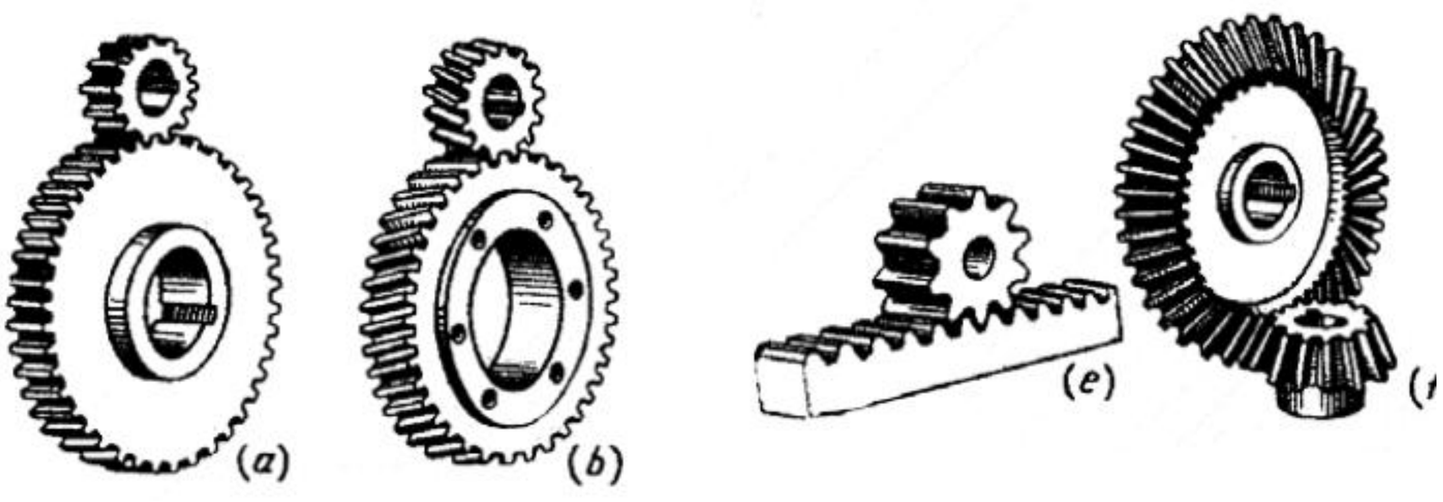
ME 308
MACHINE ELEMENTS II

CHAPTER 5

GEARS_part3



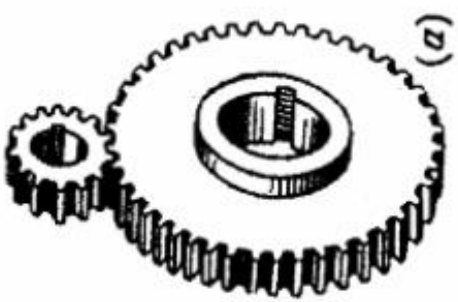
Gear trains are usually used to:

- increase speed & decrease torgue
- decrease speed & increase torgue
- change the direction of rotation.

A small gear driving a large gear is speed reducing gear train whereas a large gear driving a small gear is a speed increasing gear train

Gear trains are classified based on two criteria:

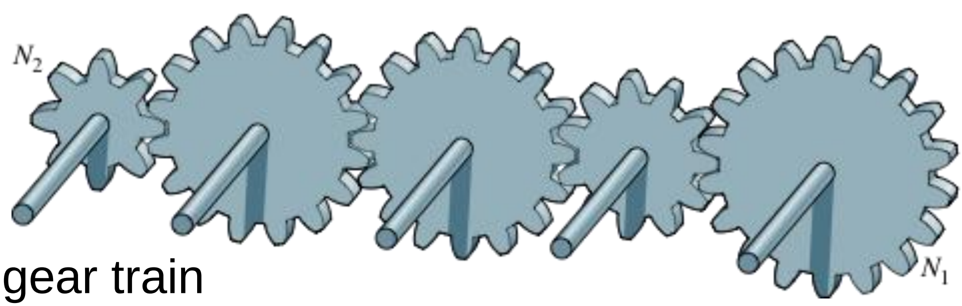
- number of stages of gear train
- positioning of the gears on shafts



There are two main types of gear trains based on number of stages

1) Single stage gear train is the one where there is only one driving and one driven gear

2) Multiple stage gear trains have more gear pairs in mesh at the same time .

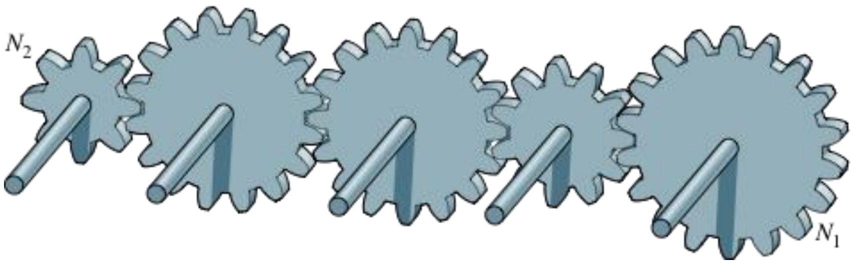


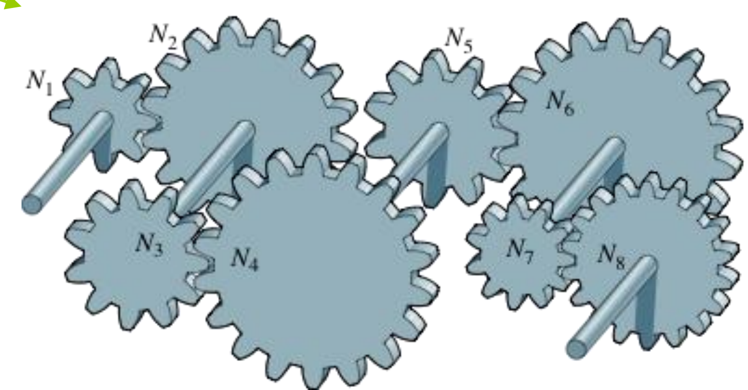
This is a 4stage gear train

There are two main types of gear trains based on positioning of the gears on shafts

1) Simple gear trains is the one where there is only one gear on each shaft (the same gear becomes both driving and driven gear on intermediary shafts)

2) compound gear trains have multiples of gears on intermediary shafts.





1) İn single stage gear trains speed ratio is inversly proportional to the radii of the gears:

Pitch line velocity=V=$w_1 r_1 = w_2 r_2$

$$\frac{W_1}{W_2}=\frac{r_2}{r_1}=\frac{mN_2/2}{mN_1/2}=\frac{N_2}{N_1}=\frac{\text{driven gear t. no}}{\text{driving gear t .no}}$$

$$W_2=\frac{N_1}{N_2}W_1=\frac{\text{driving tooth no}}{\text{driven t. no}}x\text{driving speed}$$

$$\mathrm{W}_{\text{last}}=\frac{-\text{driving t.no}}{\text{driven t.no}}xW_{first}$$

Now we also see that speed ratio is inversly proportional to the number of tooth of gears.


(a)

2) In multiple-stages trains speed ratio of each stage is inversly proportional to the (radii or) tooth numbers of the gears in that stage, hence

$$W_2 = \frac{-N_1}{N_2} W_1$$

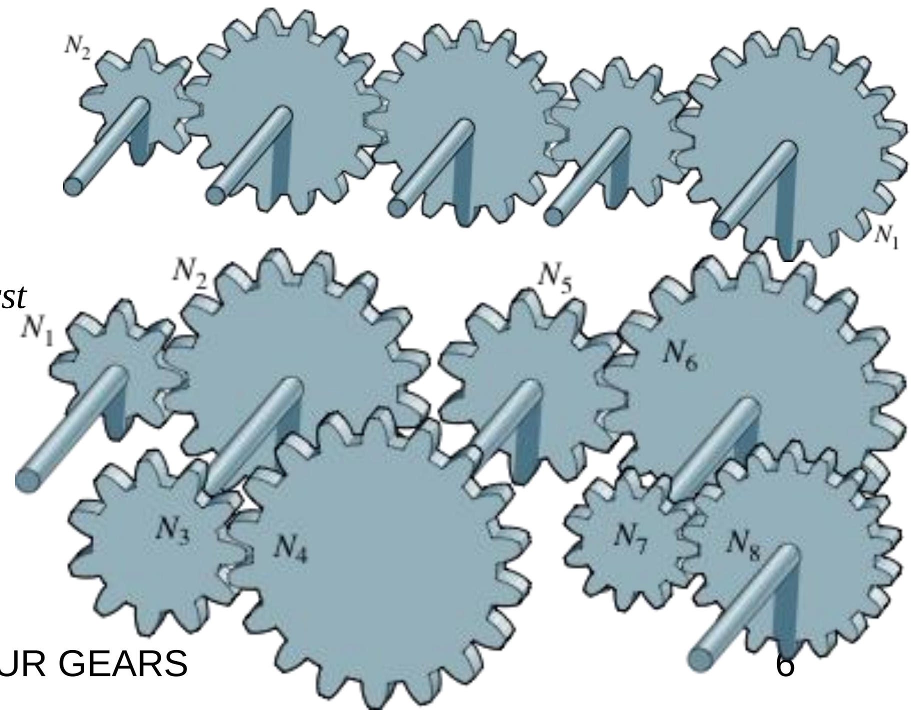
$$W_3 = \frac{-N_2}{N_3} W_2 \rightarrow \frac{-N_2}{N_3}\left(\frac{-N_1}{N_2} W_1\right) = \frac{-N_1}{N_2}\frac{-N_2}{N_3} W_1$$

$$W_3 = \frac{N_1 x N_2}{N_2 x N_3} x W_1 \rightarrow$$

$$W_{last} = \left(\frac{\text{prod. of driving t. n}}{\text{prod. driven t.n}}\right) x W_{first}$$

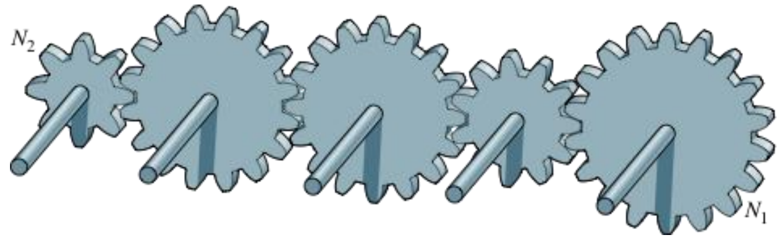
$$W_{last} = (\text{train value}) x W_{first}$$

$$W_{last} = e.W_{first}$$



A) For :simple gear trains (gears of each one on one shaft)

$$W_4 = \left(\frac{N_1}{N_2}\right)\left(\frac{N_2}{N_3}\right)\left(\frac{N_3}{N_4}\right) x W_1 = \frac{-N_1}{N_4} W_{1_1}$$



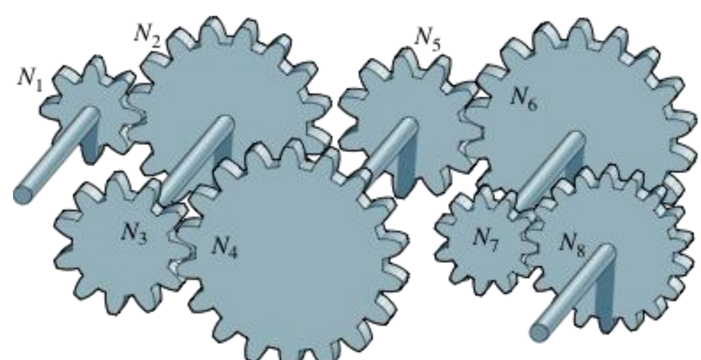
$$W_{last} = \frac{\text{tooth no on first gear}}{\text{tooth no on last gear}} x W_{first} = \frac{N_f}{N_L} x W_f$$

$$W_{last} = e.W_{first} \quad \text{e=train values}$$

B) For gears with more than one on each shaft (compound gear trains)

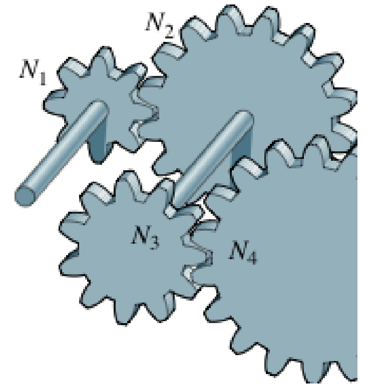
$$W_6 = \frac{N_1 x N_3 x N_5}{N_2 x N_4 x N_6} x W_1 = \frac{-N_1}{N_2} x \frac{-N_3}{N_4} x \frac{-N_5}{N_6} x W_1 = -e.W_f$$

$$W_{last} = \frac{\text{prod. of driving teeth no}}{\text{prod. of driven teeth no}} x W_{first}$$



Example: design **a two stage** compound spur gear train for an **overall ratio** of approximately 47:1

Specify tooth numbers for each gear in the train.



$e_o = T_{drvn} / T_{drvng} = T_2/T_1$

$e_o = 47/1 = e_1 * e_2$

$$\frac{n_i}{n_o} = 47 = e_1 x e_2 \ \text{ let } e_1 = e_2$$

$$\frac{W_{first}}{W_{last}} = 47:1$$

$$e_1 = \sqrt{47} \cong 6.855 (< 10)$$

a) for 20degrees pressure angle gears Tmin is theoretically 18tooth and practically 14

$$T_1 = 18 \rightarrow T_2 = e_1 x T_1 = 123.4?$$

$$T_1 = 19 \rightarrow T_2 = 130.25?$$

$$T_1 = 20 \rightarrow T_2 = 137.1?$$

$$T_1 = 21 \rightarrow T_2 = 143.955 \cong 144$$

If $T_2 = 144$

$$T_1 = 21 \rightarrow e_1 = \frac{144}{21} = 6.857$$

$$e_1 x e_2 = 47.020 \qquad \text{acceptable}$$

b)

T_1	$T_2 = T_1 x \sqrt{47}$
12	82.26
13	89.12
14	95.979
15	102.83
16	109.69
17	116.54
18	123.4
.	
.	

T1 and T2 have to be integer values

Nearest to integer (T2=96)

$$\frac{96}{14} x \frac{96}{14} = 47.020$$

$$\text{same as } \frac{144}{21} x \frac{144}{21} = 47.020$$

FORCE ANALYSIS IN SPUR GEAR SETS

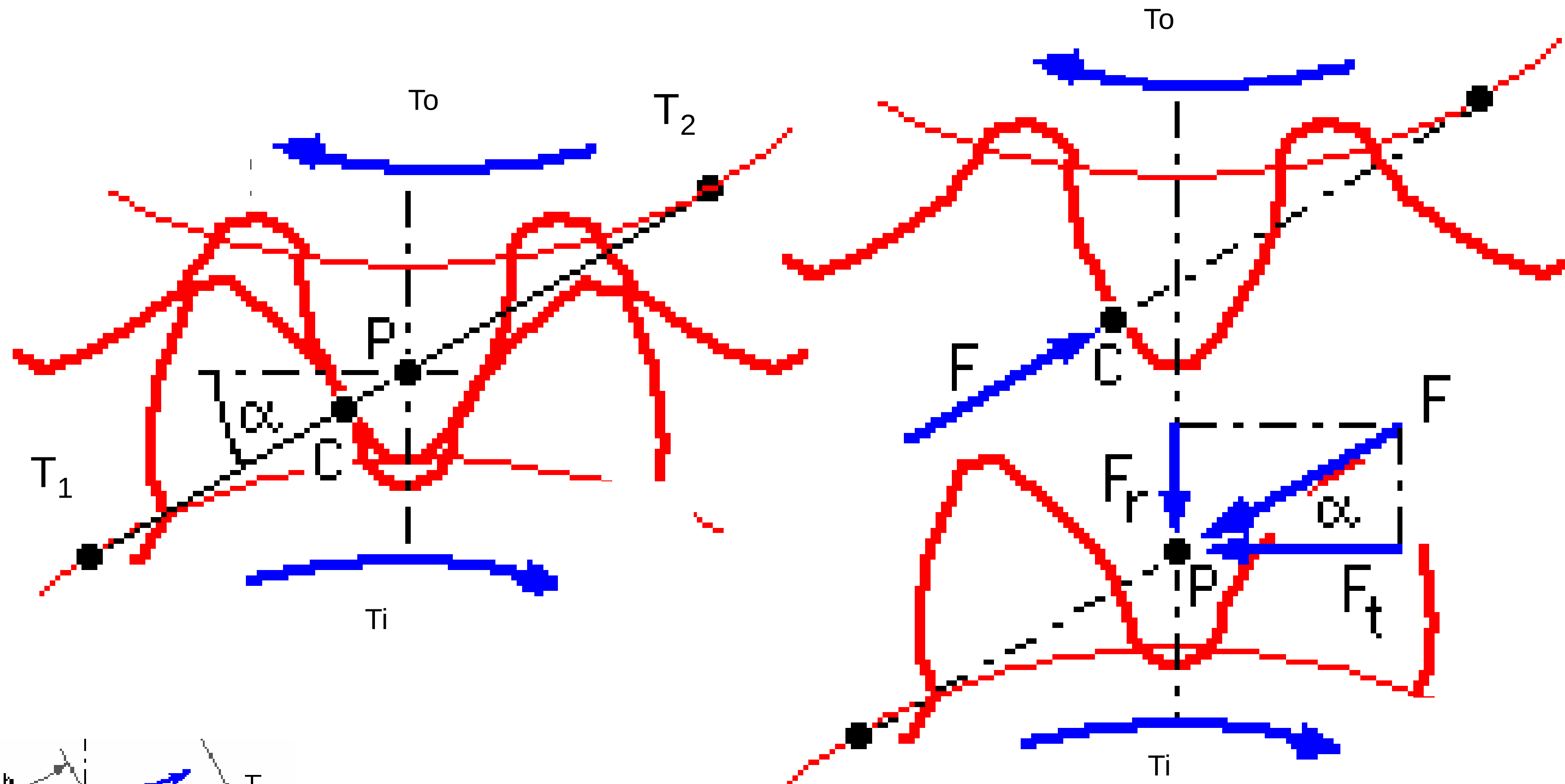


Fig. 3 Tooth forces

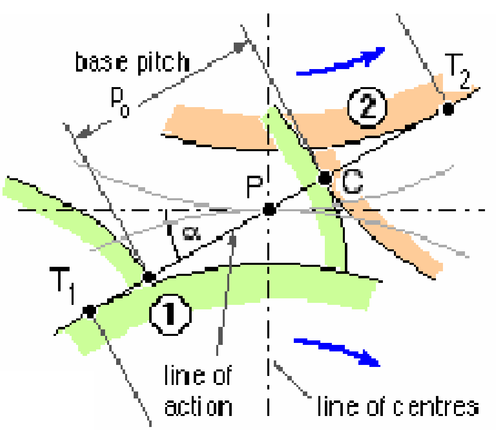
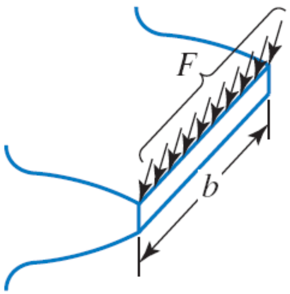
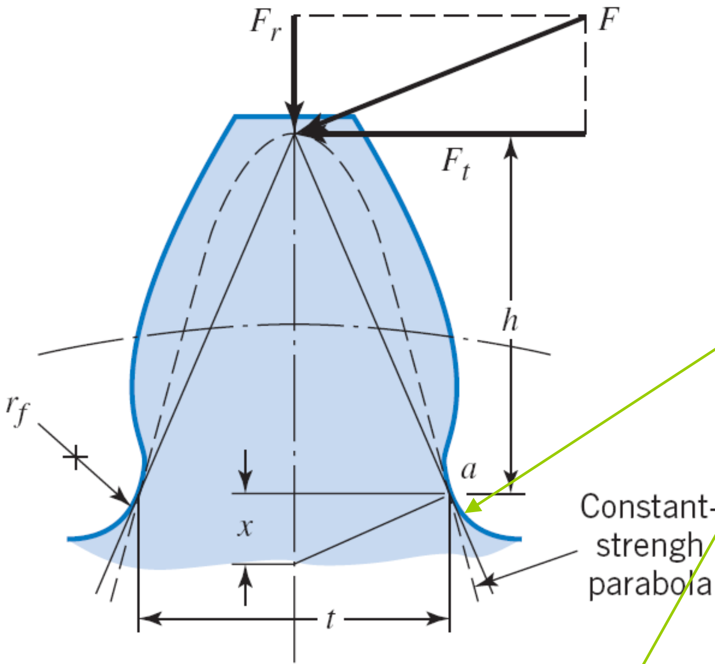


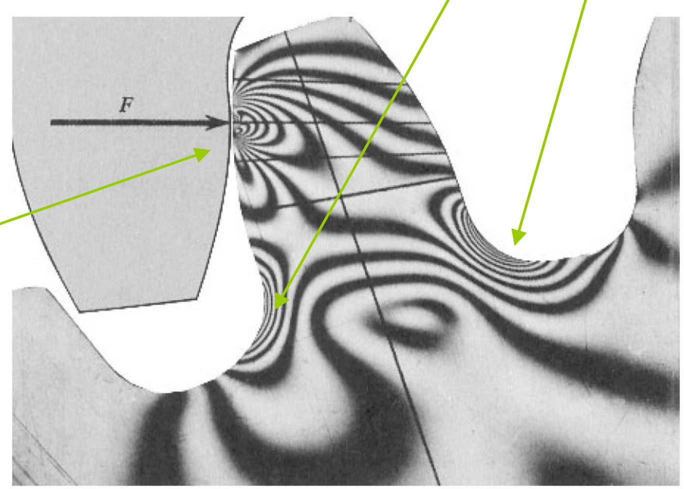
Fig 4 Involute tooth form

FORCE ANALYSIS IN SPUR GEAR SETS





Ft is the tangential component of the tooth force F and creates <u>bending stress at roots</u> of the cantilever type tooth



Tooth force F creates <u>contact (or Hertz) stresses</u> on the tooth surface

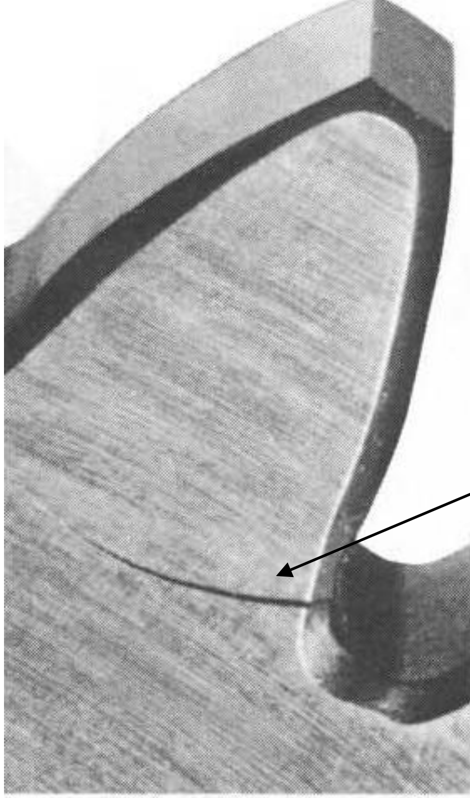
TYPES OF FAILURES OF SPUR GEARS

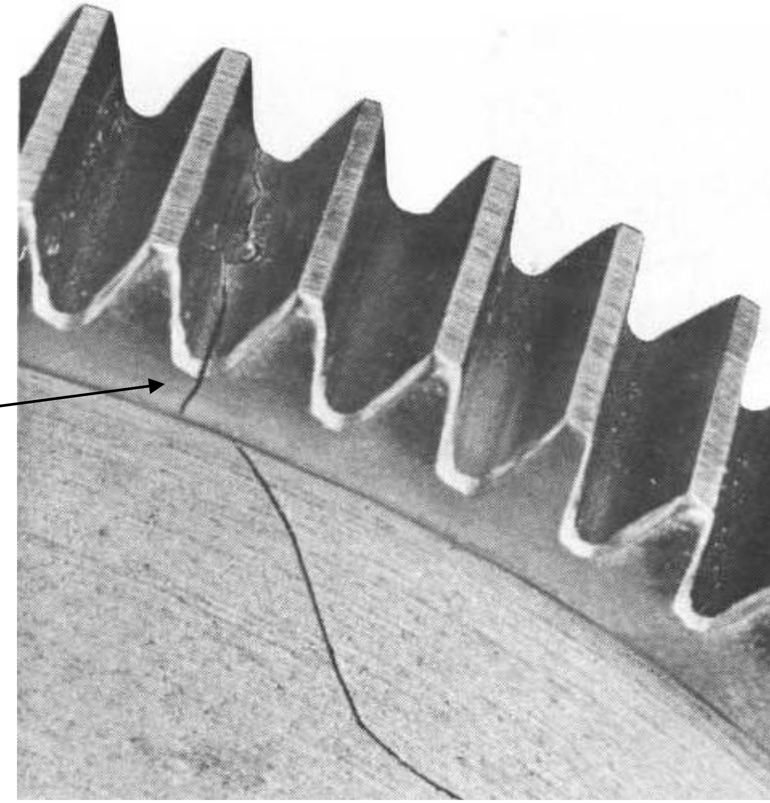
In terms of the design criteria of gears there are basically two important limiting design factors :

- Failure of tooth due to bending stress (static & fatigue)
- Failure of tooth surfaces due to contact stresses

Thus, in terms of strength of gear teeth, design is based on

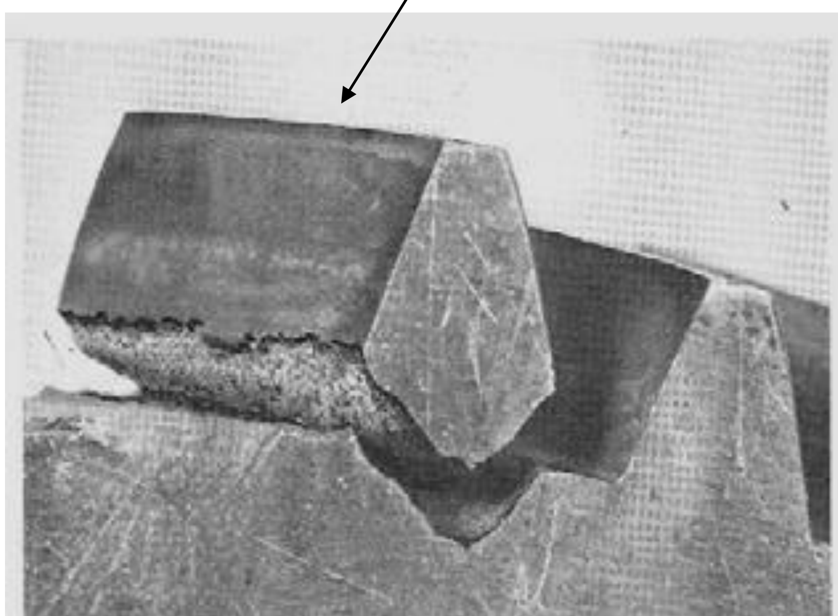
- Static (strength) failure due to bending stress
- Fatigue failure due to bending stress and
- Surface fatigue failure due to contact or Hertzian stresses

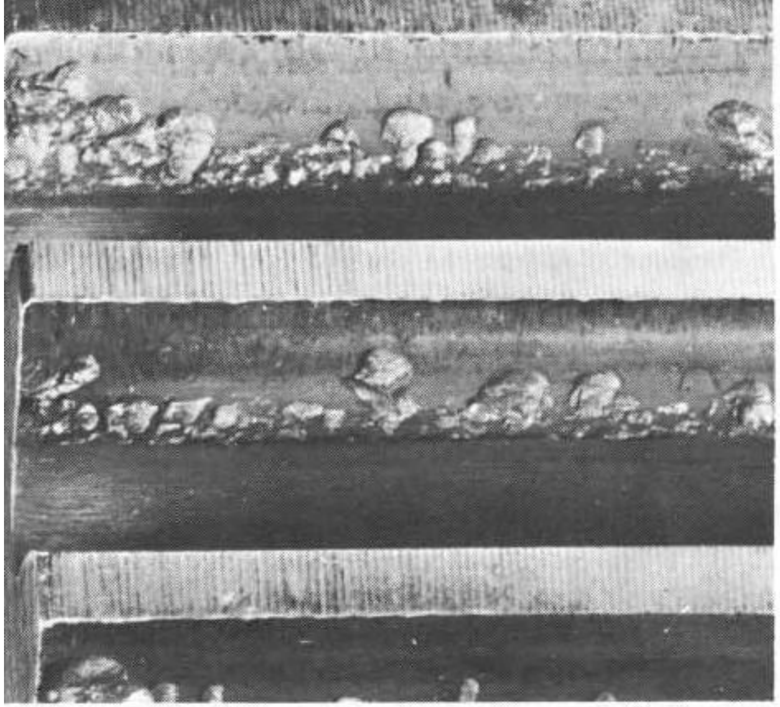


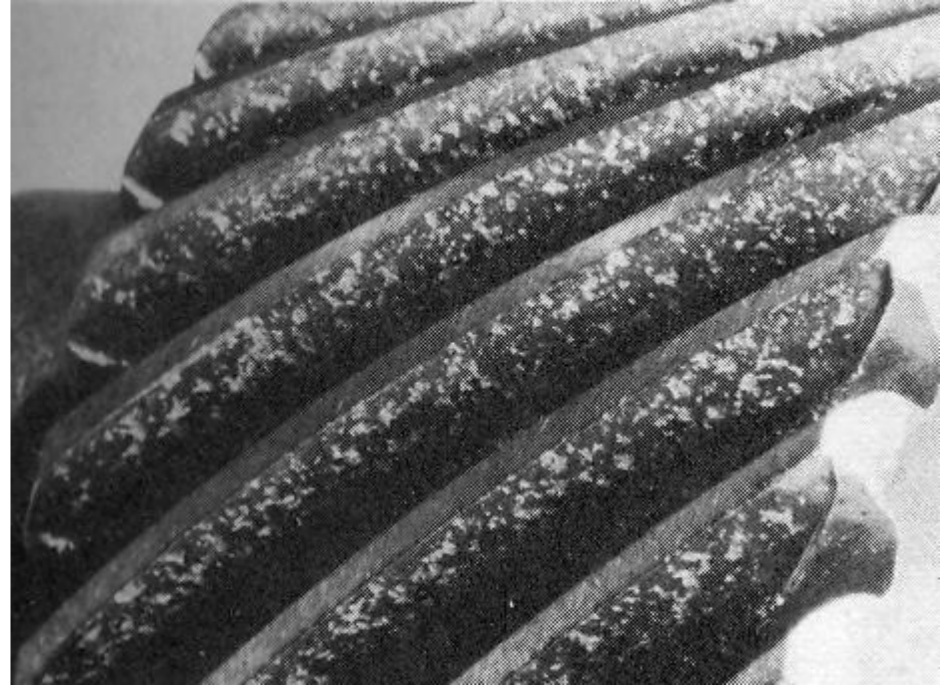


Crack due to bending stress

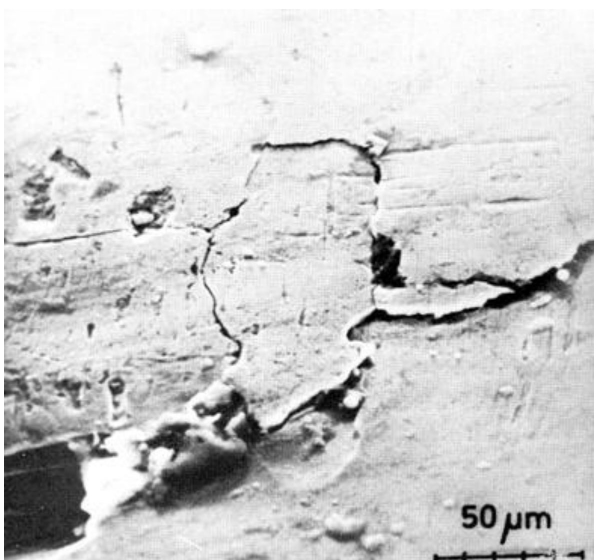
Full tooth breakage



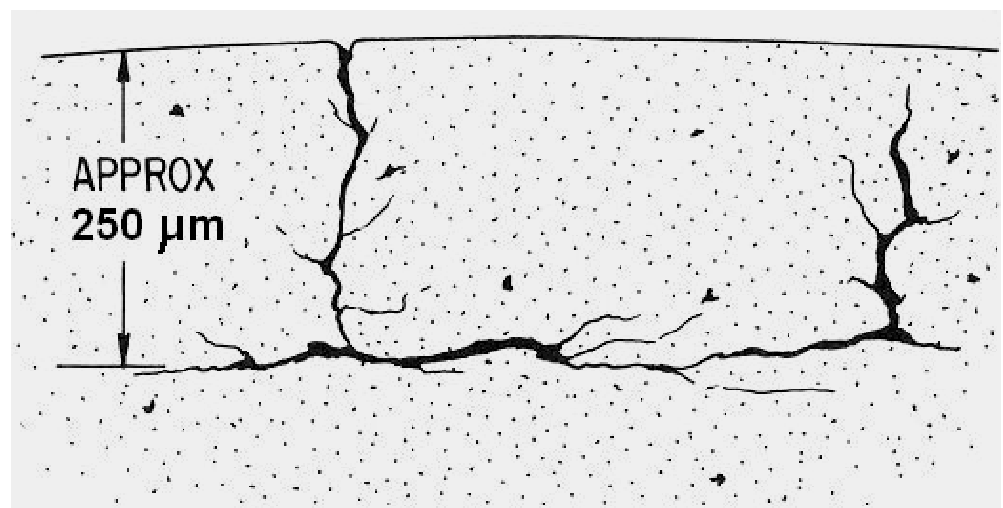




Surface failures due to contact stresses

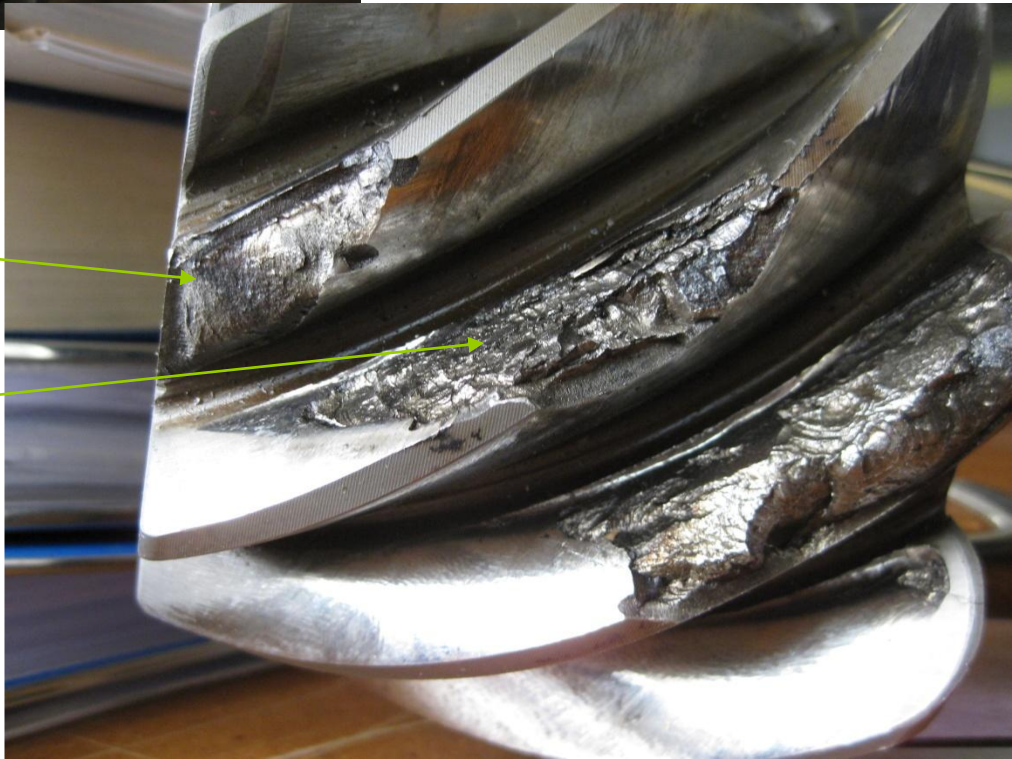


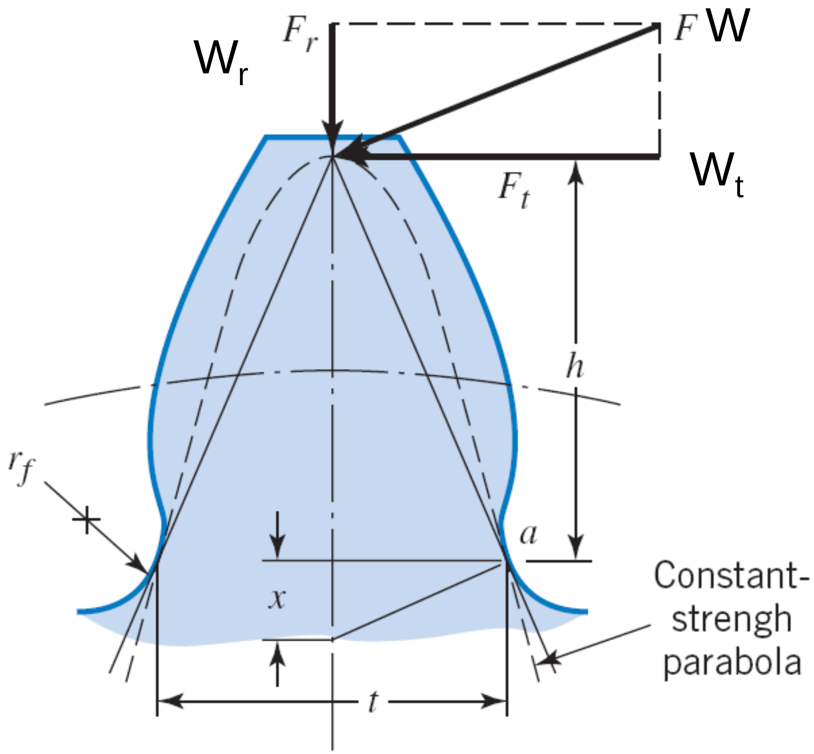
Contours of a particle to be spalled out.



Failure due to
bending stress

Failure due to
contact stress





Bending stress in tooth root (especially tensile stress) is created by the force component F_t (W_t) and calculated from general eqn:

σ=M*c/I

Here

$M=W_t*h$, c=t/2 and $I=F*t^3/12$

F is the facewidth of tooth

Force Fr (Wr) does actually help reducing the tensile bending stress, but this is not taken into consideration and the likely worst condition is analysed.

Bending stress formula for gear tooth is first derived by LEWIS and his formula is given as:

$$\sigma = \frac{Wt}{F * m * Y}$$

$$\sigma = \frac{Wt}{F * m * Y}$$

In LEWIS equation,

Wt is the tangential load,

F is the tooth facewidth,

m is the module and

Y is the modified form factor :

Lewis equation is used only for static design and is not accurate enough for dynamic conditions

The factor Y is given in tables and assumes that the load is not shared between the teeth in contact & the max. Load happens at the tip of the teeth.

However in actual case max. Load does not reach the tip of the tooth but stays somewhere below where the moment arm is not maximum.

Here again the likely worst condition is analysed

Dynamic effects

When the gears are run at high speeds small disturbances create high dynamic loads & stresses. Also at high speeds resonant speeds could be reached thus creating high vibrations hence high dynamic loads & stresses again.

To compessate for these cases a velocity factor, Kv (or called dynamic factor) is used in stress equation.

LEWIS's bending stress equation was further modified to include likely speed factor Kv for dynamic conditions and the geometry factor J (which includes bending stress concentration factor in the tooth root for fillet geometry) and re-written as:

$$\sigma = \frac{Wt}{F * m * J * Kv}$$

$$\sigma = \frac{Wt}{F * m * Y}$$

Here, (called AGMA equation)

Wt is the tangential load,

F is the tooth facewidth,

m is the module

J is the geometry factor (from AGMA) and

Kv is the speed factor:

There are different Kv equations for different gear types and gear materials (like spur, helical, bevel etc and cut or milled teeth, not carefully generated, finished by hobbing or shaping, high precision shaved or ground teeth etc)

V is the pitch line velocity of meshing gears used

$$Kv = \frac{a}{a+V}$$

$$Kv = \frac{b}{b+V}$$

$$Kv = \sqrt{\frac{c}{c+\sqrt{200V}}}$$

These are examples for different Kv equations and a, b, c can take different values based on gear type, manufacturing and material.

$$V(m/s) = \frac{\pi d n}{60}$$

d is the pitch circle diameter of the gear in meter

n is the rotational speed of the same gear in rpm

$$Wt(Newton) = \frac{Power(Watt)}{V(m/s)}$$

$$\sigma = \frac{Wt}{F * m * Y}$$

LEWIS equation is used for quick estimate of gear size when there is no risk of fatigue failure of teeth (Y - from table 13-3).

$$\sigma = \frac{Wt}{F * m * J * Kv}$$

This is called AGMA equation and is (must be) used for analysis of gears when there is the risk of bending fatigue failure of teeth (J - from table 13-4,5,6,7).

Estimating gear size

For a pair of gear transmitting power or motion the given information generally are:

-the power (watt) to be transmitted

-the speed n (rpm) of the gear to be sized and

-the speed ratio of the pair

The designer is then required to determine other parameters to satisfy a safe gear pair. For such a case, designer has to determine:

-the number of teeth (N) on the gears to be sized

-the Lewis form factor Y (table 13-3) for the gears to be sized

-the bending stress in the gear material(root bending stress)

-the gear material and its specs (Sy, Sut etc) and

-the size of the gear (specified by the parameters diameter & Facewidth).

For a safe (failure-free) operation the bending stress in the tooth root should be smaller than the strength of gear material that is:

$$\sigma = \frac{Wt}{F * m * J * Kv} \leq S_y$$

Wt can be found from power relation

$$Wt(Newton) = \frac{Power(Watt)}{V(m/s)}$$

V can be found linear pitch line velocity relation

However, V depends on diameter which then depends on tooth number and module both of which are already unknowns.

$$V(m/s) = \frac{\pi dn}{60}$$

F is also not known yet and has to be determined by designer

$$d = N * m$$

J can be found from Table if the tooth number is known (which is not determined yet)

Kv requires velocity V which was dependent on both tooth number and module (as above again)

$$Kv = \frac{a}{a + V}$$

Sy is material dependent and designer can determine it by selecting a proper material from catalogues

So many parameters depend on few unknowns like N1, N2, m & F

N1 and N2 can be determined by speed ratio (keeping in mind the undercut and minimum tooth number restriction, 18 theoretically for 20 degree press.angle).

Determining m & F, however, are not so easy, and an analytical sol'n is difficult to get. Rather an iteration technique of assuming different values m & F & then checking whether assumptions are correct or not is more suitable (as in the case of springs and RCB's).

The checking criteria are usually the strength safety and the geometric suitability: (Use a safety factor more than 2 or sometimes 3 for gear size determination).

$$\sigma = \leq \frac{S_y}{n}$$

To prevent tooth root crack or breakage due to bending

$$3p_c \leq F \leq 5p_c$$

$3p_c$: To prevent requirement of larger diameter gears

$5p_c$: To prevent mal-distribution of tooth load over face width

$$\sigma = \frac{Wt}{F * m * J * Kv} \leq S_y$$

After determining the proper tooth numbers, iteration technique starts as follows:

1) Assume a module m,
2) Calculate pitch diameters (d1and d2)
3) Calculate pitch line velocity V
4) Calculate Wt from power relation
5) Calculate Kv from relation
6) Find Y or J-factor from table ?-?
7) Calculate face width F,
8) Check if F is in limits?
9) If YES, design (gear sizing) is finished
10) If NO, re-try another iteration until satisfied

$$d = N * m$$

$$V(m/s) = \frac{\pi d n}{60}$$

$$Wt(Newton) = \frac{Power(Watt)}{V(m/s)}$$

$$Kv = \frac{a}{a+V}$$

$$F = \frac{Wt}{(\frac{Sy}{n}) * m * J * Kv}$$

$$3p_c \leq F \leq 5p_c$$

$$p_c = \pi m$$

Regarding material strength, when both pinion & gear are made of the same material the pinion is always the weaker one of the two because it has more undercut shape due to less number of teeth (& it rotates more hence loaded up more frequently when fatigue is considered).

Thus design can be done for pinion only and concluded with same F for gear.

If materials are different for pinion and gear the weaker material strength and specs are used in design and analysis processes.

Example: A steel pinion with m=4mm, 20 degree pressure angle & 22T runs at 900 rpm and transmits 12.5 hp to a 60T gear.

Calculate the bending stress on the pinion tooth using the Lewis stress eqn based on a facewidth of 38 mm.

$$T = \frac{W_t}{FmYK_V} \;;\mathrm{W_t} = \frac{60P}{\pi dn} = \frac{60x(12.5x746)}{\pi x(4x22x10^{-3})x900} = 2248N$$

$$\text{Now} \quad \mathrm{T} = \frac{2249}{38\mathrm{x}4\mathrm{x}0.31997\mathrm{x}0.591} = 78.24MP_a$$

$$Y = 0.31997 \text{ (from table 13-3 for 22T)}$$

$$\mathrm{K_V} = \frac{6}{6+V} ;V = 4.147 \text{ m/sec.}$$

Example :

A set of BS Grade cast Iron (with $S_{ut} = 180 MP_a$) gears is to be designed to transmit 1.2 kW at a pinion speed of 400 rpm and a speed reduction of 1.5:1 . Use a safety factor of 4 and determine suitable values of m, T_p, T_g, d_p, d_g, F based on the Lewis stress eqn. Use 20^o full depth teeth with b=1.25 m.

Solution:

This is a gear design problem with criteria of 1- $T \leq \frac{T_p}{n}$

$$T = \frac{W_t}{FmYK_V} = \frac{S_{ut}}{n} \quad W_t = \frac{60P}{\pi dn} \quad d=?$$

$$K_V = \frac{3}{3+V} \qquad V = \frac{\pi dn}{60} \quad d=?$$

2- $3p \leq F \leq 5p$

3- $\frac{T_2}{T_1} = 1.5 = \frac{d_2}{d_1}$

Since both d & F are unknown , iteration is suitable for

$$\theta = 20^o \rightarrow T_{\min} = 18 \rightarrow T_2 = 1.5xT_1 = 27$$

$$T_p = 18 \,\&\, T_g = 27 \text{ and}$$

$$Y = 0.29327 \quad \text{for 18T pinion}$$

m	$d_1 = T_1 x m$	V m/s	K_V	W_t, N	$F = W_t / (K_V m Y S_{ut} / n)$	$3p = 3x\pi x m$	5p	Notes
2mm	36mm	0.754	0.8	1591.5	75.37 mm	18.84	31.4	Not suitable
3mm	54mm	1.131	0.7262	1061	36.9	28.3	47.1	SUİTABLE
<u>Sol'n</u>	m=3mm							
	$d_p = 54mm$							
	$d_g = 81mm$							
	F=37mm							