EEE352 Automatic Control Systems Lab

Quiz-2 / Quiz-3: Detailed Solution

Note: Each quiz is graded out of 100 points (50 points per question). Partial credit is awarded for correctly identifying the theoretical formulas and demonstrating the proper algebraic steps, even if the final arithmetic calculation is incorrect.

Quiz 2: Time Domain Response (Experiments 3 & 4) – 100 Points Total

Question 1: Modeling Mechanical Systems (Damping Ratio ζ) (50 Points)

System: $m\ddot{x}(t) + b\dot{x}(t) + kx(t) = F(t)$
(Given $m = 1$**,** $k = 100$**, and a variable** b**)**

- **Step 1: Identify the standard characteristic equation (20 Points)**
  The standard second-order form is:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

- **Step 2: Calculate the Natural Frequency (10 Points)**
  Using the given mass and spring constant:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{1}} = 10 \text{ rad/s}$$

- **Step 3: Solve for the Damping Ratio (20 Points)**
  Equating the middle terms of the actual system to the standard form:

$$\begin{aligned} 2\zeta\omega_n &= \frac{b}{m} \\ 2\zeta(10) &= b \\ \zeta &= \frac{b}{20} \end{aligned}$$

  (Full credit requires substituting the specific group's b value into this final equation).

Question 2: Steady-State Final Value (50 Points)

Open-loop TF: $G(s) = \frac{100}{(s+P)(s+5)}$
(Given a unit step input and a variable pole P**)**

- **Step 1: State the Final Value Theorem (20 Points)**
  To find the steady-state value y_{ss} for a given input $R(s)$:

$$y_{ss} = \lim_{s \to 0} s \cdot Y(s) = \lim_{s \to 0} s \cdot G(s) \cdot R(s)$$

- **Step 2: Apply the Unit Step Input (10 Points)**
  A unit step input is $R(s) = \frac{1}{s}$. Substituting this in cancels the s:

$$y_{ss} = \lim_{s \to 0} s \cdot G(s) \cdot \frac{1}{s} = \lim_{s \to 0} G(s)$$

- **Step 3: Evaluate the Limit (20 Points)**
  Substituting $s = 0$ into the transfer function:

$$y_{ss} = G(0) = \frac{100}{(0 + P)(0 + 5)} = \frac{100}{5P} = \frac{20}{P}$$

(Full credit requires substituting the specific group's pole P into this final equation).

Quiz 3: Frequency Domain & Physical Systems (Experiments 5 & 6) – 100 Points Total

Question 1: Linear Gain Margin (50 Points)

(Given a variable magnitude $|G(j\omega_{pc})|$ at the phase crossover frequency)

- **Step 1: Define Gain Margin (30 Points)**
  The linear Gain Margin is defined as the inverse of the open-loop system's magnitude at the exact frequency where the phase shift crosses $-180°$ (ω_{pc}).

  $$GM_{linear} = \frac{1}{|G(j\omega_{pc})|}$$

- **Step 2: Calculate the Final Value (20 Points)**
  Simply take the reciprocal of the magnitude provided on the specific lab sheet. For example, if the magnitude is 0.2, the $GM = \frac{1}{0.2} = 5$.

  CRITICAL GRADING NOTE: The final answer MUST be the linear gain margin. Answers provided in decibels (dB) will NOT be accepted and will receive zero points for this final step.

Question 2: Liquid-Level System Time Constant (τ) (50 Points)

(Given a constant valve resistance $R = 5$ and a variable tank area A)

- **Step 1: Establish the Mass Balance Equation (20 Points)**
  The rate of change of fluid volume in the tank equals the difference between the volumetric inflow (Q_{in}) and outflow (Q_{out}). With cross-sectional area A and fluid height h:

  $$A\frac{dh(t)}{dt} = Q_{in}(t) - Q_{out}(t)$$

  Since the outflow through the linear valve is defined as $Q_{out}(t) = \frac{h(t)}{R}$, we substitute this to get the system's differential equation:

  $$A\frac{dh(t)}{dt} + \frac{h(t)}{R} = Q_{in}(t) \quad \Rightarrow \quad AR\frac{dh(t)}{dt} + h(t) = RQ_{in}(t)$$

- **Step 2: Derive the Transfer Function (15 Points)**
  Taking the Laplace transform of both sides (assuming zero initial conditions) yields:

  $$(ARs + 1)H(s) = RQ_{in}(s) \quad \Rightarrow \quad \frac{H(s)}{Q_{in}(s)} = \frac{R}{ARs + 1}$$

  Comparing this to the standard first-order form $\frac{K}{\tau s+1}$, we identify the system's time constant as $\tau = AR$.

- **Step 3: Calculate the Time Constant τ (15 Points)**
  Substitute the given valve resistance ($R = 5$) and the specific group's tank area (A) into the derived formula:

  $$\tau = 5 \times A$$

  (Full credit requires substituting the specific group's area A into this equation to find the final time in seconds).