



ME 444

MATLAB® FOR ENGINEERS

Lecturer:

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CHAPTER 9

SYMBOLIC OPERATIONS in MATLAB

Symbolic Operations in MATLAB

- To find the solution of an equation in Matlab, for example, we can solve the equation $x^2-2x-15=0$ by writing the necessary codes in an m file.
- The m file of this program can be as follows:

```
a=input('a = ');
b=input('b = ');
c=input('c = ');
delta=b*b-4*a*c;
if delta>0
x1=(-b-delta^0.5)/(2*a);
x2=(-b+delta^0.5)/(2*a);
fprintf('İki reel kök; x1 = %f      x2 =%f',x1,x2);
elseif delta==0
fprintf('One root exists; x1 = x2= %f ',-b/(2*a));
else
fprintf('Kökler sanal ');
end;
```

- When the program is run, we find the roots of the equation as -3 and 5 by entering the values of a b c coefficients 1, -2 and -15 respectively.
- Since we know the formulas that give the solution of a quadratic equation, we were able to write its program (although not very easily).
- What if the equation is of 3rd order, 4th or 5th order, or is like $x \cdot \sin x = 1/5$ or like $x^x = 64$... how can we solve these equations?

Symbolic Operations in MATLAB

- Can we simplify a given algebraic expression like $\frac{(x^3-8)(x^2+7x)}{(2x^2+4x+8)(x^2-2x)}$
- Can we find the limit, derivative and integral of a function in Matlab?
- In order to do such operations, we should use the Symbolic Logic (Symbolic Object) concept in Matlab. To get explanation and help about this issue, it is enough to write *help symbolic* on the command line.
- For all the operations mentioned above, it is necessary to convert the mathematical expression into a symbolic one first by using `syms` command.
- General syntax of the command is:

`syms x`

```
syms x
f(x)=x^2+3;
fprintf('f=%s \n', f(x))
-----
f=x^2 + 3
```

To assign a value to the x variable, the following additions can be made to the code given below.

```
syms x
f(x)=x^2+3;
fprintf('f=%f \n', f(2))
-----
f=7
```

Difference of Numerical and Symbolic Equations

- The two examples below essentially do the same calculation.
- However, in the example on the left, the values of x and y are given first and the addition is done later.
- In the example on the left, the symbols are first collected and then the numerical values are replaced and the process is completed.

Numerical operation:

```
x=5.2;  
y=8.3;  
f=x+y  
-----  
f =  
  
13
```

Symbolic operation:

```
syms x y;  
f(x,y)=x+y;  
f(5,8)  
-----  
f=  
  
13
```

If more than one variable is to be used for symbolic operation with the `syms` command, no commas are inserted.

Finding Roots of Symbolic Equations

- The `solve` command can be used to find the roots of a given n degree polynomial.
- Command syntax is:

`solve(equation, variable)`

- Here, the `equation` is the n degree polynomial and the `variable` indicates that this function will be solved for this variable. If no variable is given, the command will be

`solve(equation)`

```
>> syms x
>> a=solve(x^2 - 4*x + 2==0,x)
a =
2 - 2^(1/2)
2^(1/2) + 2
```

```
>> syms x
>> a=solve(x^2 - 4*x + 2==0,x);
>> double(a)
ans =
    0.5858
    3.4142
```

On the left, we see that the results are fractional. We use the `double` command to show the results in decimal.

Finding Roots of Symbolic Equations

- The `subs` command is used to find the result of a symbolic expression for a value given to a variable.
- Command syntax is:

`subs(equation, variable, value)`

EXAMPLE: $y = f(x) = \frac{x^2 - x + 7}{3x + 5}$ function is given. First find the $f(2005)$ value, then the function composition $f(g(x))$ where $z = g(x) = \frac{x^3 - 2}{x + 3}$.

```
%First we need the numerical expressions into  
%symbolic  
syms x;  
%polynomials  
f = (x^2 - 2*x + 7) / (3*x + 5);  
g = (x^3 - 2) / (x + 3);  
%f(2005)  
subs(f, x, 2005)
```



```
Command Window  
  
>> Untitled4  
  
ans =  
  
2008011/3010
```

Finding Roots of Symbolic Equations

EXAMPLE: $y = f(x) = \frac{x^2 - x + 7}{3x + 5}$ function is given. First find the $f(2005)$ value, then the function composition $f(g(x))$ where $z = g(x) = \frac{x^3 - 2}{x + 3}$.

```
%First we need the numerical expressions into  
%symbolic  
syms x;  
%polynomials  
f = (x^2 - 2*x + 7) / (3*x + 5);  
g = (x^3 - 2) / (x + 3);  
%f(g(x))  
subs(f, x, g)
```



```
ans =  
  
((x^3 - 2)^2 / (x + 3)^2 - (2*(x^3 - 2)) / (x + 3) + 7) / ((3*(x^3 - 2)) / (x + 3) + 5)
```

Finding Roots of Symbolic Equations

Let's make the result clear and compact by using `pretty` command.

```
%First we need the numerical expressions into
%symbolic
syms x;
%polynomials
f=(x^2-2*x+7)/(3*x+5);
g=(x^3-2)/(x+3);
%f(g(x))
pretty(subs(f,x,g))
```



```
>> untitled
      3      2      3
(x  - 2)    2 (x  - 2)
----- - ----- + 7
      2      x + 3
(x + 3)
-----
      3
      3 (x  - 2)
----- + 5
      x + 3
```

Symbolic Differentiation



- We can derive a symbolically given equation with the `diff` command.
- Command syntax is:

`diff(equation, variable)`

- Here, while the `equation` represents a symbolically defined equation, the `variable` indicates according to which variable the derivative will be taken.

```
>> syms x
>> f=x^3-3*x+4;
>> diff(f,x)
-----
ans =
3*x^2 - 3
```

```
>> syms x
>> f=x^3+5*sin(x);
>> diff(f,x)
-----
ans =
5*cos(x)+3*x^2
```

- Example: Let's take the derivative of the equation $F(x,y)=x^3*y^4+y*\cos(x)$ first with respect to x and then with respect to y .

```
>> syms x y
>> f=x^3*y^4+y*cos(x);
>> diff(f,x)

ans =
      3*x^2*y^4-y*sin(x)

>> diff(f,y)

ans =
      cos(x)+4*x^3*y^3
```

- The symbolic integration function `int` is used for symbolic integration operation.
- `int(s,x)` : Takes the indefinite integral of the symbolic expression `s` with respect to `x`.
- `int(s, x, a, b)` : Takes the definite integral of `s` with respect to `x` from `a` to `b`

```
>> syms x
>> f=x^5-2*x^4+2*x^3+3*x^2+4;
>> int(f,x)

ans =
      x^6/6 - (2*x^5)/5 + x^4/2 + x^3 + 4*x
```

Symbolic Integration



- Let's solve the integration in $[1,2]$

```
>> syms x
>> f=x^5-2*x^4+2*x^3+3*x^2+4;
>> int(f,x,1,2)

ans =
      83/5
```

Solving Equations with Multiple Unknowns

- We can find the unknowns of n equations with n unknowns, again using the solve command. General use of:

`solve(equations, variables)`

- Here, `equations` indicate a symbolically defined equation, while `variables` indicate which unknowns to be found.

```
>> syms x y z
>> f1=3*x +2*y -z -10;
>> f2=-x +2*y -z;
>> f3=x -y -z +1;

>> solve(f1,f2,f3)

ans =

    struct with fields:

        x: [1×1 sym]
        y: [1×1 sym]
        z: [1×1 sym]
```

On the left, we see that the result is given symbolically. In order to get the numerical values of the results, we use "." before the relevant variable as shown on the right.

```
>> syms x y z
>> f1=3*x +2*y -z -10;
>> f2=-x +2*y -z;
>> f3=x -y -z +1;
>> soln=solve(f1,f2,f3);
>> soln.x
>> soln.y
>> soln.z

ans =
    -2
ans =
     5
ans =
    -6
```

Solution of Systems of Linear Equations

- An n-order system of linear equations is defined as follows:

$$\begin{array}{rcl}
 a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n & = & b_1 \\
 a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n & = & b_2 \\
 \hline
 \hline
 a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n & = & b_n
 \end{array}$$

- This system of equations can also be written in matrix form as $[A]*[X]=[B]$. Here, A is the coefficients matrix, B is the result vector, and X is the vector of the variables whose solution is desired.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}_{n \times n}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix}_{n \times 1}$$

$$B = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{bmatrix}_{n \times 1}$$

- If $[A]*[X]=[B]$ then $X=inv(A)*B$ (!! A must be a square matrix to get the inverse of A matrix.)

Solution of Systems of Linear Equations

- Example: Solve the system of equations given below.

$$\begin{array}{rcrcrcrcrcrl} x_1 & + & 4x_2 & - & x_3 & + & x_4 & = & 2 \\ 2x_1 & + & 7x_2 & + & x_3 & - & 2x_4 & = & 16 \\ x_1 & + & 4x_2 & - & x_3 & + & 2x_4 & = & 1 \\ 3x_1 & - & 10x_2 & - & 2x_3 & + & 5x_4 & = & -15 \end{array}$$

- Solution:

```
>> A=[1 3 -1 1; 2 7 1 -2; 1 4 -1 2; 3 -10 -  
2 5];  
>> B=[2;16;1;-16];  
>> X=inv(A)*B
```

X =

```
    2.0000  
    1.0000  
    1.0000  
   -2.0000
```



**END OF THE
COURSE**