ME 209 NUMERICAL METHODS
2nd EXAM QUESTIONS

Name Surname: **15.12.2025**
Student No: **Duration: 90 min**

1. The upward velocity of a rocket is measured several times instant during its ascent. The experimental data are given in table below.

t (s)	0	10	15	20	22.5	30
$v(t)$ (m/s)	0	227.04	362.78	517.35	602.97	901.67

a) (20 p) Using the data points at t = 0, 10, 15, 20 s, construct Newton's Divided Difference Table as shown below:

x	$f()$	$f[,]$	$f[,,]$	$f[,,,]$

b) (20 p) Using the table, construct a linear, a quadratic and a cubic interpolation polynomial equation.

c) (10 p) Using the interpolation polynomials constructed in (b), estimate the rocket velocity $v(t)$ at t = 13 s.

2. (25 p) Fit a straight line to the x and y values in the given table:

x	1	2	3	4	5	6	7
y	0.5	2.5	2.0	4.0	3.5	6.0	5.5

3. (25 p) Using the velocity-time table given in the 1st question, estimate the acceleration of the rocket at t=15 s. Use forward, backward and two-step methos for numerical differentiation. Comment briefly on the results.

SOME OF THE IMPORTANT FORMULATIONS

Newton's Interpolating Polynomials: $$f_n(x) = f(x_0) + (x - x_0)f[x_1, x_0] + (x - x_0)(x - x_1)f[x_2, x_1, x_0] + \cdots + (x - x_0)(x - x_1)\cdots(x - x_{n-1})f[x_n, x_{n-1}, \ldots, x_0]$$

$$f[x_i, x_j] = \frac{f(x_i) - f(x_j)}{x_i - x_j} \qquad f[x_i, x_j, x_k] = \frac{f[x_i, x_j] - f[x_j, x_k]}{x_i - x_k}$$

$$y = a_0 + a_1 x \qquad a_0 = \bar{y} - a_1\bar{x} \qquad a_1 = \frac{n\Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n\Sigma x_i^2 - (\Sigma x_i)^2}$$

$\bar{x}$ and $\bar{y}$ are the mean values.

Forward Difference Equation	Backward Diff.Eqn.	2-Step Method
$\frac{df(x)}{dx} \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$	$\frac{df(x)}{dx} \approx \frac{f(x) - f(x-\Delta x)}{\Delta x}$	$\frac{df(x)}{dx} \approx \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$

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1.

Zeroth-order differences

$$f[t_0] = 0, f[t_1] = 227.04, f[t_2] = 362.78, f[t_3] = 517.35$$

First divided differences

$$f[t_0, t_1] = \frac{227.04 - 0}{10 - 0} = 22.704 \quad 5$$

$$f[t_1, t_2] = \frac{362.78 - 227.04}{15 - 10} = \frac{135.74}{5} = 27.148 \quad 3$$

$$f[t_2, t_3] = \frac{517.35 - 362.78}{20 - 15} = \frac{154.57}{5} = 30.914 \quad 3$$

Second divided differences

$$f[t_0, t_1, t_2] = \frac{27.148 - 22.704}{15 - 0} = \frac{4.444}{15} = 0.2963 \quad 3$$

$$f[t_1, t_2, t_3] = \frac{30.914 - 27.148}{20 - 10} = \frac{3.766}{10} = 0.3766 \quad 3$$

Third divided difference

$$f[t_0, t_1, t_2, t_3] = \frac{0.3766 - 0.2963}{20 - 0} = \frac{0.0803}{20} = 0.00402 \quad 3$$

Newton Divided Difference Table

t	$f[t]$	1st DD	2nd DD	3rd DD
0	0	22.704	0.2963	0.00402
10	227.04	27.148	0.3766	
15	362.78	30.914		
20	517.35			

20 (a)

(b) Interpolation Polynomials

Linear equation should be btw. 10-15 s!

Linear interpolation (10–15 s)

$$v_1(t) = 227.04 + 27.148(t - 10) \quad 7$$

At $t = 13$:

$$v_1(13) = 227.04 + 27.148(3) = \boxed{308.48 \text{ m/s}} \quad 3$$

Quadratic interpolation (10–15–20 s)

OR $v_2(t) = t(22.704) + t(t-10)(0.2963)$
$= 306.708$

Newton form (starting at $t_0 = 10$):

$$v_2(t) = 227.04 + 27.148(t - 10) + 0.3766(t - 10)(t - 15) \quad 7$$

At $t = 13$:

$$v_2(13) = 227.04 + 27.148(3) + 0.3766(3)(-2)$$

$$v_2(13) = 227.04 + 81.444 - 2.260 = \boxed{306.22 \text{ m/s}} \quad 3$$

Cubic interpolation (0–10–15–20 s)

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Newton form:

$$v_3(t) = \begin{array}{c} 0 + 22.704(t-0) + 0.2963(t-0)(t-10) \\ +0.00402(t-0)(t-10)(t-15) \end{array}$$ 7

At $t = 13$:

$$v_3(13) = 22.704(13) + 0.2963(13)(3) + 0.00402(13)(3)(-2)$$
$$v_3(13) = 295.15 + 11.56 - 0.31 = \boxed{306.40 \text{ m/s}}$$ 3

(c) Final Results

Method	$v(13)$(m/s)
Linear	308.48
Quadratic	306.22
Cubic	306.40

2.

② $y = a_0 + a_1 x$ where $a_0 = \bar{y} - a_1 \bar{x}$ and $a_1 = \dfrac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}$

n	x_i	y_i	x_i^2	$x_i y_i$
1	1	0.5	1	0.5
	2	2.5	4	5.0
	3	2.0	9	6.0
	4	4.0	16	16.0
	5	3.5	25	17.5
	6	6.0	36	36.0
	7	5.5	49	38.5
Σ =	28	24	140	119.5

$\bar{x} = \dfrac{\sum x_i}{n} = \dfrac{28}{7} = 4$ 5/7

$\bar{y} = \dfrac{\sum y_i}{n} = \dfrac{24}{7} = 3.428571$ 5/7

• $a_1 = \dfrac{7(119.5) - (28)(24)}{7(140) - (28)^2}$

$= 0.8392857$ 5/7

• $a_0 = 3.428571 - (0.8392857)(4)$

$= 0.07142857$ 5/7

Therefore, the equation $\boxed{y = 0.07142857 + 0.8392857x}$ 5/7

Result with too few digits ~~[illegible]~~
lead to incorrect equation.
Therefore, results with few digits
gets −5.

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3. The time step is uniform: $h = 5$ s

1. Forward difference method $(O(h))$

$$\frac{dv}{dt}\Big|_{t=t_i} \approx \frac{v(t_{i+1}) - v(t_i)}{h}$$

At $t = 15$s: $a_f(15) = \frac{v(20)-v(15)}{5} = \frac{517.35-362.78}{5} = \frac{154.57}{5} = \boxed{30.91 \text{ m/s}^2}$

2. Backward difference method $(O(h))$

$$\frac{dv}{dt}\Big|_{t=t_i} \approx \frac{v(t_i) - v(t_{i-1})}{h}$$

At $t = 15$s: $a_b(15) = \frac{v(15)-v(10)}{5} = \frac{362.78-22\ .04}{5} = \frac{135.74}{5} = \boxed{27.15 \text{ m/s}^2}$

3. Two-step (central difference) method $(O(h^2))$

$$\frac{dv}{dt}\Big|_{t=t_i} \approx \frac{v(t_{i+1}) - v(t_{i-1})}{2h}$$

At $t = 15$s:

$$a_c(15) = \frac{517.35 - 227.04}{2(5)} = \frac{290.31}{10} = \boxed{29.03 \text{ m/s}^2}$$

Summary of Results

Method	Acceleration at $t = 15$s (m/s²)
Forward difference	30.91
Backward difference	27.15
Two-step (central) difference	29.03

Comments

- The forward and backward difference methods are first-order accurate $(O(h))$and give noticeably different results.
- The two-step (central difference) method is second-order accurate $(O(h^2))$and generally provides a more reliable estimate.
- Since $t = 15$s is an interior point and the data spacing is uniform, the central difference result $a(15) \approx 29.0$ m/s^{2}is the preferred estimate.

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