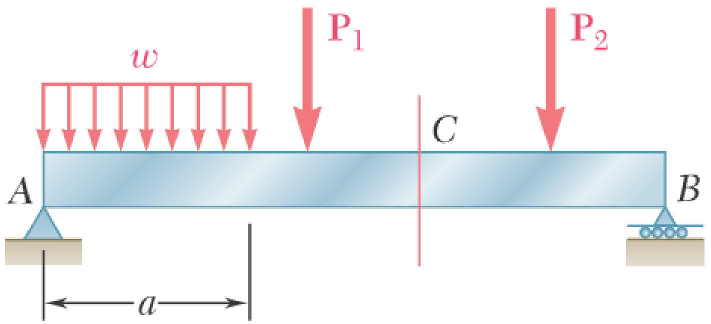
Ch.6 Shearing Stresses in Beams and Thin-Walled Members

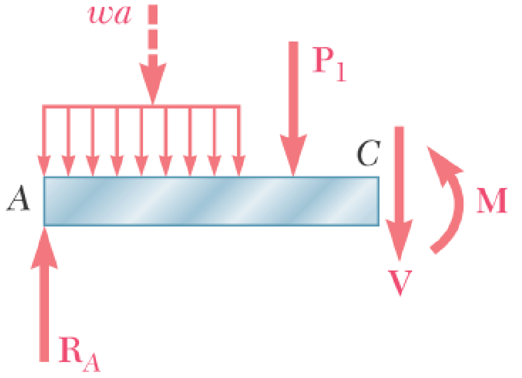
Part I

Objectives

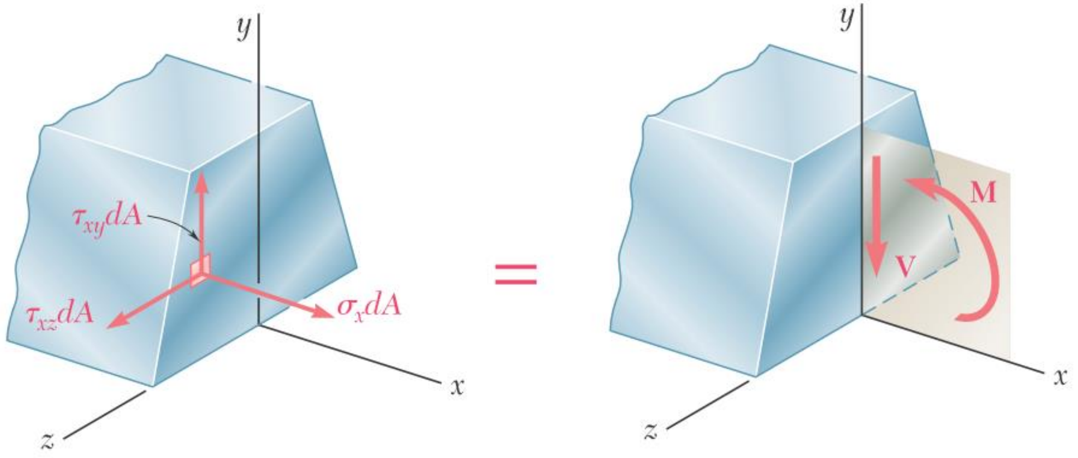
- how transverse loads on a beam generate shearing stresses.
- the stresses and shear flow on a horizontal section in a beam.
- the shearing stresses in a thin-walled beam.
- cases of symmetric and unsymmetric loading.
- shear flow to determine the location of the shear center in unsymmetric beams.



- Consider a prismatic beam AB with a vertical plane of symmetry that supports various concentrated and distributed loads
- Six equations can be written to express the equilirium state.



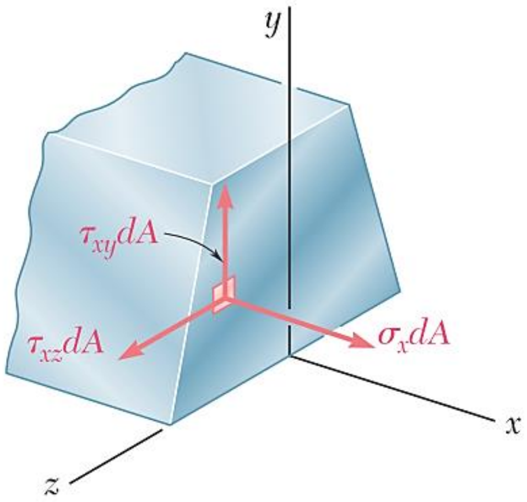
- Three of these equations involve only the normal forces $\sigma_x dA$



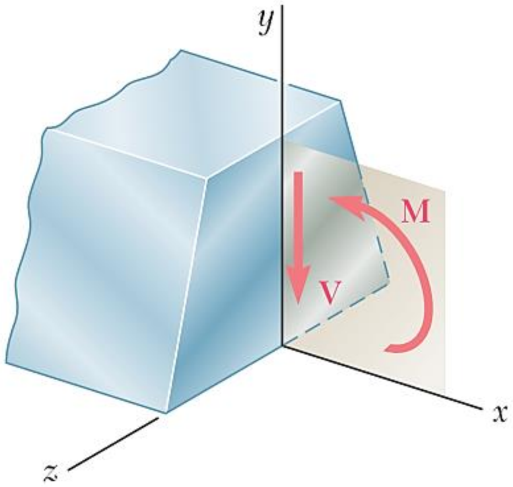
$$\Sigma F_x = 0 \rightarrow \int \sigma_x dA = 0$$

$$\Sigma M_y = 0 \rightarrow \int z\sigma_x dA = 0$$

$$\Sigma M_z = 0 \rightarrow \int -y\sigma_x dA = M$$



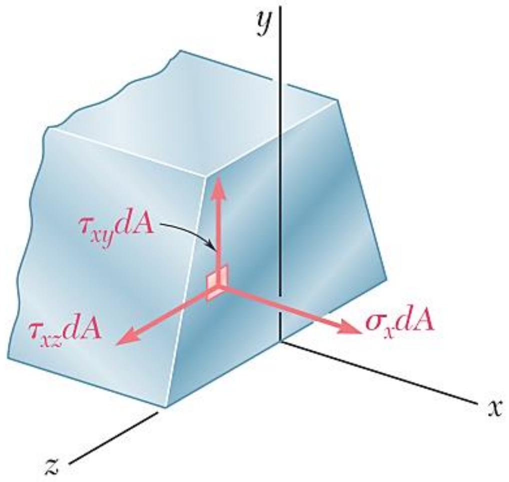
- Three more equations involving the shearing forces $\tau_{xy}dA$ and $\tau_{xz}dA$ now can be written.
- One equation expresses that the sum of the moments of the shearing forces about the x axis is zero and can be dismissed as trivial in view of the symmetry of the beam with respect to the xy plane.



- The other two involve the y and z components of the elementary forces and are

$$\int \tau_{xy}\, dA = -V$$

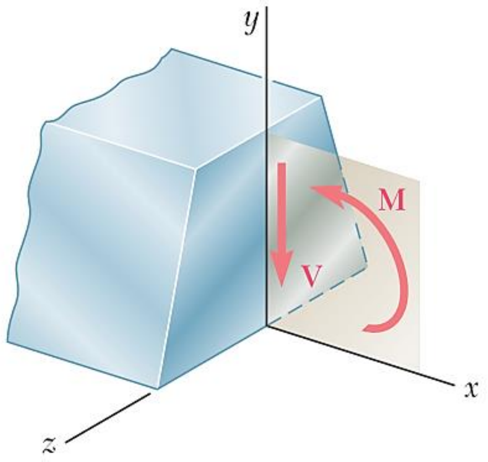
$$\int \tau_{xz}\, dA = 0$$



$$\int \tau_{xy}\, dA = -V$$

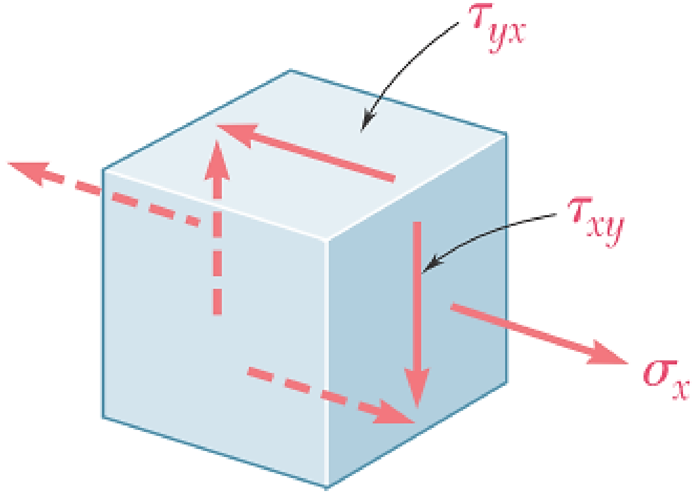
- Equation shows that vertical shearing stresses must exist in a transverse section of a beam under transverse loading.

$$\int \tau_{xz}\, dA = 0$$



- Equation indicates that the average lateral shearing stress in any section is zero. However, this does not mean that the shearing stress τ_{xz} is zero everywhere.

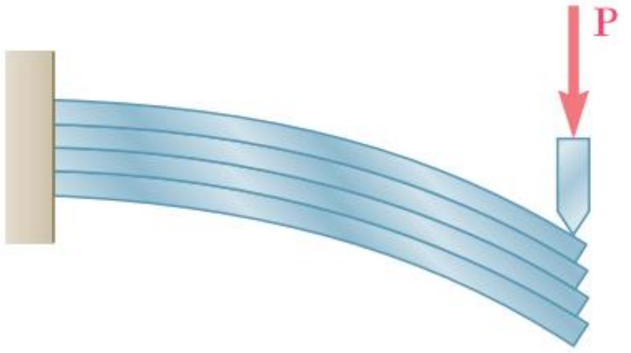
- Now consider a small cubic element located in the vertical plane of symmetry of the beam (where τ_{xz} must be zero) and examine the stresses exerted on its faces. A normal stress σ_x and a shearing stress τ_{xy} are exerted on each of the two faces perpendicular to the x axis.



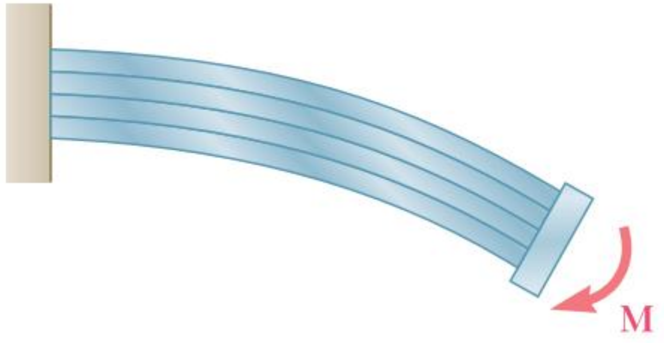
- The longitudinal shearing stresses must exist in any member subjected to a transverse loading.

- This is verified by considering a cantilever beam made of separate planks clamped together at the fixed end.

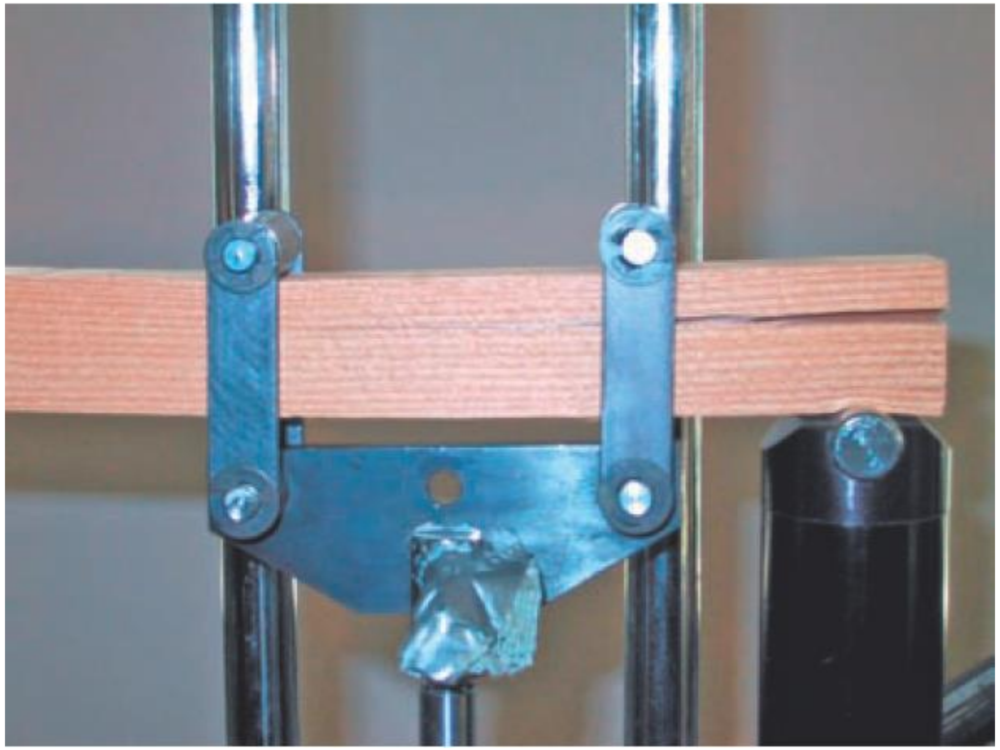
- When a transverse load P is applied to the free end of this composite beam, the planks slide with respect to each other.



- In contrast, if a couple M is applied to the free end of the same composite beam, the various planks bend into circular concentric arcs and do not slide with respect to each other. This **verifies the fact that shear does not occur in a beam subjected to pure bending**.



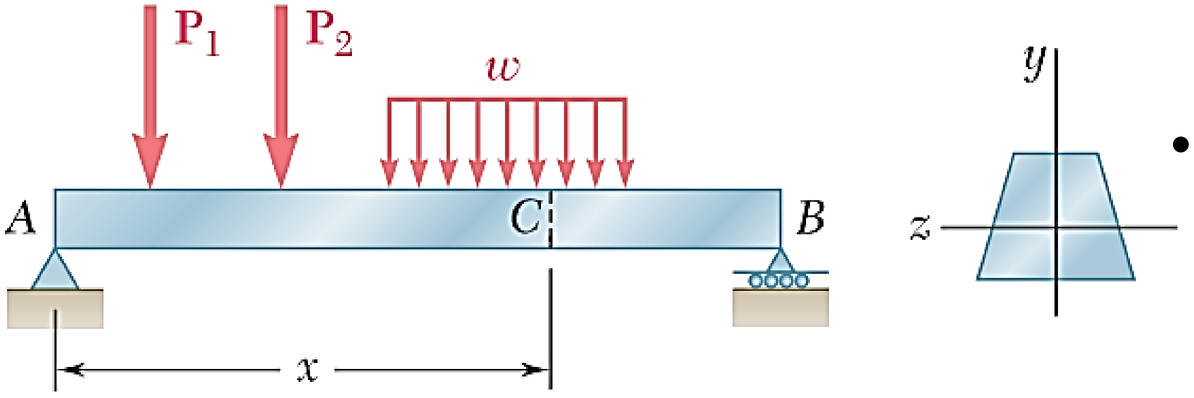
- While sliding does not actually take place when a transverse load P is applied to a beam made of a homogeneous and cohesive material such as steel, the tendency to slide exists, showing that stresses occur on horizontal longitudinal planes as well as on vertical transverse planes.

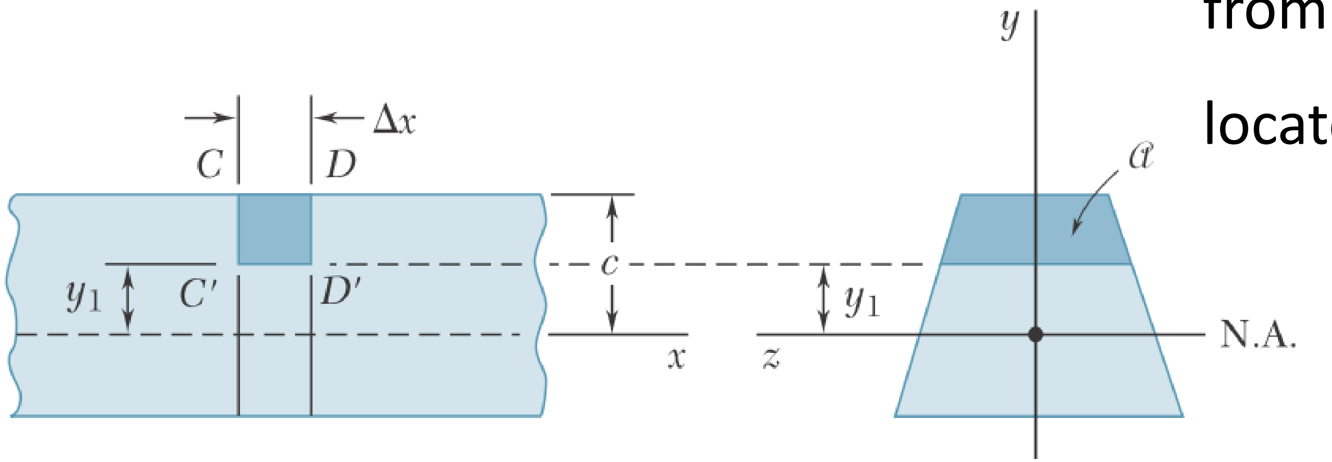


In timber beams, whose resistance to shear is weaker between fibers, failure due to shear occurs along a longitudinal plane rather than a transverse plane.

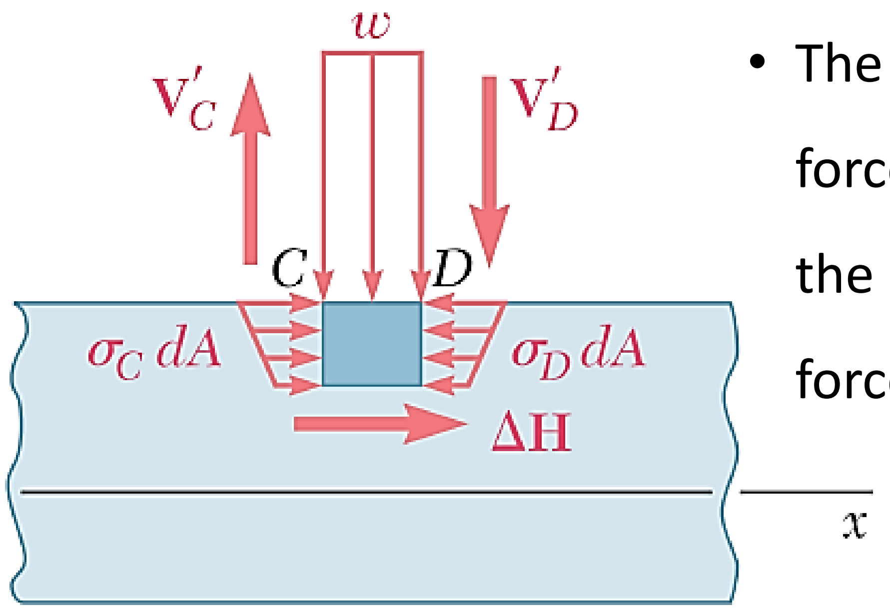
Horizontal Shearing Stress In Beams

Shear on the Horizontal Face of a Beam Element

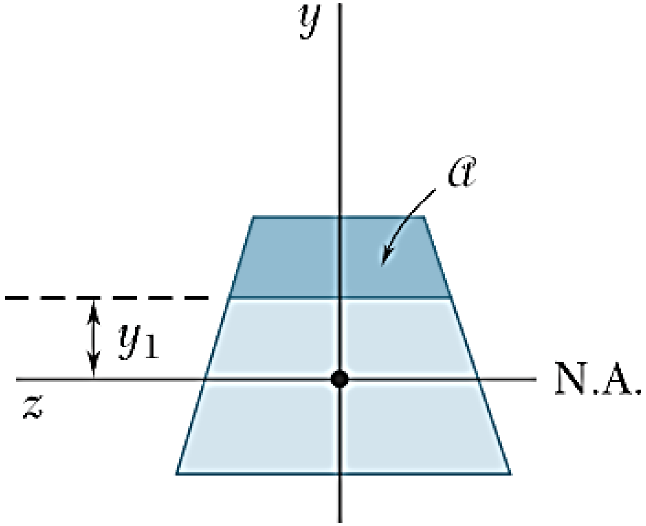




- At a distance x from end A, we detach from the beam an element *CDD'C'* with length of Δx extending across the width of the beam from the upper surface to a horizontal plane located at a distance y_1 from the neutral axis.



- The forces exerted on this element consist of vertical shearing forces V'_C and V'_D, a horizontal shearing force ΔH exerted on the lower face of the element, elementary horizontal normal forces $\sigma_C dA$ and $\sigma_D dA$, and possibly a load $w\Delta x$.



- The equilibrium equation for horizontal forces is

$$\sum F_x = 0 \rightarrow \Delta H + \int_a (\sigma_c - \sigma_D) dA = 0$$

- where the integral extends over the shaded area A of the section located above the line $y = y_1$.

- Solving the equation for ΔH and using $\sigma = \frac{My}{I}$ to express the normal stresses in terms of the bending moments at C and D, provides

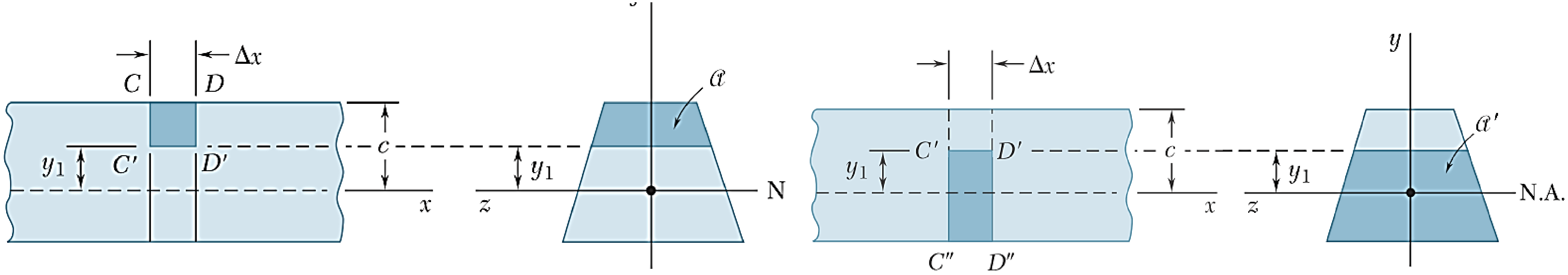
$$\Delta H = \frac{M_D - M_C}{I} \int_a y dA$$

- The integral represents the **first moment** with respect to the neutral axis of the **portion $\boldsymbol{a}$ of the cross section** of the beam that is located above the line $y = y_1$ and will be denoted by $\boldsymbol{Q}$. On the other hand, the increment $M_D - M_C$ of the bending moment is

$$M_D - M_C = \Delta M = (dM/dx)\Delta x = V\Delta x$$

- Substituting this, the horizontal shear exerted on the beam element become

$$\Delta H = \frac{VQ}{I}\Delta x$$



- This leads us to observe that the first moment Q of the portion a' of the cross section located below the line $y = y_1$ is equal in magnitude and opposite in sign to the first moment of the portion a located above that line.

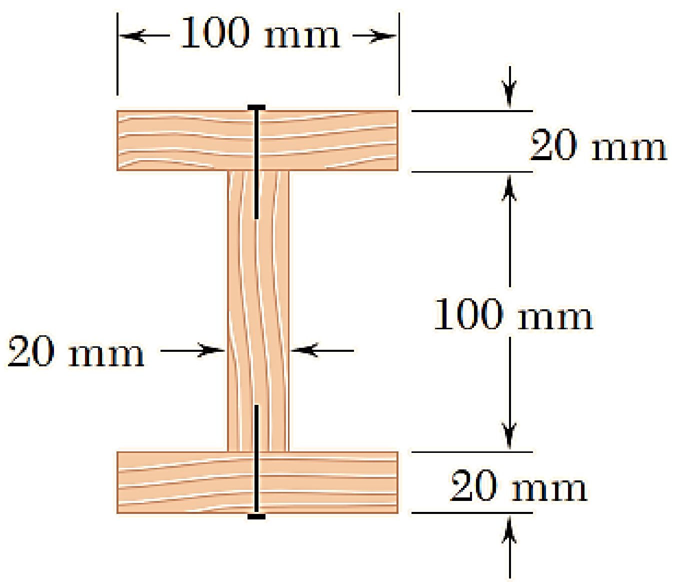
- The **horizontal shear per unit length**, which will be denoted by q, is obtained by dividing ΔH to Δx :

$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I}$$

- Recall that $\boldsymbol{Q}$ **is the first moment with respect to the neutral axis** of the portion of the cross section located either above or below the point at which q is being computed and that $\boldsymbol{I}$ **is the centroidal moment of inertia of the entire cross-sectional area**.

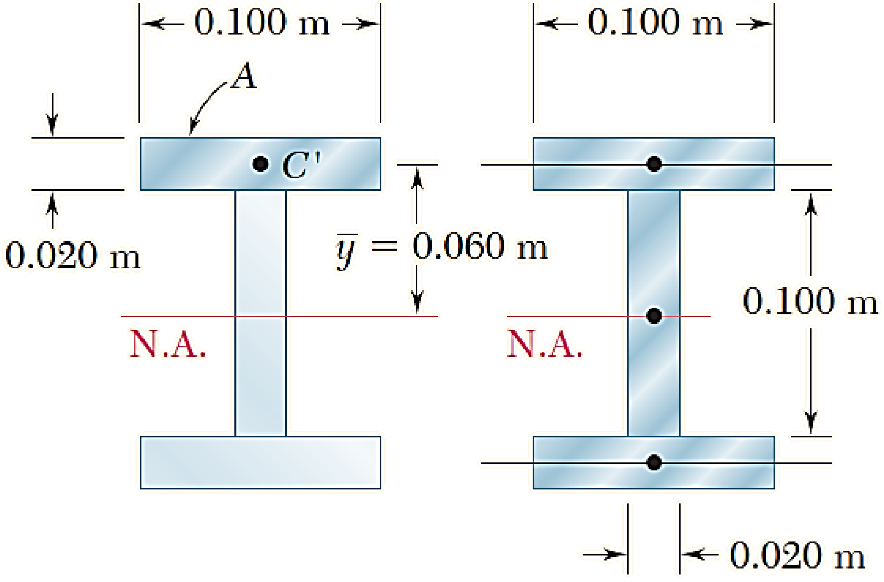
- The **horizontal shear per unit length** $\boldsymbol{q}$ **is also called the shear flow**.

Concept Application 6.1



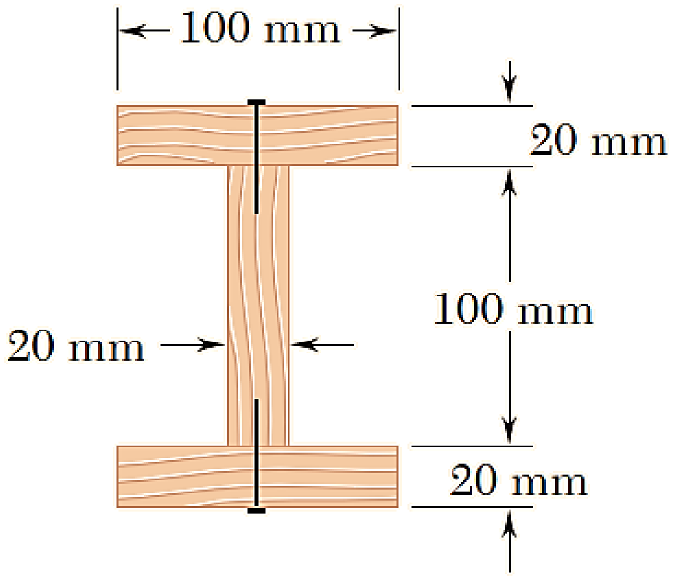
A beam is made of three planks, 20 by 100 mm in cross section, and nailed together. Knowing that the spacing between nails is 25 mm and the vertical shear in the beam is V = 500 N, determine the shearing force in each nail. Determine the horizontal force per unit length q exerted o the n lower face of the upper plank.

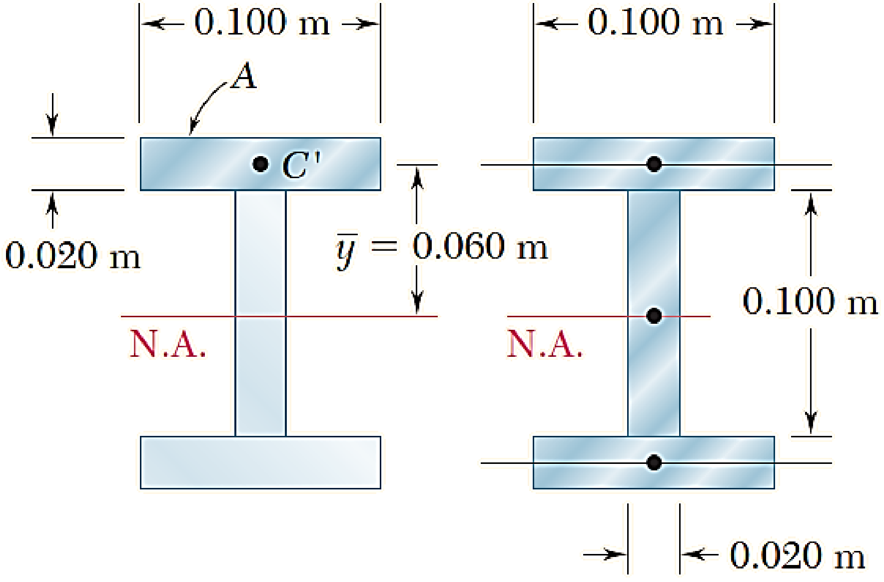
We will use $q = \dfrac{\Delta H}{\Delta x} = \dfrac{VQ}{I}$



where Q represents the first moment with respect to the neutral axis of the shaded area A,
and I is the moment of inertia about the same axis of the entire cross-sectional area.

Concept Application 6.1



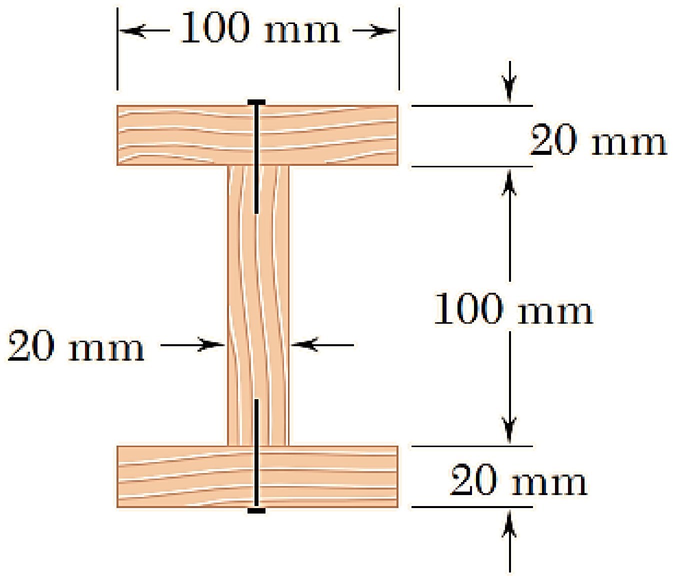


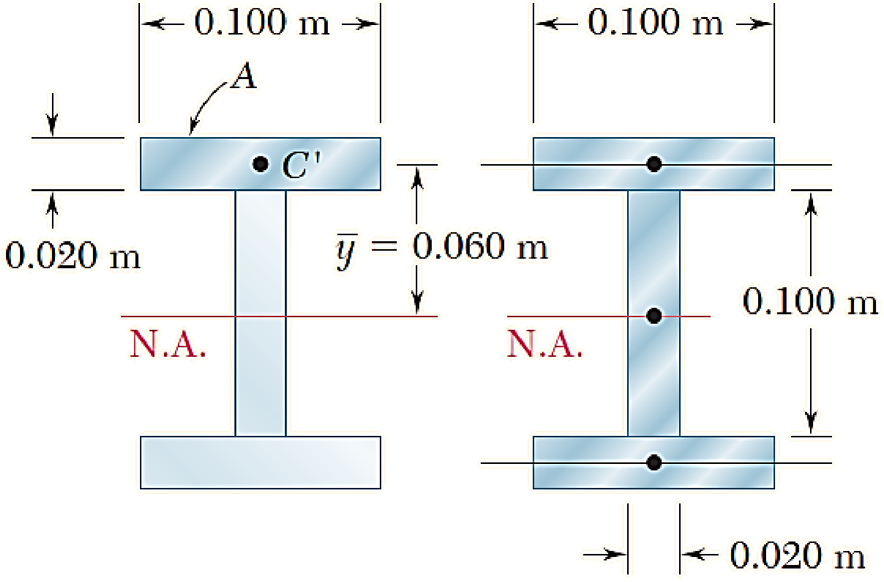
$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I}$$

$$Q = A\bar{y} = (0.020\text{ m} \times 0.100\text{ m})(0.060\text{ m})$$
$$= 120 \times 10^{-6}\text{ m}^3$$

$$I = \tfrac{1}{12}(0.020\text{ m})(0.100\text{ m})^3$$
$$+2[\tfrac{1}{12}(0.100\text{ m})(0.020\text{ m})^3$$
$$+(0.020\text{ m} \times 0.100\text{ m})(0.060\text{ m})^2]$$
$$= 1.667 \times 10^{-6} + 2(0.0667 + 7.2)10^{-6}$$
$$= 16.20 \times 10^{-6}\text{ m}^4$$

Concept Application 6.1





Substituting into Eq.

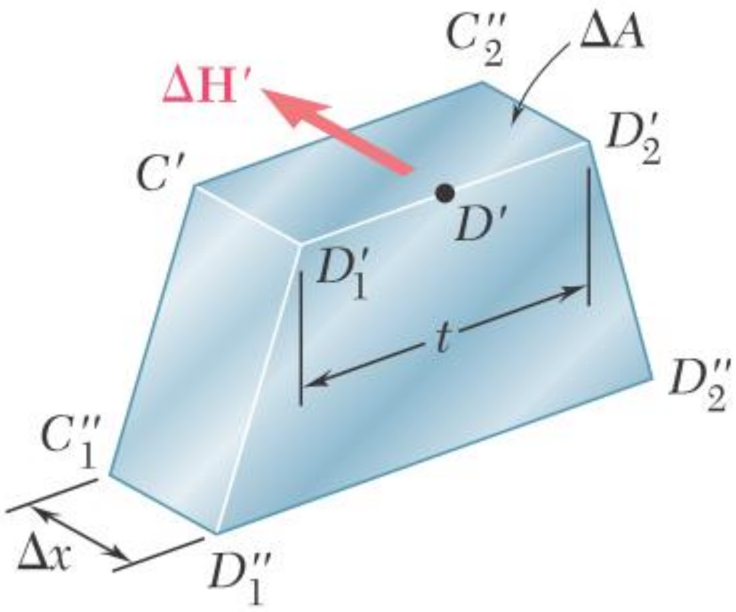
$$q = \frac{VQ}{I} = \frac{(500\ \text{N})(120 \times 10^{-6}\ \text{m}^3)}{16.20 \times 10^{-6}\ \text{m}^4} = 3704\ \text{N/m}$$

Since the spacing between the nails is 25 mm, the shearing force in each nail is

$$F = (0.025\ \text{m})q = (0.025\ \text{m})(3704\ \text{N/m})$$

$$= 92.6\ \text{N}$$

Shearing Stresses in a Beam

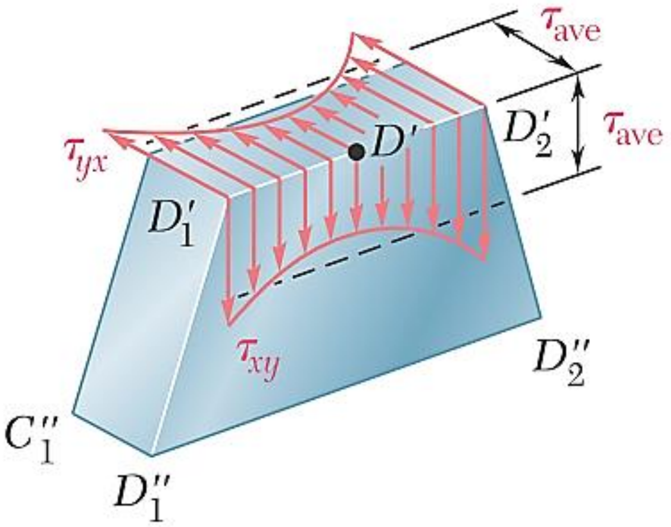


- Consider again a beam with a vertical plane of symmetry that is subjected to various concentrated or distributed loads applied in that plane. If, through two vertical cuts and one horizontal cut, an element of length Δx is detached from the beam.

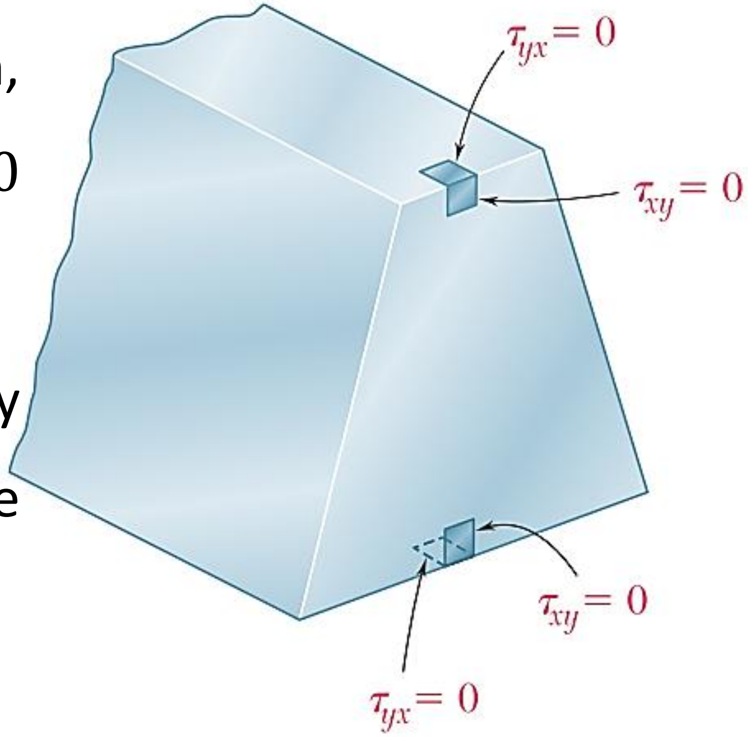
The average shearing stress τ_{ave} on that face of the element is obtained by dividing ΔH by the area ΔA of the face. Observing that $\Delta A = t\Delta x$, where t is the width of the element at the cut, we write

$$\tau_{ave} = \frac{\Delta H}{\Delta A} = \frac{VQ}{I}\frac{\Delta x}{t\Delta x} = \frac{VQ}{It}$$

- Note that since the shearing stresses τ_{xy} and τ_{yx} exerted on a transverse and a horizontal plane through D' are equal, the expression also represents the average value of τ_{xy} along the line $D_1' D_2'$.

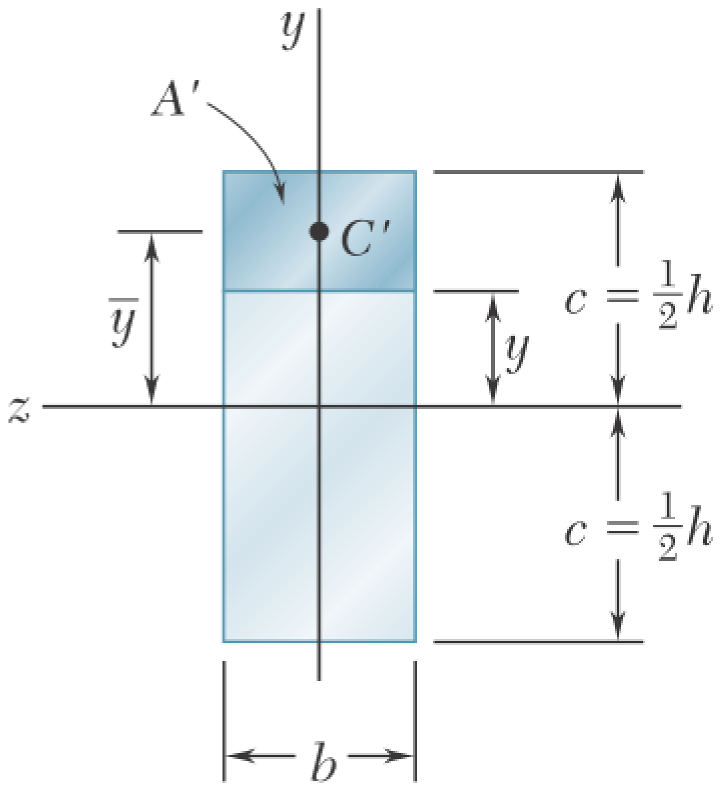


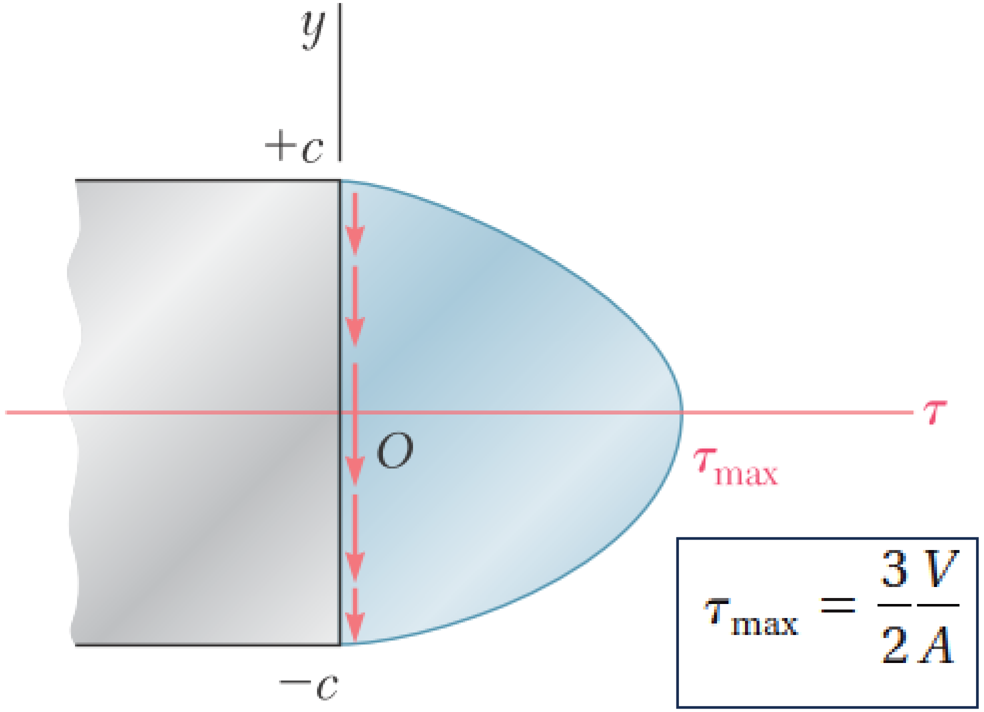
- Observe that $\tau_{yx} = 0$ on the upper and lower faces of the beam, since no forces are exerted on these faces. It follows that $\tau_{xy} = 0$ along the upper and lower edges of the transverse section.



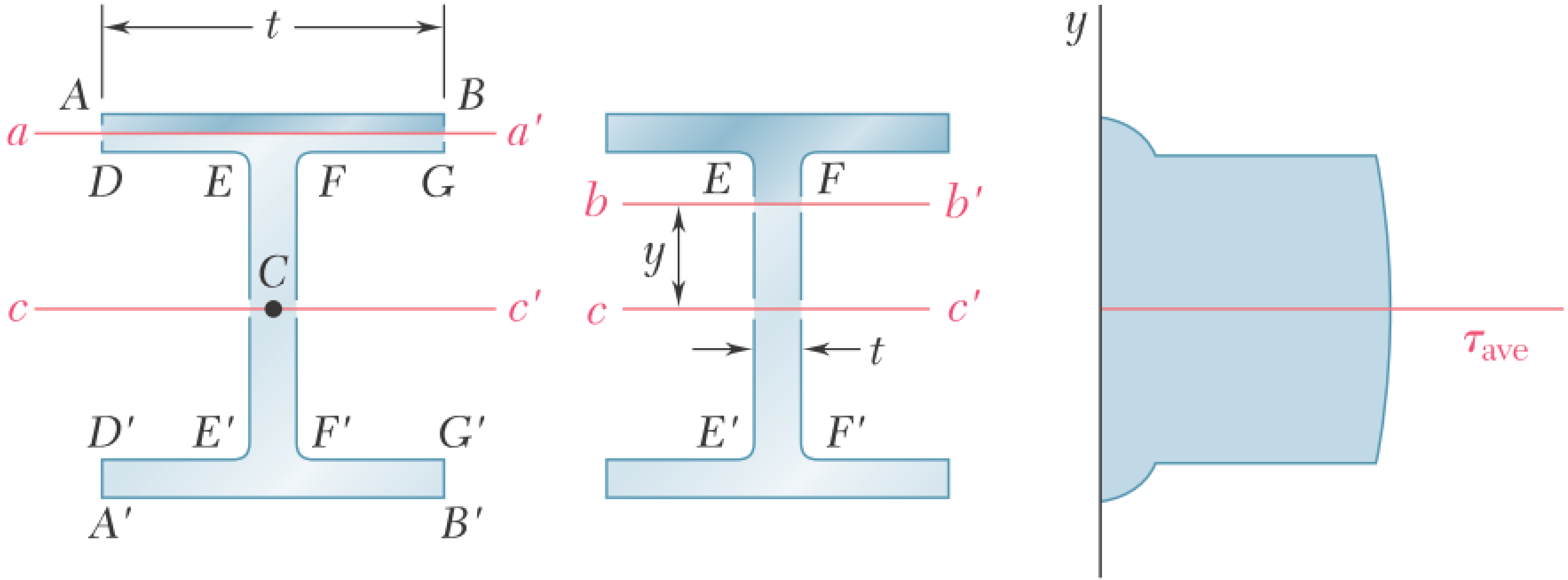
- Also note that while Q is maximum for $y = 0$ (i.e. at N.A), τ_{ave} may not be maximum along the neutral axis, since τ_{ave} depends upon the width t of the section as well as upon Q.

Shearing Stresses τ_{xy} In Common Beam Types





$$\tau_{max} = \frac{3}{2}\frac{V}{A}$$



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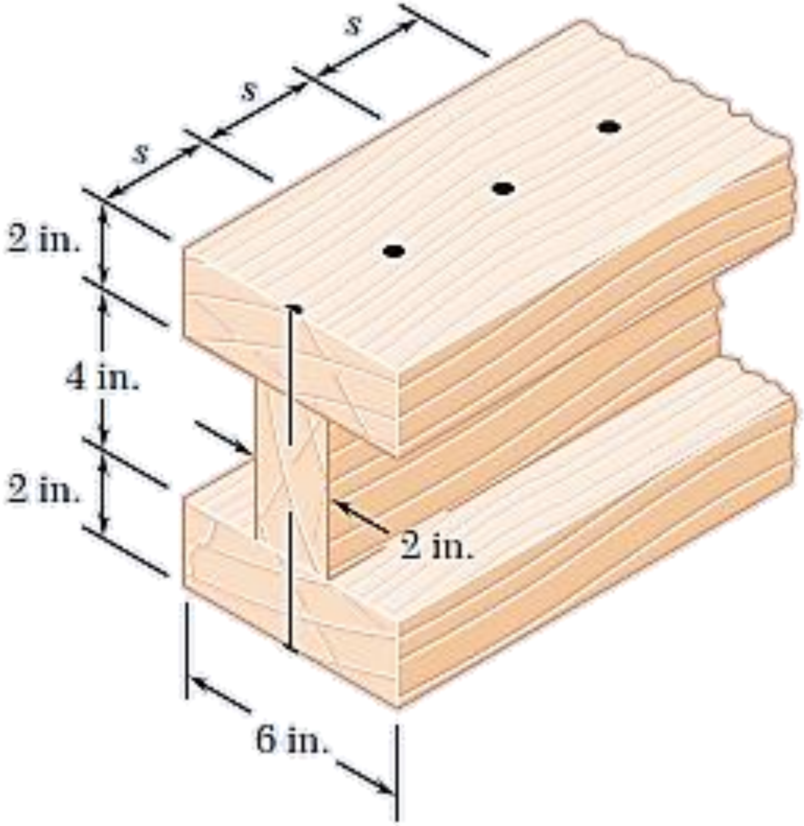
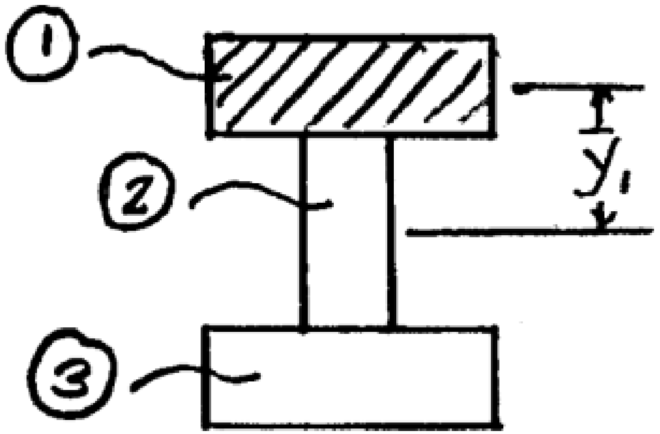
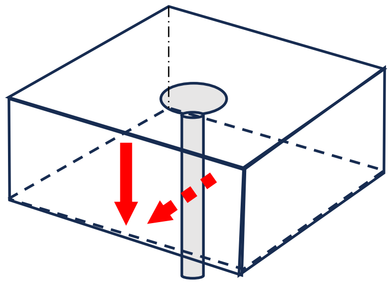
Example

Three boards, each 2 in. thick, are nailed together to form a beam that is subjected to a vertical shear. Knowing that the allowable shearing force in each nail is 150 lb, determine the allowable shear if the spacing ***s*** between the nails is 3 in.







$$Q = A_1\bar{y}_1 = (6)(2)(3) = 36 \text{ in}^3$$

$$qs = F_{\text{nail}} \quad (1)$$

$$q = \frac{VQ}{I} \quad (2)$$

Using equation (1) in (2):

$$\frac{1}{s} = \frac{VQ}{F_{\text{nail}} I}$$

$$V = \frac{F_{\text{nail}} I}{Qs} = \frac{(150)(234.667)}{(36)(3)}$$

$$I_1 = \frac{1}{12}bh^3 + Ad^2$$

$$= \frac{1}{12}(6)(2)^3 + (6)(2)(3)^2 = 112 \text{ in}^4$$

$$I_2 = \frac{1}{12}bh^3 = \frac{1}{12}(2)(4)^3 = 10.667 \text{ in}^4$$

$$I_3 = I_1 = 112 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 234.667 \text{ in}^4$$