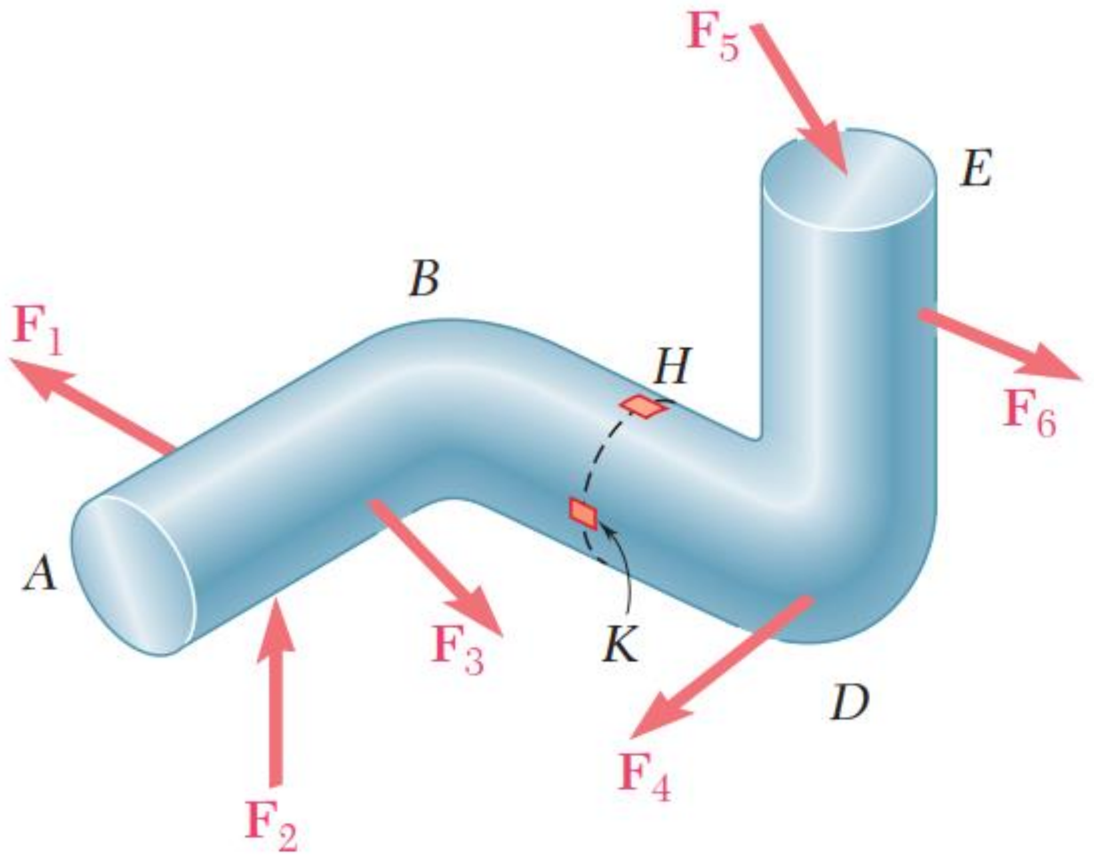
Combined Loading

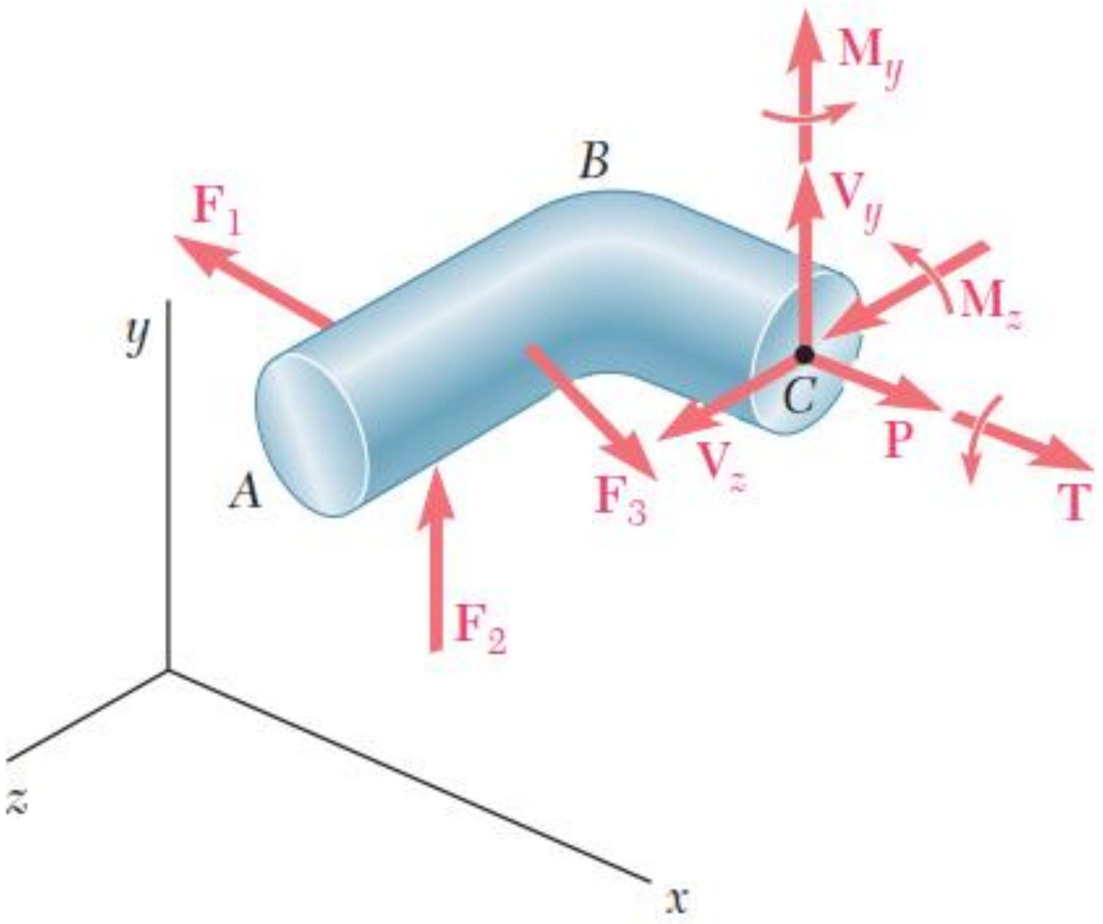
8

Objectives

- ✓ Determine stresses developed in a member's cross section when axial load, torsion, bending and shear occur simultaneously.

Stresses Under Combined Loads





Stresses Under Combined Loads

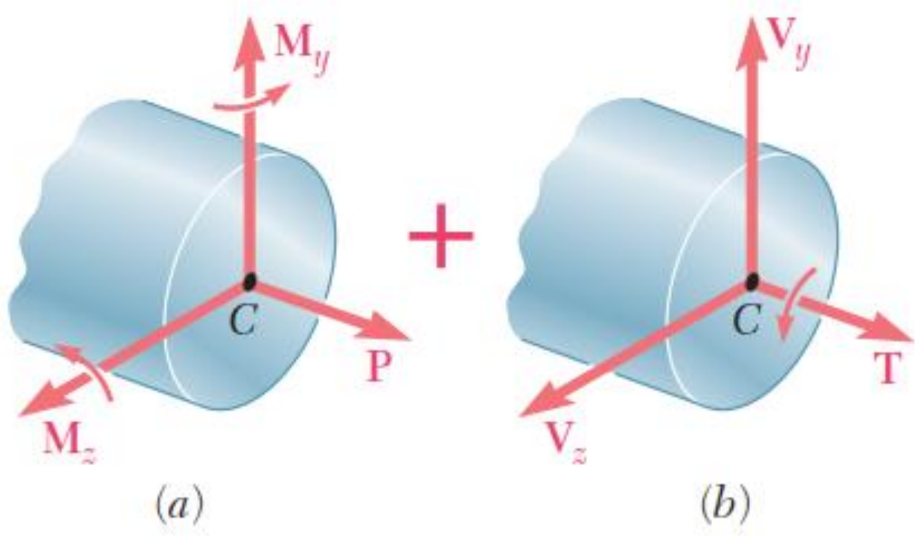


Fig. 8.17 Internal forces and couple vectors separated into (a) those causing normal stresses and (b) those causing shearing stresses.

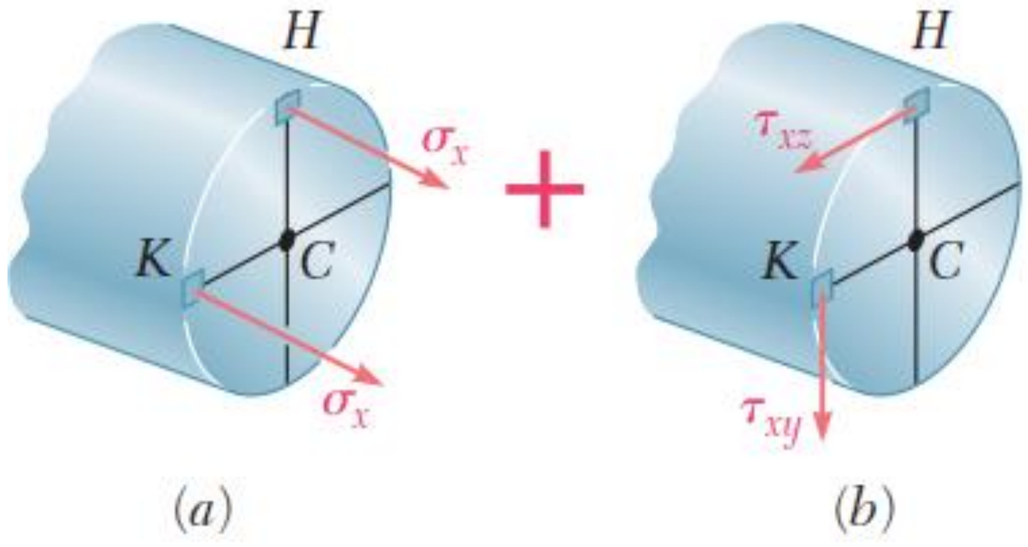


Fig. 8.18 Normal and shearing stresses at points H and K.

Stresses Under Combined Loads

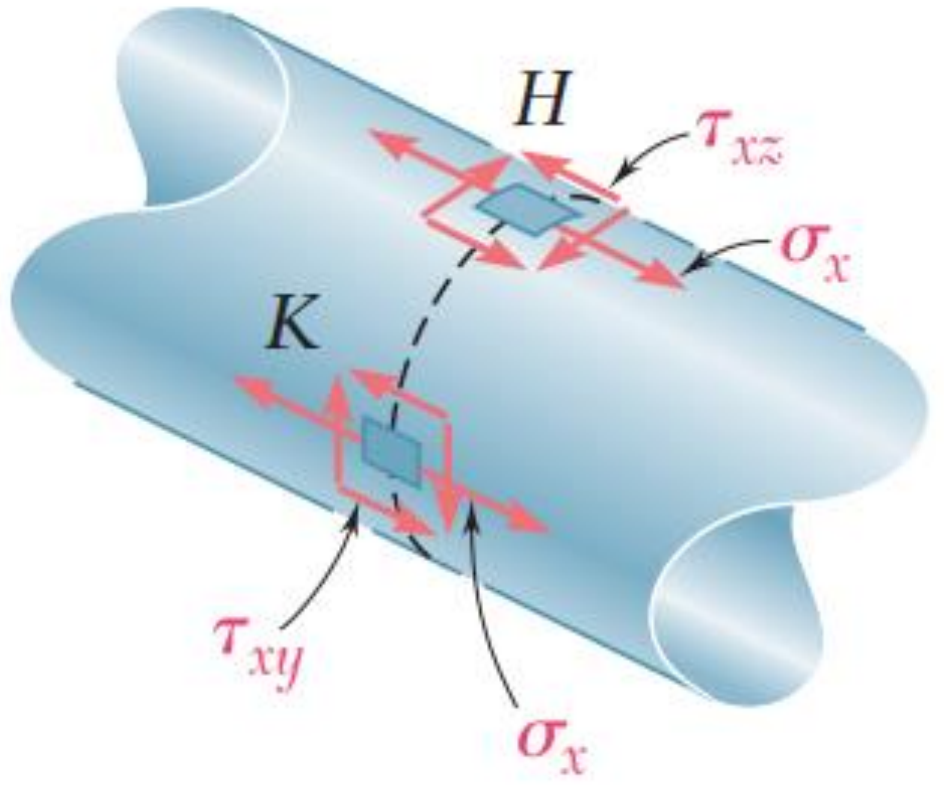
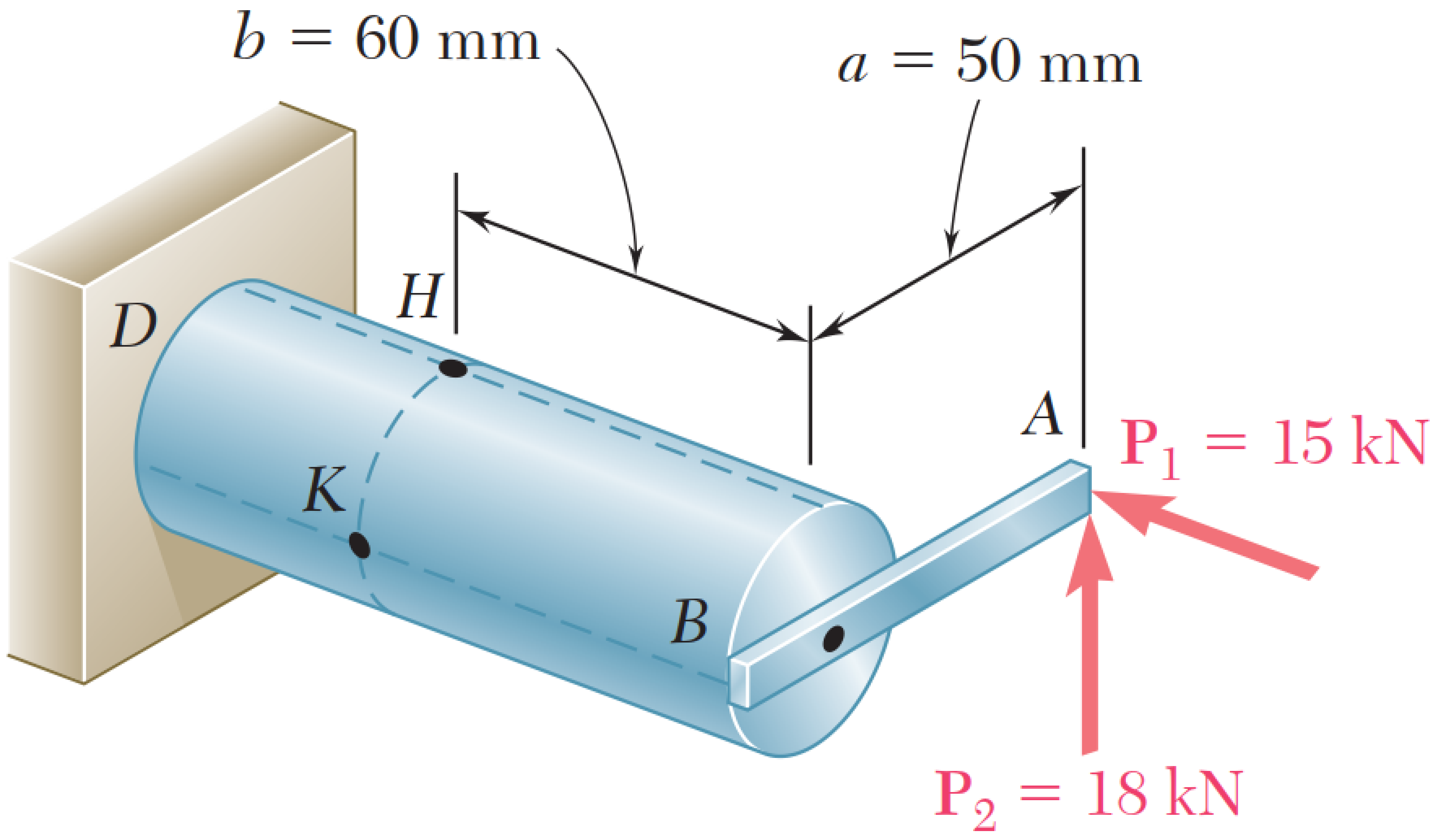


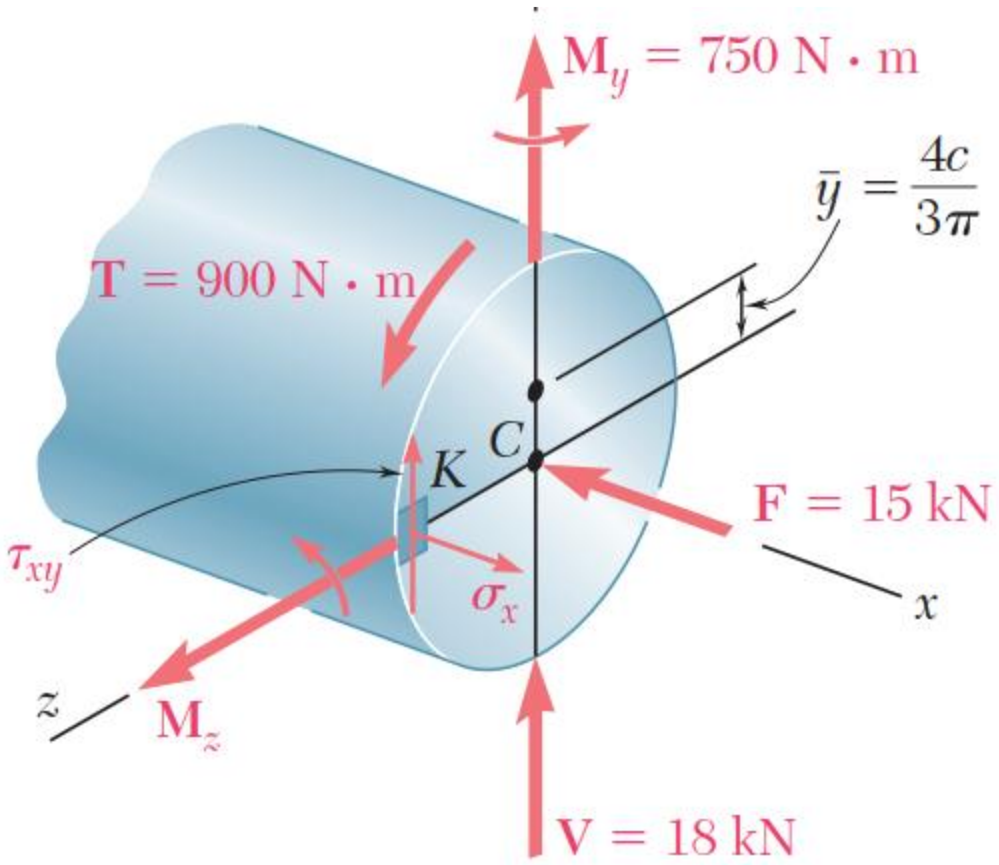
Fig. 8.19 Elements at points H and K showing combined stresses.

Example



Determine (*a*) the normal and shearing stresses at point *K* of the transverse section of member *BD* located at a distance *b* = 60 mm from end *B*, ~~(*b*) the principal axes and principal stresses at *K*, and (*c*) the maximum shearing stress at *K*.~~

Solution



$$F = P_1 = 15 \text{ kN}$$

$$V = P_2 = 18 \text{ kN}$$

$$T = P_2 a = (18 \text{ kN})(50 \text{ mm}) = 900 \text{ N·m}$$

$$M_y = P_1 a = (15 \text{ kN})(50 \text{ mm}) = 750 \text{ N·m}$$

$$M_z = P_2 b = (18 \text{ kN})(60 \text{ mm}) = 1080 \text{ N·m}$$

Solution

Geometric Properties of the Section

$$A = \pi c^2 = \pi(0.020\text{ m})^2 = 1.257 \times 10^{-3}\text{ m}^2$$

$$I_y = I_z = \tfrac{1}{4}\pi c^4 = \tfrac{1}{4}\pi(0.020\text{ m})^4 = 125.7 \times 10^{-9}\text{ m}^4$$

$$J_C = \tfrac{1}{2}\pi c^4 = \tfrac{1}{2}\pi(0.020\text{ m})^4 = 251.3 \times 10^{-9}\text{ m}^4$$

$$Q = A'\bar{y} = \left(\frac{1}{2}\pi c^2\right)\left(\frac{4c}{3\pi}\right) = \frac{2}{3}c^3 = \frac{2}{3}(0.020\text{ m})^3$$

$$= 5.33 \times 10^{-6}\text{ m}^3$$

$$t = 2c = 2(0.020\text{ m}) = 0.040\text{ m}$$

Solution

Normal Stresses.

$$\sigma_x = -\frac{F}{A} + \frac{M_y c}{I_y} = -11.9\text{ MPa} + \frac{(750\text{ N}\cdot\text{m})(0.020\text{ m})}{125.7 \times 10^{-9}\text{ m}^4}$$

$$= -11.9\text{ MPa} + 119.3\text{ MPa}$$

$$\sigma_x = +107.4\text{ MPa}$$

Shearing Stresses.

$$(\tau_{xy})_V = +\frac{VQ}{I_z t} = +\frac{(18 \times 10^3\text{ N})(5.33 \times 10^{-6}\text{ m}^3)}{(125.7 \times 10^{-9}\text{ m}^4)(0.040\text{ m})}$$

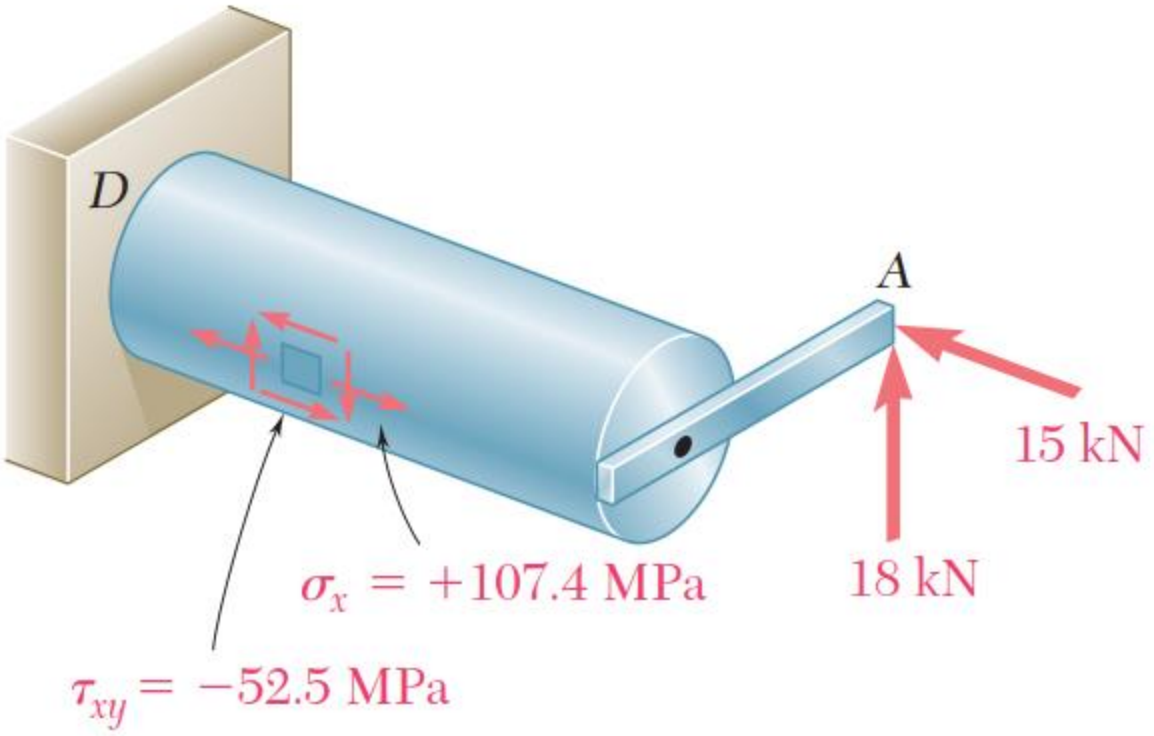
$$= +19.1\text{ MPa}$$

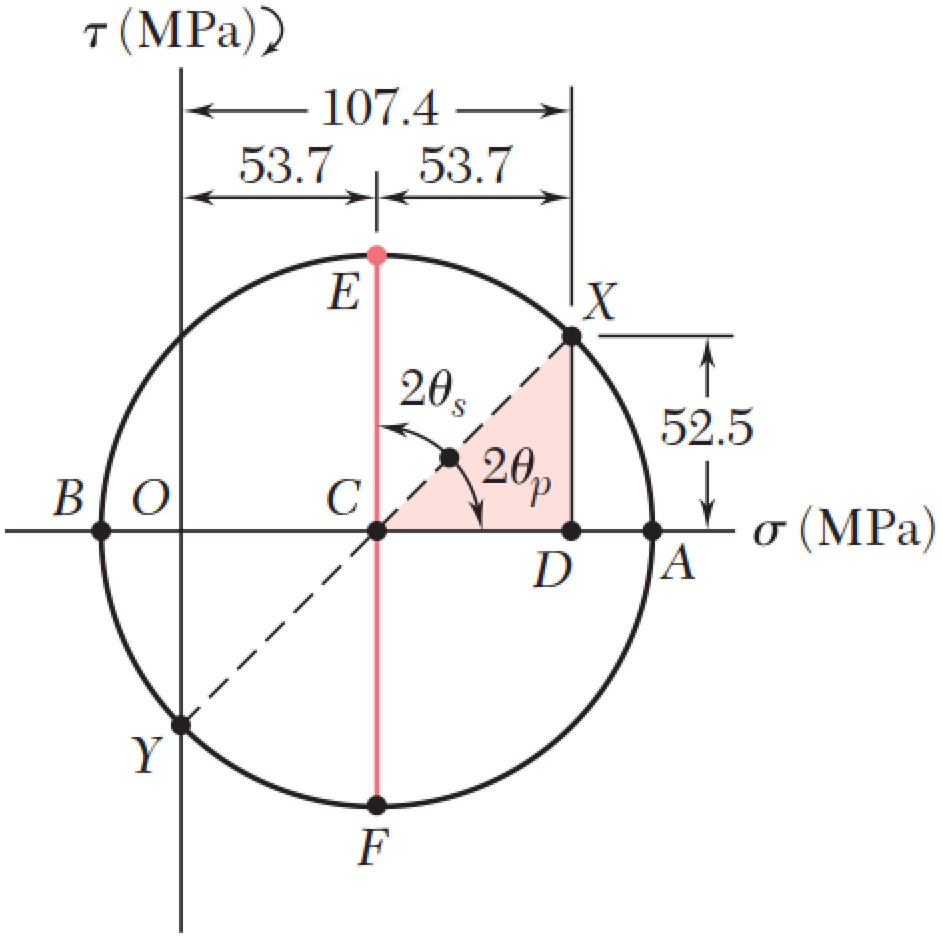
$$(\tau_{xy})_{\text{twist}} = -\frac{Tc}{J_C} = -\frac{(900\text{ N}\cdot\text{m})(0.020\text{ m})}{251.3 \times 10^{-9}\text{ m}^4} = -71.6\text{ MPa}$$

$$\tau_{xy} = (\tau_{xy})_V + (\tau_{xy})_{\text{twist}} = +19.1\text{ MPa} - 71.6\text{ MPa}$$

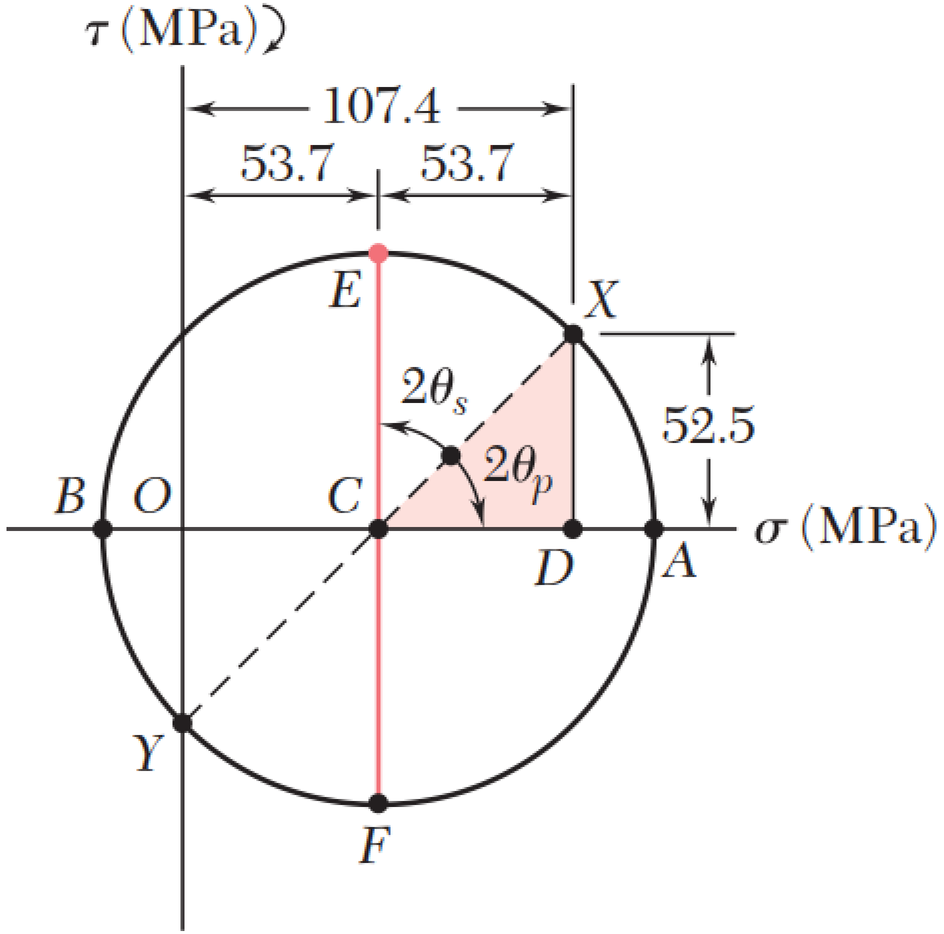
$$\tau_{xy} = -52.5\text{ MPa}$$

Solution


D
A
15 kN
18 kN
$\sigma_x = +107.4$ MPa
$\tau_{xy} = -52.5$ MPa


τ (MPa)
107.4
53.7
53.7
E
X
$2\theta_s$
$2\theta_p$
52.5
B
O
C
D
A
σ (MPa)
Y
F

Solution



Principal Planes and Principal Stresses at Point *K*.

$$\tan 2\theta_p = \frac{DX}{CD} = \frac{52.5}{53.7} = 0.97765 \qquad 2\theta_p = 44.4^\circ \curvearrowright$$

$$\theta_p = 22.2^\circ \curvearrowright$$

$$R = \sqrt{(53.7)^2 + (52.5)^2} = 75.1 \text{ MPa}$$

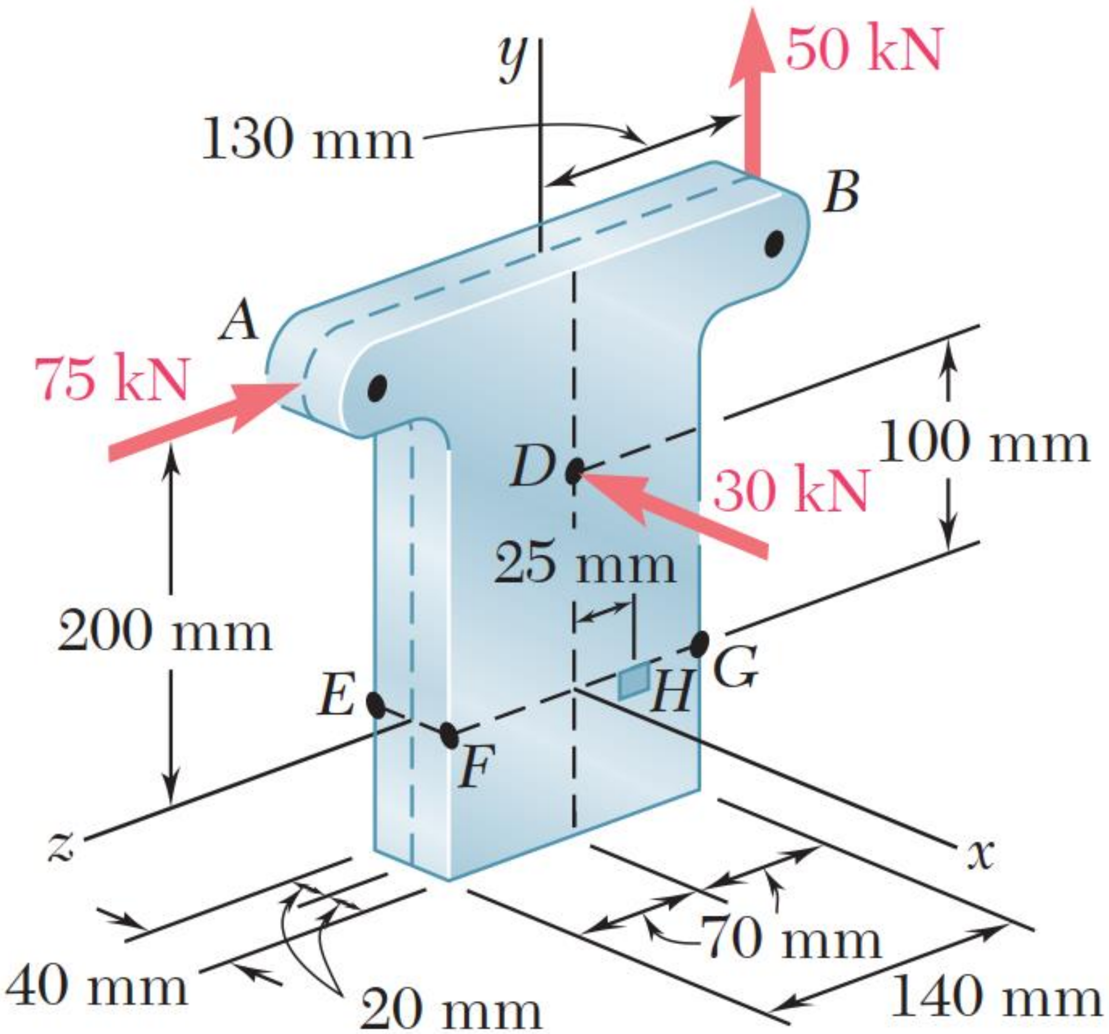
$$\sigma_{\max} = OC + R = 53.7 + 75.1 = 128.8 \text{ MPa}$$

$$\sigma_{\min} = OC - R = 53.7 - 75.1 = -21.4 \text{ MPa}$$

Maximum Shearing Stress at Point *K*.

$$\tau_{\max} = CE = R = 75.1 \text{ MPa} \qquad \theta_s = 22.8^\circ \curvearrowleft$$

Example 2



For the loading shown determine the principal stresses, principal planes, and maximum shearing stress at point *H*.

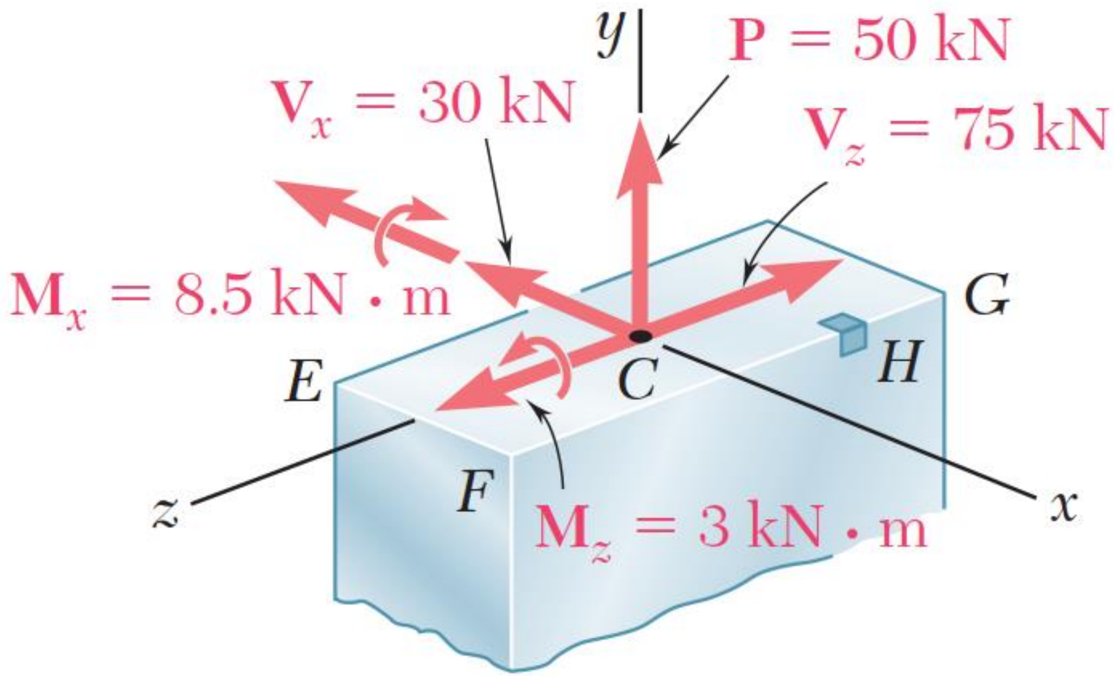
Solution

Internal Forces in Section *EFG*.

$$V_x = -30\text{ kN} \qquad P = 50\text{ kN} \qquad V_z = -75\text{ kN}$$

$$M_x = (50\text{ kN})(0.130\text{ m}) - (75\text{ kN})(0.200\text{ m}) = -8.5\text{ kN}\cdot\text{m}$$

$$M_y = 0 \qquad M_z = (30\text{ kN})(0.100\text{ m}) = 3\text{ kN}\cdot\text{m}$$

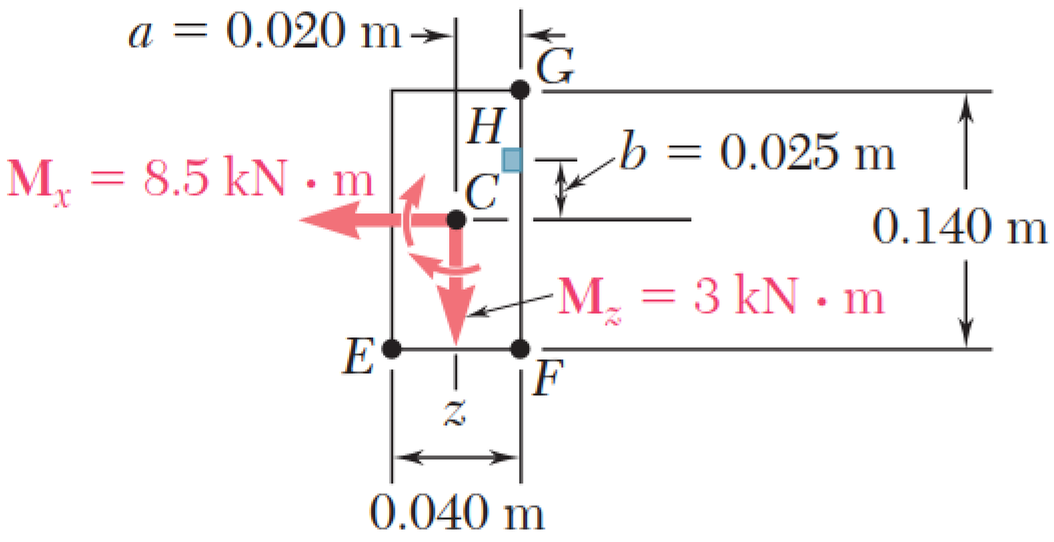


$$A = (0.040\text{ m})(0.140\text{ m}) = 5.6 \times 10^{-3}\text{ m}^2$$

$$I_x = \tfrac{1}{12}(0.040\text{ m})(0.140\text{ m})^3 = 9.15 \times 10^{-6}\text{ m}^4$$

$$I_z = \tfrac{1}{12}(0.140\text{ m})(0.040\text{ m})^3 = 0.747 \times 10^{-6}\text{ m}^4$$

Solution



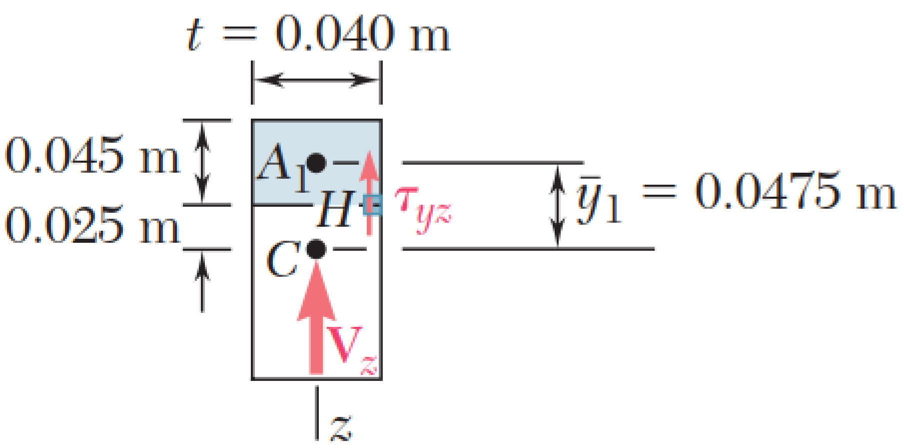
Normal Stress at *H*.

$$\sigma_y = +\frac{P}{A} + \frac{|M_z|a}{I_z} - \frac{|M_x|b}{I_x}$$

$$= \frac{50\text{ kN}}{5.6 \times 10^{-3}\text{ m}^2} + \frac{(3\text{ kN·m})(0.020\text{ m})}{0.747 \times 10^{-6}\text{ m}^4} - \frac{(8.5\text{ kN·m})(0.025\text{ m})}{9.15 \times 10^{-6}\text{ m}^4}$$

$$\sigma_y = 8.93\text{ MPa} + 80.3\text{ MPa} - 23.2\text{ MPa} \qquad \sigma_y = 66.0\text{ MPa}$$

Solution



Considering the shearing force $\mathbf{V}_x$, we note that $Q = 0$ with respect to the z axis, since H is on the edge of the cross section. Thus, $\mathbf{V}_x$ produces no shearing stress at H.

The shearing force $\mathbf{V}_z$ does produce a shearing stress at H

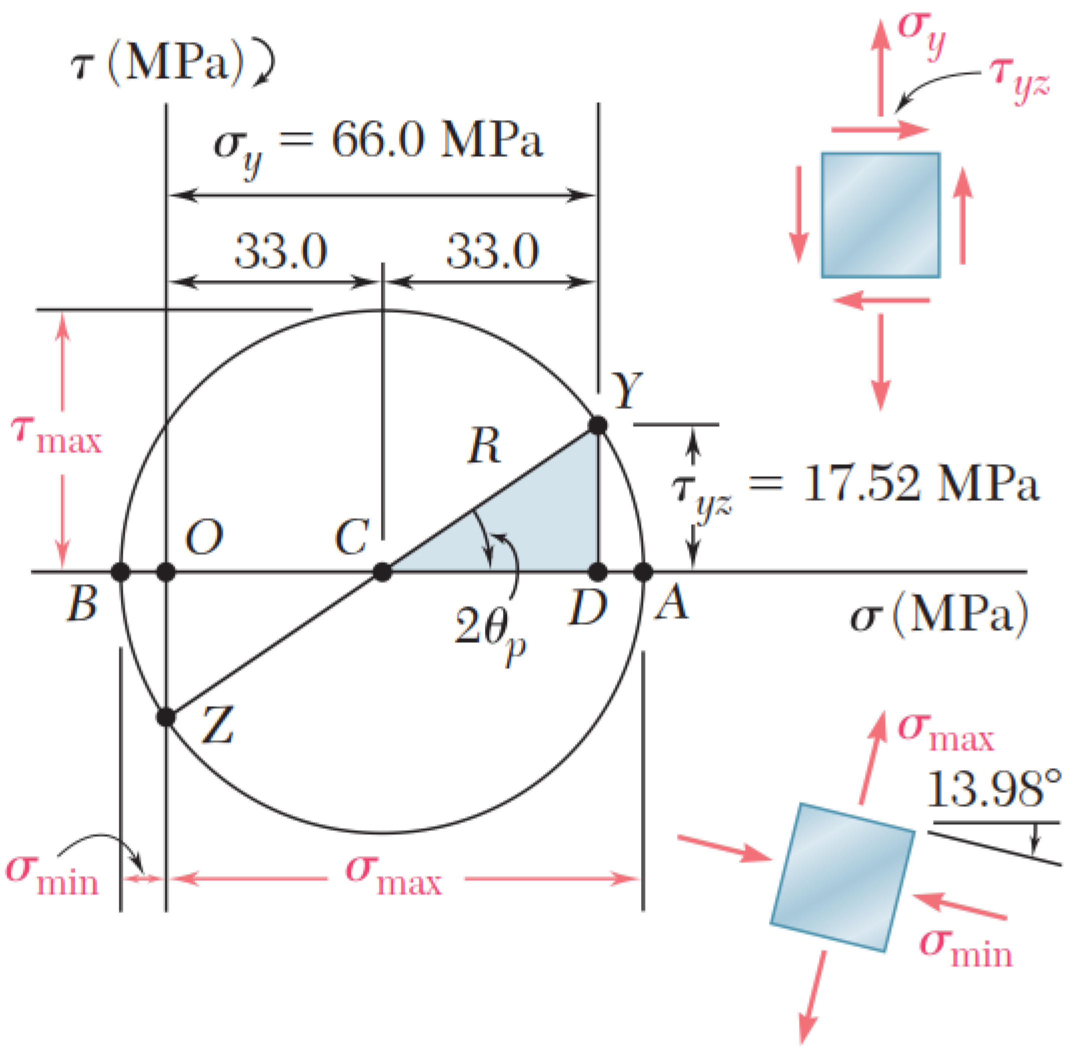
Shearing Stress at *H*.

$$Q = A_1\bar{y}_1 = [(0.040\ \text{m})(0.045\ \text{m})](0.0475\ \text{m}) = 85.5 \times 10^{-6}\ \text{m}^3$$

$$\tau_{yz} = \frac{V_z Q}{I_x t} = \frac{(75\ \text{kN})(85.5 \times 10^{-6}\ \text{m}^3)}{(9.15 \times 10^{-6}\ \text{m}^4)(0.040\ \text{m})} \qquad \tau_{yz} = 17.52\ \text{MPa}$$

Solution

Principal Stresses, Principal Planes, and Maximum Shearing Stress at *H*.



$$\tan 2\theta_p = \frac{17.52}{33.0} \qquad 2\theta_p = 27.96^\circ$$

$$R = \sqrt{(33.0)^2 + (17.52)^2} = 37.4 \text{ MPa}$$

$$\sigma_{\max} = OA = OC + R = 33.0 + 37.4$$

$$\sigma_{\min} = OB = OC - R = 33.0 - 37.4$$

$$\theta_p = 13.98^\circ$$

$$\tau_{\max} = 37.4 \text{ MPa}$$

$$\sigma_{\max} = 70.4 \text{ MPa}$$

$$\sigma_{\min} = -7.4 \text{ MPa}$$

Problem 8.38

Two forces are applied to the pipe AB as shown. Knowing that the pipe has inner and outer diameters equal to 35 and 42 mm, respectively, determine the normal and shearing stresses at (a) point a, (b) point b.

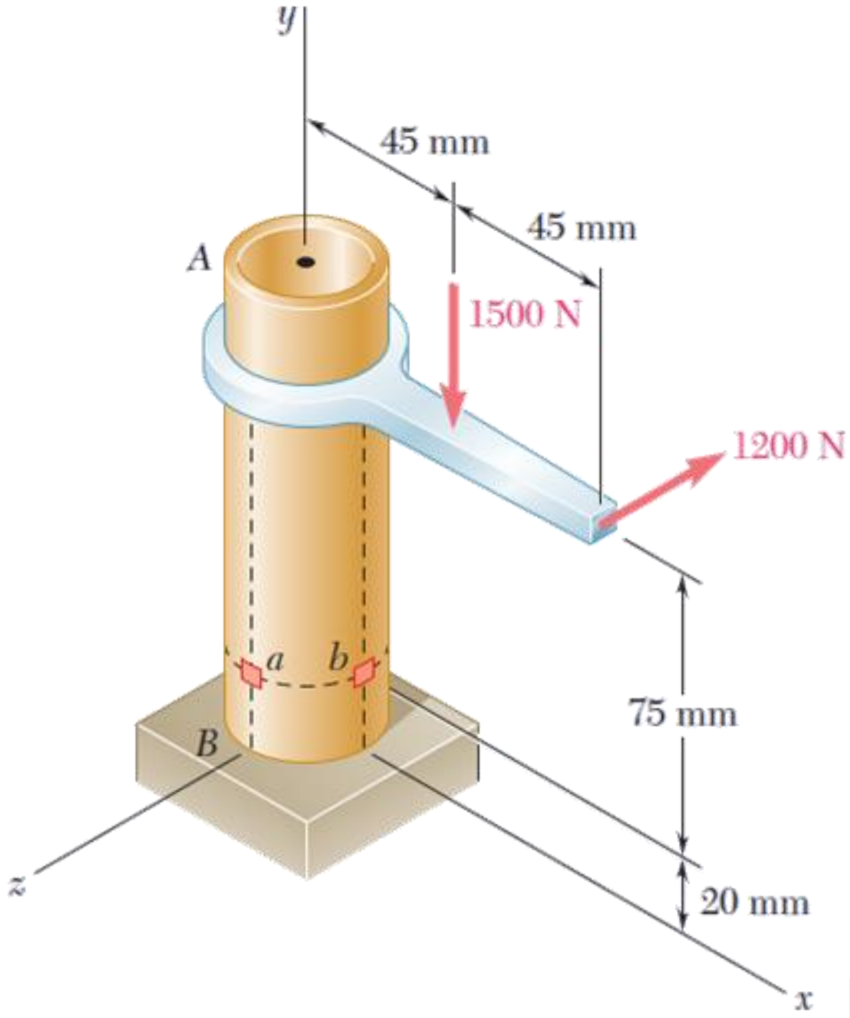
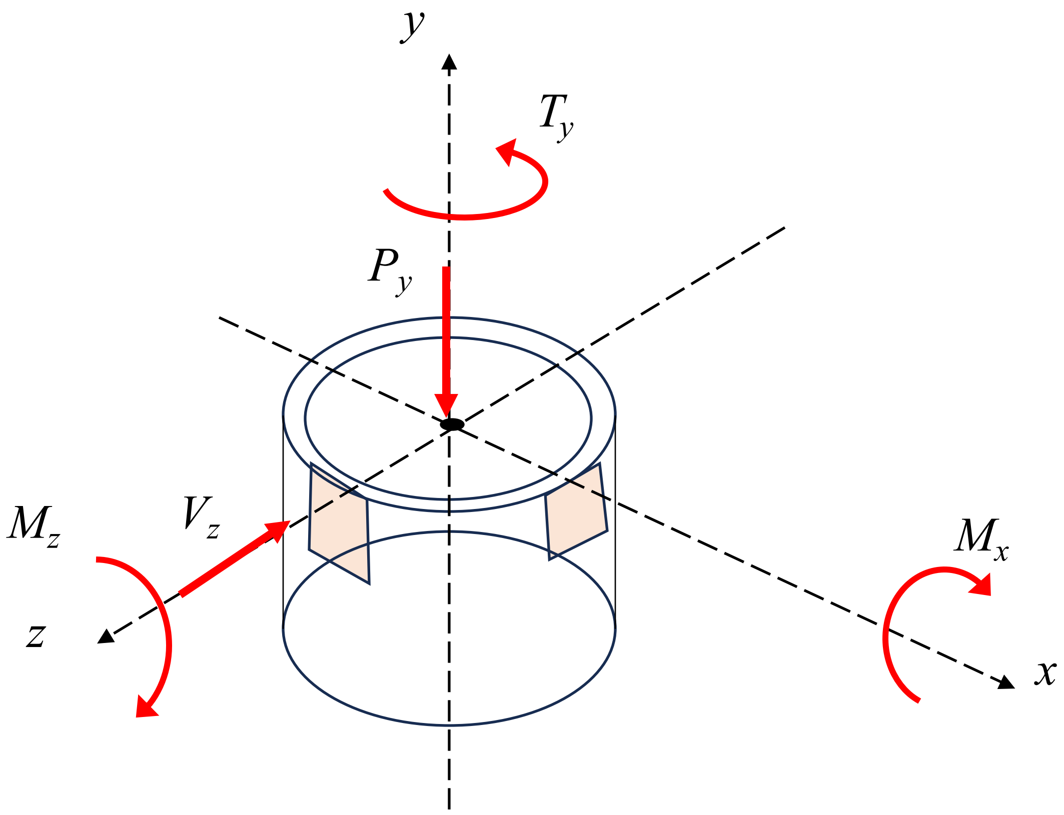


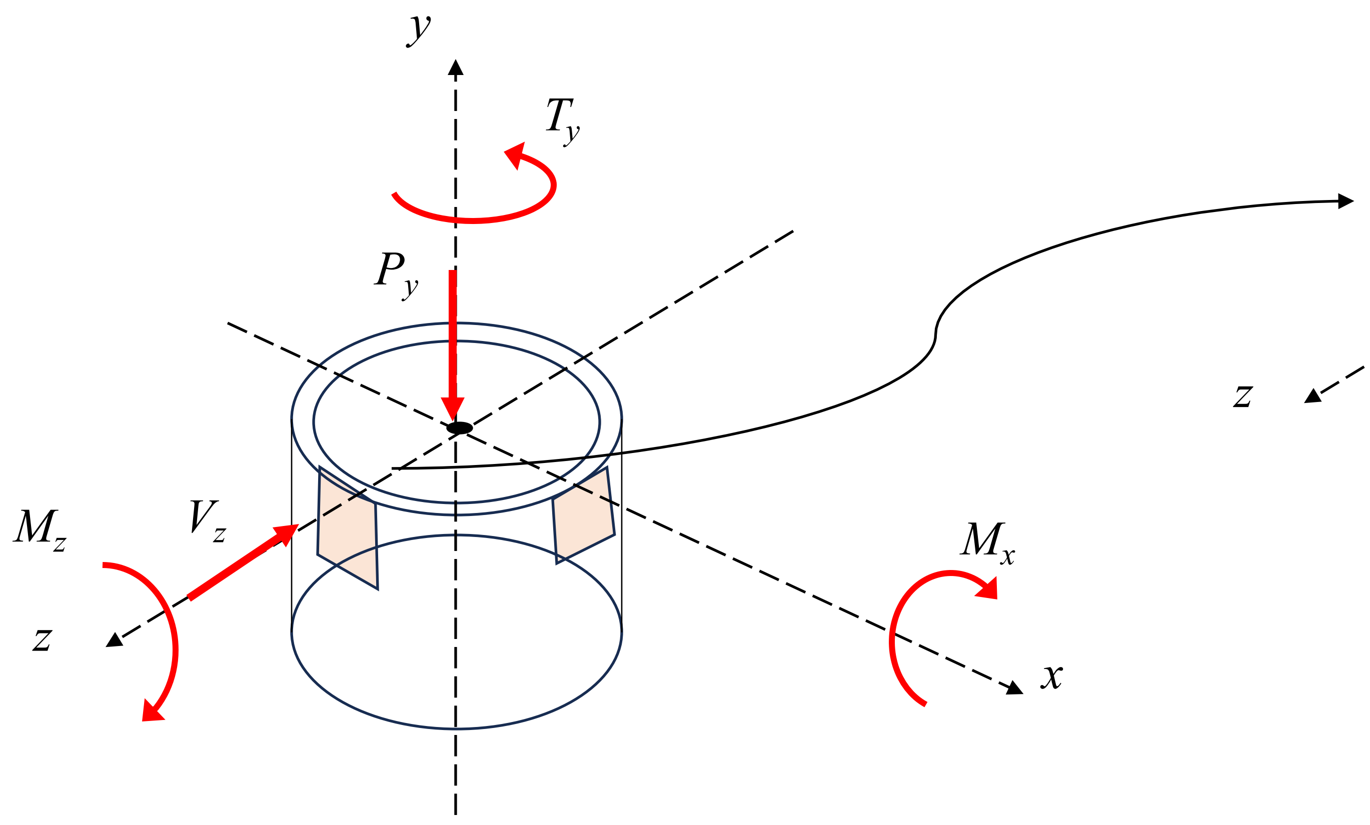
Fig. P8.38

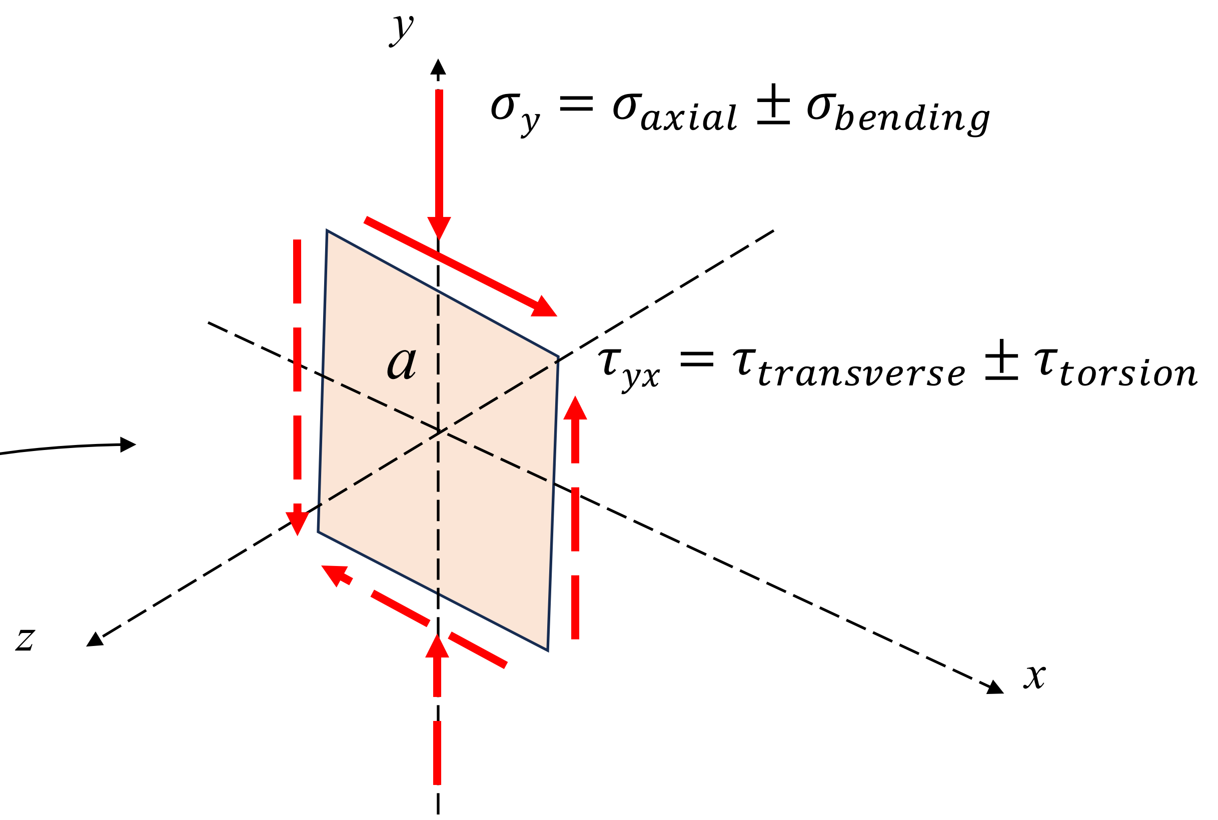


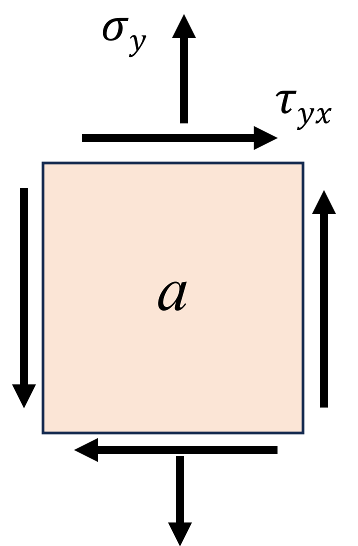
Reaction loads on the crossection should be found first (i.e, applied loads need to be moved to center of crossection)

(*a*) 20.4 MPa; 14.34 MPa. (*b*) −21.5 MPa; 19.98 MPa.

Problem 8.38



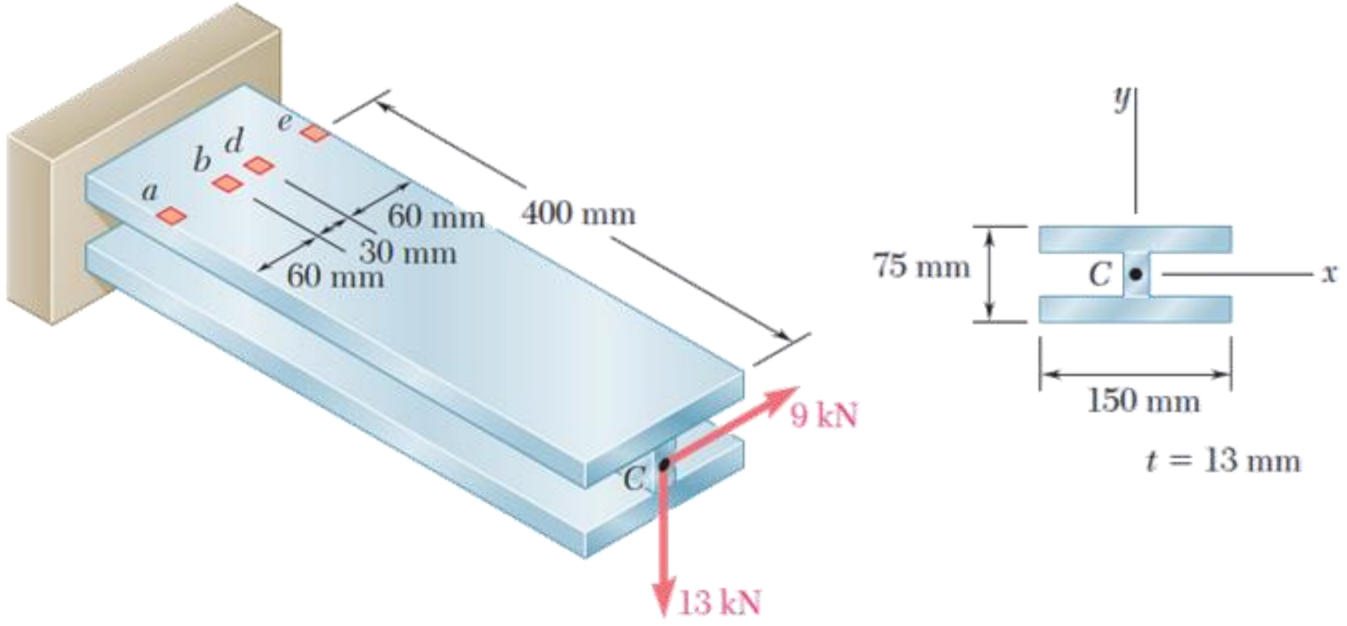




Stress state of point a

Problem 8.53

Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points a and b.



(a) 86.5 MPa; 0. (b) 57.0 MPa; 9.47 MPa.