

HOMEWORK-3

(Submit it at the start of the next lecture, 19 Oct 2026)

Develop computer codes for the following questions and compare your analytical (handwritten) results with your code results. Draw the functions as plots.

1. Using Taylor's series, derive first-order forward-difference and rearward-difference expressions for $\partial u / \partial y$.
2. Using Taylor's series, derive the second-order central difference for $\partial u / \partial y$.
3. Consider the function $\phi(x, y) = e^x + e^y$. Consider the point $(x, y) = (1, 1)$.
 - (a) Calculate the *exact* values of $\partial \phi / \partial x$ and $\partial \phi / \partial y$ at this point.
 - (b) Use first-order forward differences, with $\Delta x = \Delta y = 0.1$, to calculate approximate values of $\partial \phi / \partial x$ and $\partial \phi / \partial y$ at point $(1, 1)$. Calculate the percentage difference when compared with the exact values from part (a).
 - (c) Use first-order rearward differences, with $\Delta x = \Delta y = 0.1$, to calculate approximate values of $\partial \phi / \partial x$ and $\partial \phi / \partial y$ at point $(1, 1)$. Calculate the percentage difference when compared with the exact values from part (a).
 - (d) Use second-order central differences, with $\Delta x = \Delta y = 0.1$, to calculate approximate values for $\partial \phi / \partial x$ and $\partial \phi / \partial y$ at point $(1, 1)$. Calculate the percentage difference when compared with the exact value from part (a).
4. Repeat Prob.3, but with $\Delta x = \Delta y = 0.01$. Compare the accuracy of the finite-difference results obtained here with those obtained in Prob.3.
5. Derive the following expression, which is a third-order-accurate one-sided difference.

$$\left(\frac{\partial u}{\partial y} \right)_{i,j} = \frac{1}{6\Delta y} (-11u_{i,j} + 18u_{i,j+1} - 9u_{i,j+2} + 2u_{i,j+3})$$

NOTE: The report should be concise and should not exceed **10 pages**, excluding the source code.