Oct 5th, 2026

HOMEWORK-2

(Submit it at the start of the next lecture, 12 Oct 2026)

Q1) For a fully developed laminar flow between two parallel plates, the solution of the momentum equation, subject to the boundary conditions

$$y = 0, \quad u = 0; \quad y = 2h, \quad u = u_0 = \text{const}$$

is

$$u = u_0 \frac{y}{2h} - \frac{h^2}{2\mu}\frac{dp}{dx}\frac{y}{h}\left(2 - \frac{y}{h}\right) \tag{1}$$

Eq. (1) is known as Couette flow. When both plates are stationary ($u_o = 0$), Couette flow becomes

$$u = -\frac{h^2}{2\mu}\frac{dp}{dx}\frac{y}{h}\left(2 - \frac{y}{h}\right) \tag{2}$$

which is the case of plane Poiseuille flow. Thus Couette flow comprises the linear velocity distribution of Eq. (2) given above due to the shear flow, between the two plates with no imposed pressure gradient, together with the quadratic velocity distribution caused by the pressure parameter, P:

$$P = -(h^2/2\mu u_0)(p_0 - p_i)/L \tag{3}$$

The flow represented by Eq. (3) is similar to that existing in the narrow clearance between a journal bearing and a stationary shaft, where inertial effects are negligible and the oil is forced along the annulus. Write a code and compare your code results with analytical solution for the following questions. Show that;

a. the average velocity U_m is related to the maximum velocity U_{max} by

$$\frac{u_{\max}}{u_m} = \frac{3}{2} \tag{4}$$

b. and the dimensionless pressure drop can be written as

$$\frac{p_i - p_0}{\rho u_m^2} = \frac{3}{R_h}\frac{L}{h} \quad \text{where} \quad R_h = \frac{u_m h}{\nu}. \tag{5}$$

c. Plot u/u_o against $y/2h$ from Eq. (PS.3) for values of P = -2, -1, 0, 1, 2 (from Eq. (3)) to illustrate the effect of pressure gradient on the velocity profile.

Q2)

a. Consider Laplace's equation, given by

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

Show that this is an elliptic equation.

b. Show that the second-order wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

is a hyperbolic equation.

c. Show that the first-order wave equation

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

is a hyperbolic equation.

Draw all the equations in a plot using a computer program. Compare and discuss their differences.

NOTE: The report should be concise and should not exceed **10 pages**, excluding the source code. No CFD flow solution is required.