ME 209
NUMERICAL METHODS

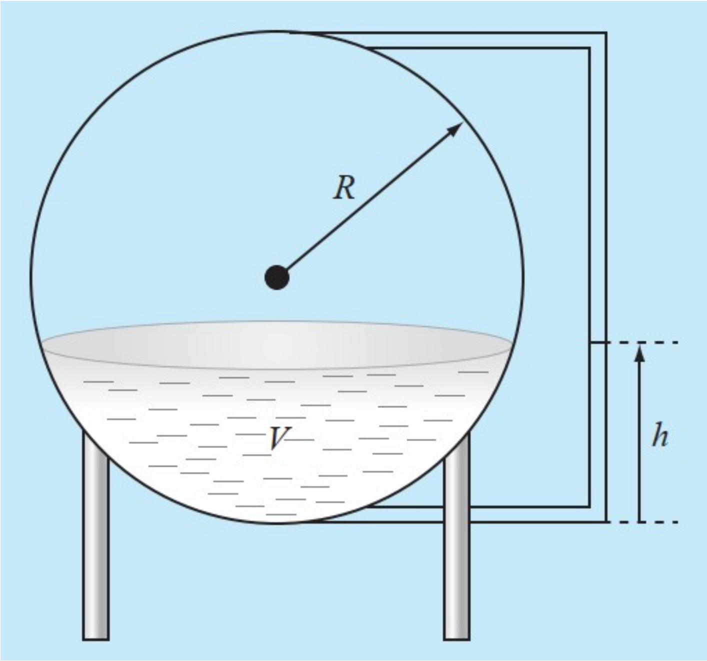
PROBLEM HOUR I

CH1 to CH4

Q1. You are designing a spherical tank in given figure to hold water for a small village in a developing country. The volume of liquid it can hold can be computed as

where V = volume [m^3], h = depth of water in tank [m], and R = the tank radius [m].

If R = 3 m, to what depth must the tank be filled so that it holds 30 m^3? Use three iterations of the false-position method to determine your answer. Determine the approximate relative error after each iteration. Employ initial guesses of 0 and R.



$$V = \pi h^2 \frac{[3R - h]}{3}$$

Q2. Water is flowing in a trapezoidal channel at a rate of Q = 20 m^3/s. The critical depth y for such a channel must satisfy the equation

$$0 = 1 - \frac{Q^2}{gA_c^3}B$$

where g = 9.81 m/s^2, A_c = the cross-sectional area (m^2), and B = the width of the channel at the surface (m). For this case, the width and the cross-sectional area can be related to depth y by

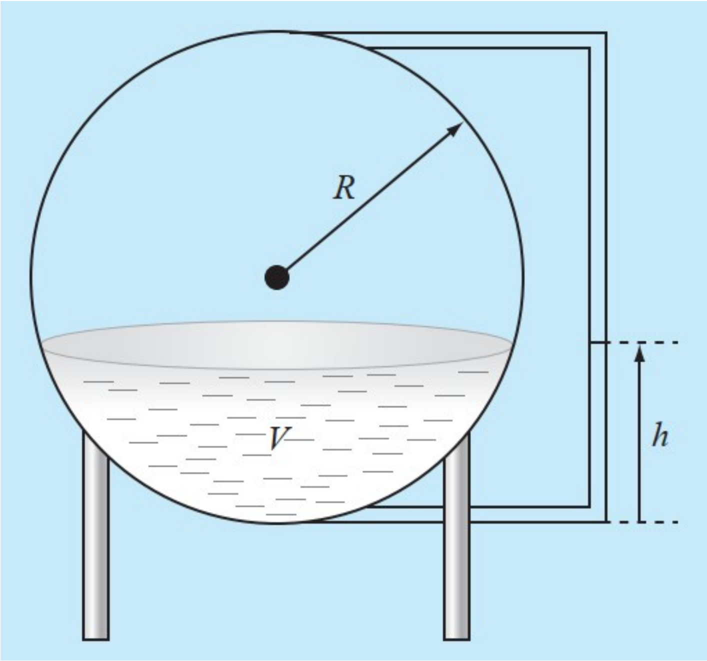
$$B = 3 + y \qquad \text{and} \qquad A_c = 3y + \frac{y^2}{2}$$

Solve for the critical depth using **(a)** bisection method, and **(b)** false position. Use initial guesses of x_l = 0.5 and x_u = 2.5, and iterate until the approximate error falls below 2% or the number of iterations exceeds 6. Discuss your results.

Q3. You are designing a spherical tank in given figure to hold water for a small village in a developing country. The volume of liquid it can hold can be computed as

where V = volume [m^3], h = depth of water in tank [m], and R = the tank radius [m].

If R = 3 m, to what depth must the tank be filled so that it holds 30 m^3? Use three iterations of the Newton-Raphson method to determine your answer. Determine the approximate relative error after each iteration. Note that an initial guess of R will always converge.



$$V = \pi h^2 \frac{[3R - h]}{3}$$

Q4. Use **(a)** fixed-point iteration and **(b)** the Newton-Raphson method to determine a root of $f(x) = -x^2 + 1.8x + 2.5$ using $x_0 = 5$. Perform the computation until ε_a is less than $\varepsilon_s = 0.75\%$. Also perform an error check of your final answer.