Question 4. Use the **Gauss–Seidel method** to solve the following system of simultaneous linear algebraic equations:
$3x_1$+-$7x_2$+-x_3=19
-$9x_1$+x_2-$2x_3$==-37
-$2x_1$-x_2+$8x_3$=28

Use initial guesses of:
$x_1 = 0$
$x_2 = 0$

$x_3 = 0$

Perform iterations until the absolute approximate relative percentage error for all unknowns falls below $\mathbf{5\%}$.

If necessary, rearrange the equations to ensure convergence.

Three blocks are placed on a frictionless inclined plane and connected by linear springs. The first and third blocks are also connected to fixed supports by springs.

Solution 4.

REARRANGEMENT OF EQUATIONS

To ensure convergence, rearrange the equations to obtain a diagonally dominant system:
$-9x_1 + x_2 - 2x_3 = -37$
$3x_1 - 7x_2 - x_3 = 19$
$-2x_1 - x_2 + 8x_3 = 28$
Diagonal dominance check:
9 > 1 + 2 OK
7 > 3 + 1 OK
8 > 2 + 1 OK
Gauss–Seidel iteration equations:
$x_1 = (37 + x_2 - 2x_3)/9$
$x_2 = (3x_1 - x_3 - 19)/7$
$x_3 = (28 + 2x_1 + x_2)/8$
Iteration 0
$x_1^{(0)} = 0$
$x_2^{(0)} = 0$
$x_3^{(0)} = 0$

Iteration 1
$x_1^{(1)} = [37 + 0 - 2(0)]/9 = 4.111111$
$x_2^{(1)} = [3(4.111111) - 0 - 19]/7 = -0.952381$
$x_3^{(1)} = [28 + 2(4.111111) - 0.952381]/8 = 4.408730$

$\varepsilon_a(x_1) = |(4.111111 - 0)/4.111111| \times 100 = 100\% > 5\%$
$\varepsilon_a(x_2) = |(-0.952381 - 0)/(-0.952381)| \times 100 = 100\% > 5\%$
$\varepsilon_a(x_3) = |(4.408730 - 0)/4.408730| \times 100 = 100\% > 5\%$

Iteration 2

$x_1^{(2)} = [37 - 0.952381 - 2(4.408730)]/9 = 3.025573$
$x_2^{(2)} = [3(3.025573) - 4.408730 - 19]/7 = -2.047430$
$x_3^{(2)} = [28 + 2(3.025573) - 2.047430]/8 = 4.000465$

$\varepsilon_a(x_1) = |(3.025573 - 4.111111)/3.025573| \times 100 = 35.8788\% > 5\%$
$\varepsilon_a(x_2) = |(-2.047430 + 0.952381)/(-2.047430)| \times 100 = 53.4841\% > 5\%$
$\varepsilon_a(x_3) = |(4.000465 - 4.408730)/4.000465| \times 100 = 10.2055\% > 5\%$

Iteration 3

$x_1^{(3)} = [37 - 2.047430 - 2(4.000465)]/9 = 2.994627$
$x_2^{(3)} = [3(2.994627) - 4.000465 - 19]/7 = -2.002369$
$x_3^{(3)} = [28 + 2(2.994627) - 2.002369]/8 = 3.998361$

$\varepsilon_a(x_1) = |(2.994627 - 3.025573)/2.994627| \times 100 = 1.0334\% < 5\%$ OK
$\varepsilon_a(x_2) = |(-2.002369 + 2.047430)/(-2.002369)| \times 100 = 2.2504\% < 5\%$ OK
$\varepsilon_a(x_3) = |(3.998361 - 4.000465)/3.998361| \times 100 = 0.0526\% < 5\%$ OK

The solution vector is:

$x_1 = 2.994627$
$x_2 = -2.002369$
$x_3 = 3.998361$

the exact solution is:
$x_1 = 3$
$x_2 = -2$
$x_3 = 4$