ME 209
Numerical Methods

Problem Hour 3

INTERPOLATION METHODS

Summary

Linear Interpolation

$$f_1(x) = b_0 + b_1(x - x_0) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$

Quadratic Interpolation

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

or, collecting terms,

$$f_2(x) = a_0 + a_1 x + a_2 x^2 \quad \text{where} \quad \begin{aligned} a_0 &= b_0 - b_1 x_0 + b_2 x_0 x_1 \\ a_1 &= b_1 - b_2 x_0 - b_2 x_1 \\ a_2 &= b_2 \end{aligned}$$

For b_0, $x = x_0$ $\qquad b_0 = f(x_0)$

For b_1, $x = x_1$ $\qquad b_1 = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

For b_2, $x = x_2$ $\qquad b_2 = \dfrac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$

Summary

General Form of Newton's Interpolating Polynomials

The nth-order polynomial is

$$f_n(x) = b_0 + b_1(x - x_0) + \cdots + b_n(x - x_0)(x - x_1)\cdots(x - x_{n-1})$$

the coefficients:

$$b_0 = f(x_0)$$
$$b_1 = f[x_1, x_0]$$
$$b_2 = f[x_2, x_1, x_0]$$
$$\vdots$$
$$b_n = f[x_n, x_{n-1}, \ldots, x_1, x_0]$$

where the bracketed function evaluations are finite divided differences.

For example,

$$f[x_i, x_j] = \frac{f(x_i) - f(x_j)}{x_i - x_j}$$

$$f[x_i, x_j, x_k] = \frac{f[x_i, x_j] - f[x_j, x_k]}{x_i - x_k}$$

$$f[x_n, x_{n-1}, \ldots, x_1, x_0] = \frac{f[x_n, x_{n-1}, \ldots, x_1] - f[x_{n-1}, x_{n-2}, \ldots, x_0]}{x_n - x_0}$$

1. Estimate the common logarithm of 5 using linear interpolation. **(a)** Interpolate between log 4=0.60206 and log 6= 0.7781513. **(b)** Interpolate between log 4.5= 0.6532125 and log 5.5=0.7403627. For each of the interpolations, compute the percent relative error based on the true value (log 5= 0.69897).

2. Fit a second-order Newton's interpolating polynomial to estimate log 5 using the data from Prob. 1 at x 4, 4.5, and 5. Compute the true percent relative error.

3. Fit a third-order Newton's interpolating polynomial to estimate log 5 using the data from Prob. 1.

4. Given the data

x	1.6	2	2.5	3.2	4	4.5
$f(x)$	2	8	14	15	8	2

Calculate $f(2.8)$ using Newton's interpolating polynomials of order 1 through 3. Choose the sequence of the points for your estimates to attain the best possible accuracy. First, fill the finite divided difference table below.

Finite Divided Difference Table

i	x_i	$f(x_i)$	$f[x_{i+1}, x_i]$	$f[x_{i+2}, x_{i+1}, x_i]$	$f[x_{i+3}, x_{i+2}, x_{i+1}, x_i]$
0	1.6	2			
1	2.0	8			
2	2.5	14			
3	3.2	15			
4	4.0	8			
5	4.5	2			