ME 209
Numerical Methods

Problem Hour 4

INTERPOLATION-REGRESSION-DIFFERENTIATION

1. Use the portion of the given steam table for superheated H_2O at 200 MPa to
   a) find the corresponding entropy ***s*** for a specific volume ***v*** of 0.108 m^3/kg with linear interpolation
   b) find the same corresponding entropy using quadratic interpolation

v (m^3/kg)	0.10377	0.11144	0.1254
s (kJ/(kg.K))	6.4147	6.5453	6.7664

2. Rather than using the base-***e*** exponential model, a common alternative is to use a base-10 model,

$$y = \propto_5 10^{\beta_5 x}$$

Use this model given above to fit the following data:

Table 1. Experimental data.

x	0.4	0.8	1.2	1.6	2	2.3
y	800	975	1500	1950	2900	3600

Hint:

$$y = a_0 + a_1 x$$

$$a_1 = \frac{n\sum_1^n x_i y_i - \sum_1^n x_i \sum_1^n y_i}{n\sum_1^n x_i^2 - (\sum_1^n x_i)^2}$$

$$a_0 = \bar{y} - a_1 \bar{x}$$

3. Compute forward and backward difference approximations of $O(h)$ (1st order approximation) and $O(h^2)$ (2nd order approximation), and central difference (two-step size) approximations of $O(h^2)$ (1st order approximation for centered) and $O(h^4)$ (2nd order approximation for centered) for the first derivative of $y = \sin x$ at $x = \pi/4$ using a value of $h = \pi/12$. Estimate the true percent relative error ε_t for each approximation.

	x	$f(x)$
x_{i-2}	$\pi/12$	0.259
x_{i-1}	$\pi/6$	0.5
x_i	$\pi/4$	0.707
x_{i+1}	$\pi/3$	0.866
x_{i+2}	$5\pi/12$	0.966

true = $\cos(\pi/4) = 0.707$

	first order	second order
Forward	0.607 (14.5%)	0.720 (−1.8%)
Back	0.791 (−10.6%)	0.726 (−2.6%)
Center	0.699 (1.1%)	0.7069 (0.014%)

Forward finite-divided-difference formulas: two versions are presented for each derivative. The latter version incorporates more terms of the Taylor series expansion and is, consequently, more accurate.

First Derivative	Error
$f'(x_i) = \dfrac{f(x_{i+1}) - f(x_i)}{h}$	$O(h)$
$f'(x_i) = \dfrac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h}$	$O(h^2)$

Backward finite-divided-difference formulas: two versions are presented for each

First Derivative	Error
$f'(x_i) = \dfrac{f(x_i) - f(x_{i-1})}{h}$	$O(h)$
$f'(x_i) = \dfrac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{2h}$	$O(h^2)$

Centered finite-divided-difference formulas: two versions are presented for each

First Derivative

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$$

$$f'(x_i) = \frac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{12h}$$