AC Steady-State Sinusoidal Analysis

Study Tutorial with Solutions

Part I: Sinusoidal Waveform Fundamentals

Identify and compute basic properties of sinusoidal signals: amplitude, frequency, period, phase angle, and angular frequency.

1. A voltage is given by $v(t) = 120 \cos(377t + 30°)$ V. Find the amplitude, angular frequency, frequency, period, and phase angle.

Answer: Amplitude $V_m = 120$ V; $\omega = 377$ rad/s; $f = \omega/2\pi = 60$ Hz; $T = 1/f = 16.67$ ms; $\varphi = +30°$

2. Write the expression for a sinusoidal current with peak value 5 A, frequency 50 Hz, and phase angle −45°.

Answer: $i(t) = 5 \cos(100\pi t - 45°)$ A [or equivalently $5 \cos(314.16t - 45°)$ A]

3. Convert $v(t) = 10 \sin(1000t + 60°)$ V to cosine form.

Answer: $v(t) = 10 \cos(1000t + 60° - 90°) = 10 \cos(1000t - 30°)$ V

4. Find the RMS value of $v(t) = 170 \cos(377t)$ V.

Answer: $V_{rms} = V_m/\sqrt{2} = 170/\sqrt{2} \approx 120.2$ V

5. Two voltages are $v_1 = 4 \cos(\omega t + 30°)$ and $v_2 = 6 \cos(\omega t - 15°)$. Which leads and by how much?

Answer: v_1 leads v_2 by $30° - (-15°) = 45°$

6. A current $i(t) = 8 \cos(2\pi \cdot 400t - 25°)$ mA. Find its frequency, period, and RMS value.

Answer: $f = 400$ Hz; $T = 2.5$ ms; $I_{rms} = 8/\sqrt{2} \approx 5.66$ mA

7. Convert $i(t) = -6 \cos(\omega t + 40°)$ to a positive cosine expression.

Answer: $i(t) = 6 \cos(\omega t + 40° + 180°) = 6 \cos(\omega t + 220°)$ [or $6 \cos(\omega t - 140°)$] A

8. A 60 Hz sinusoidal voltage has a peak-to-peak value of 340 V. Write its time-domain expression (zero phase).

Answer: $V_{pp} = 340$ V → $V_m = 170$ V; $v(t) = 170 \cos(120\pi t)$ V

9. What is the phase difference between $v_1 = 10 \cos(\omega t)$ and $v_2 = 10 \sin(\omega t)$? Which leads?

Answer: $\sin(\omega t) = \cos(\omega t - 90°)$, so v_1 leads v_2 by 90°

10. Find the average power delivered by $v(t) = 170 \cos(377t)$ V across a 10 Ω purely resistive load.

Answer: $P = V_{rms}^2/R = (170/\sqrt{2})^2/10 = 14450/10 = 1445$ W

Part II: Phasors — Representation and Conversion

Convert between time-domain sinusoids and phasors. Perform phasor addition, subtraction, and represent signals in polar and rectangular form.

11. Convert $v(t) = 50 \cos(\omega t + 25°)$ V to phasor form.

Answer: $V = 50\angle 25°$ V (or in rectangular: $50\cos 25° + j50\sin 25° = 45.32 + j21.13$ V)

12. Convert the phasor $I = 3\angle -40°$ A to time-domain at $\omega = 1000$ rad/s.

Answer: $i(t) = 3 \cos(1000t - 40°)$ A

13. Add the phasors $V_1 = 10\angle 30°$ V and $V_2 = 8\angle -45°$ V. Express result in polar form.

Answer: $V_1 = 8.66+j5$; $V_2 = 5.66-j5.66$; Sum $= 14.32-j0.66 = 14.33\angle -2.64°$ V

14. Subtract phasors: $V = V_1 - V_2$ where $V_1 = 6\angle 60°$ and $V_2 = 4\angle 30°$. Give in rectangular and polar form.

Answer: $V_1 = 3+j5.196$; $V_2 = 3.464+j2$; $V = -0.464+j3.196 = 3.23\angle 98.24°$ V

15. Express $I = (3 + j4)$ A in polar form and then as a time-domain current at $\omega = 500$ rad/s.

Answer: $I = 5\angle 53.13°$ A; $i(t) = 5 \cos(500t + 53.13°)$ A

16. Find the sum $i = i_1 + i_2$ where $i_1 = 4\cos(\omega t + 30°)$ and $i_2 = 3\cos(\omega t - 60°)$ using phasors.
Answer: $I_1 = 4\angle 30°$; $I_2 = 3\angle -60°$; $I = 3.464+j2 + 1.5-j2.598 = 4.964-j0.598 = 5\angle -6.87°$ A; $i = 5\cos(\omega t - 6.87°)$ A

17. Convert $V = 12\angle 150°$ V to rectangular form.
Answer: $V = 12\cos 150° + j12\sin 150° = -10.39 + j6$ V

18. Multiply phasors: $Z_1 = 4\angle 30°$ and $Z_2 = 2\angle 45°$.
Answer: $Z = Z_1 \cdot Z_2 = (4\times 2)\angle(30°+45°) = 8\angle 75°$

19. Divide phasors: $V = 20\angle 60°$ V, $I = 4\angle 15°$ A. Find $Z = V/I$.
Answer: $Z = (20/4)\angle(60°-15°) = 5\angle 45°\ \Omega$

20. Three phasors: $I_1 = 5\angle 0°$, $I_2 = 3\angle 90°$, $I_3 = 4\angle -90°$. Find $I\_total = I_1 + I_2 + I_3$.
Answer: $I = 5 + j3 - j4 = 5 - j1 = 5.1\angle -11.31°$ A

Part III: Impedance of R, L, and C Elements

Compute the impedance of resistors, inductors, and capacitors at given frequencies.

21. Find the impedance of a 47 Ω resistor.
Answer: $Z\_R = 47\angle 0°\ \Omega = 47 + j0\ \Omega$ (purely real, no phase shift)

22. Find the impedance of a 10 mH inductor at f = 60 Hz.
Answer: $X\_L = \omega L = 2\pi(60)(0.01) = 3.77\ \Omega$; $Z\_L = j3.77 = 3.77\angle 90°\ \Omega$

23. Find the impedance of a 100 μF capacitor at ω = 1000 rad/s.
Answer: $X\_C = 1/(\omega C) = 1/(1000\times 100\times 10^{-6}) = 10\ \Omega$; $Z\_C = -j10 = 10\angle -90°\ \Omega$

24. At what frequency does a 50 mH inductor have an impedance of magnitude 100 Ω?
Answer: $X\_L = \omega L = 100$; $\omega = 100/0.05 = 2000$ rad/s; $f = 2000/2\pi \approx 318.3$ Hz

25. At what frequency does a 10 μF capacitor have an impedance magnitude of 50 Ω?
Answer: $X\_C = 1/(\omega C) = 50$; $\omega = 1/(50\times 10\times 10^{-6}) = 2000$ rad/s; $f \approx 318.3$ Hz

26. Find the impedance of a series R-L circuit with R = 30 Ω and L = 40 mH at ω = 500 rad/s.
Answer: $Z = R + j\omega L = 30 + j(500\times 0.04) = 30 + j20 = 36.06\angle 33.69°\ \Omega$

27. Find the impedance of a series R-C circuit with R = 6 Ω and C = 200 μF at f = 50 Hz.
Answer: $X\_C = 1/(2\pi\times 50\times 200\times 10^{-6}) = 15.92\ \Omega$; $Z = 6 - j15.92 = 17.02\angle -69.35°\ \Omega$

28. Find the total impedance of a series R-L-C circuit: R = 10 Ω, L = 30 mH, C = 100 μF at ω = 1000 rad/s.
Answer: $X\_L = 30\ \Omega$; $X\_C = 10\ \Omega$; $Z = 10 + j(30-10) = 10 + j20 = 22.36\angle 63.43°\ \Omega$

29. A capacitor C = 50 μF is in parallel with R = 200 Ω at ω = 400 rad/s. Find Z_parallel.
Answer: $Z\_C = -j/(\omega C) = -j50\ \Omega$; $Z = (200\times(-j50))/(200-j50) = -j10000/(200-j50)$; $|denom|^2 = 42500$; $Z = 11.76 - j47.06 = 48.51\angle -76°\ \Omega$

30. Find impedance of parallel R-L combination: R = 8 Ω, X_L = 6 Ω.
Answer: $Z\_L = j6$; $Z = (8\times j6)/(8+j6) = j48/(8+j6)$; multiply by conj: $Z = (48\times 6 + j48\times 8)/100 = (288+j384)/100 = 2.88 + j3.84 = 4.8\angle 53.13°\ \Omega$

Part IV: Ohm's Law and KVL/KCL in Phasor Domain

Apply Ohm's Law V = IZ, Kirchhoff's Voltage Law, and Kirchhoff's Current Law in the phasor (frequency) domain.

31. A current $I = 2\angle 30°$ A flows through $Z = 5\angle 45°\ \Omega$. Find the voltage V.
Answer: $V = IZ = (2\times 5)\angle(30°+45°) = 10\angle 75°$ V

32. $V = 120\angle 0°$ V is applied across $Z = 40 + j30\ \Omega$. Find the current I.

Answer: $|Z| = \sqrt{40^2+30^2} = 50\ \Omega$; $\angle Z = \arctan(30/40) = 36.87°$; $I = V/Z = 120/50\angle-36.87° = 2.4\angle-36.87°$ A

33. Apply KVL: $V_s = 100\angle 0°$ V in a series circuit with $V_R = 60\angle 0°$ V and V_L. Find V_L.

Answer: $V_L = V_s - V_R = 100 - 60 = 40\angle 0°$ V... If V_R is purely real and V_L is 90° ahead: $V_L = 100\angle 0° - 60\angle 0° = 40 + j0$ V → $40\angle 0°$ V (or solve with actual impedances if given)

34. A series R-L-C circuit has $V_R = 30\angle 0°$, $V_L = 60\angle 90°$, $V_C = 20\angle -90°$ V. Find total source voltage V_s.

Answer: $V_s = 30 + j60 - j20 = 30 + j40 = 50\angle 53.13°$ V

35. Apply KCL at a node: $I_s = 5\angle 0°$ A, $I_1 = 3\angle 30°$ A. Find $I_2 = I_s - I_1$.

Answer: $I_1 = 2.598 + j1.5$; $I_2 = 5 - 2.598 - j1.5 = 2.402 - j1.5 = 2.836\angle -32°$ A

36. Two impedances $Z_1 = 3 + j4\ \Omega$ and $Z_2 = 5 - j2\ \Omega$ are in series. A voltage $V = 100\angle 0°$ V is applied. Find I.

Answer: $Z_{total} = 8 + j2 = 8.246\angle 14.04°\ \Omega$; $I = 100/8.246\angle -14.04° = 12.13\angle -14.04°$ A

37. For the same circuit above, find V_{Z1} and V_{Z2}.

Answer: $V_{Z1} = I \cdot Z_1 = 12.13\angle -14.04° \times 5\angle 53.13° = 60.65\angle 39.09°$ V; $V_{Z2} = 12.13\angle -14.04° \times 5.385\angle -21.8° = 65.31\angle -35.84°$ V

38. Three impedances in parallel: $Z_1 = 10\angle 0°$, $Z_2 = 5\angle 90°$, $Z_3 = 20\angle -90°\ \Omega$; source $50\angle 0°$ V. Find total current.

Answer: $I_1 = 5\angle 0°$, $I_2 = 10\angle -90°$, $I_3 = 2.5\angle 90°$; $I_T = 5 - j10 + j2.5 = 5 - j7.5 = 9.01\angle -56.31°$ A

39. $V_s = 50\angle 30°$ V drives a circuit with $Z = 10\angle -60°\ \Omega$. Write i(t) at $\omega = 377$ rad/s.

Answer: $I = 50/10\angle(30°-(-60°)) = 5\angle 90°$ A; $i(t) = 5\cos(377t + 90°)$ A

40. In a parallel R-L circuit, $R = 4\ \Omega$, $X_L = 3\ \Omega$, $V = 12\angle 0°$ V. Find I_R, I_L, and I_{total}.

Answer: $I_R = 12/4\angle 0° = 3\angle 0°$ A; $I_L = 12/3\angle -90° = 4\angle -90°$ A; $I_T = 3 - j4 = 5\angle -53.13°$ A

Part V: Nodal and Mesh Analysis in AC Circuits

Apply nodal voltage and mesh current methods to AC circuits in phasor domain.

41. A simple two-node circuit: node 1 connected to source $I_s = 2\angle 0°$ A, shunt $Z_1 = j5\ \Omega$ to ground, and $Z_2 = 10\ \Omega$ to ground. Find V_1.

Answer: $Y = 1/Z_1 + 1/Z_2 = 1/(j5) + 1/10 = 0.1 - j0.2$; $V_1 = I_s/Y = 2/(0.1-j0.2) = 2(0.1+j0.2)/0.05 = 4 + j8 = 8.944\angle 63.43°$ V

42. Write the nodal equation for a circuit with two nodes (V_1, V_2) and admittances $Y_{11} = 0.5$, $Y_{12} = -0.2$, $Y_{22} = 0.4$ S with $I_{s1} = 3$ A and $I_{s2} = 1$ A.

Answer: $0.5V_1 - 0.2V_2 = 3$ and $-0.2V_1 + 0.4V_2 = 1$; Solving: $V_1 = 7.5$ V, $V_2 = 6.25$ V

43. Mesh analysis: single mesh with $V_s = 50\angle 0°$ V, $R = 6\ \Omega$, $X_L = 8\ \Omega$ in series. Find mesh current I.

Answer: $Z = 6 + j8 = 10\angle 53.13°\ \Omega$; $I = 50/10\angle -53.13° = 5\angle -53.13°$ A

44. Two-mesh circuit: Mesh 1 has $V_s = 100\angle 0°$, $Z_1 = 10\Omega$, $Z_{12} = j5\Omega$ (mutual). Mesh 2 has $Z_2 = 8\Omega$, same Z_{12}. Set up mesh equations.

Answer: $(10+j5)I_1 - j5I_2 = 100$ and $-j5I_1 + (8+j5)I_2 = 0$; These are the two mesh equations (solve with Cramer's rule or substitution)

45. Using superposition, find V_x due to two sources: $V_{s1} = 10\angle 0°$ V (alone gives $V_x' = 4\angle 30°$ V) and $V_{s2} = 6\angle 90°$ V (alone gives $V_x'' = 2\angle 45°$ V). Find total V_x.

Answer: $V_x = V_x' + V_x'' = 4\angle 30° + 2\angle 45° = 3.464+j2 + 1.414+j1.414 = 4.878 + j3.414 = 5.93\angle 34.96°$ V

46. Find the Thevenin equivalent of a circuit seen from terminals AB: $V_{oc} = 40\angle 30°$ V and $Z_{th} = 3 + j4\ \Omega$.

Answer: $V_{th} = 40\angle 30°$ V; $Z_{th} = 3 + j4 = 5\angle 53.13°\ \Omega$. These fully define the Thevenin equivalent.

47. A load $Z_L = 4 - j3\ \Omega$ is connected to a Thevenin source $V_{th} = 20\angle 0°$ V, $Z_{th} = 4 + j3\ \Omega$. Find I_L.

Answer: $Z_{total} = (4+j3)+(4-j3) = 8\ \Omega$; $I_L = 20/8\angle 0° = 2.5\angle 0°$ A

48. For maximum power transfer in AC, what must Z_L equal given $Z_{th} = 5 + j8\ \Omega$?

Answer: For maximum average power: $Z_L = Z_{th}^* = 5 - j8\ \Omega$ (complex conjugate of source impedance)

49. Find Norton equivalent: $I_{sc} = 4\angle -20°$ A, $Z_N = 10\angle 60°\ \Omega$.

Answer: $I_N = 4\angle -20°$ A; $Z_N = 10\angle 60°\ \Omega$. The Norton equivalent is a current source $4\angle -20°$ A in parallel with $10\angle 60°\ \Omega$.

50. Convert Thevenin ($V_{th} = 50\angle 45°$ V, $Z_{th} = 5\angle 30°\ \Omega$) to Norton equivalent.

Answer: $I_N = V_{th}/Z_{th} = 50/5\angle(45°-30°) = 10\angle 15°$ A; $Z_N = Z_{th} = 5\angle 30°\ \Omega$

Part VI: AC Power — Real, Reactive, and Apparent

Calculate real power P (W), reactive power Q (VAR), apparent power S (VA), power factor, and complex power S = P + jQ.

51. A load draws $I = 5\angle -36.87°$ A from $V = 100\angle 0°$ V. Find P, Q, S, and power factor.

Answer: $\theta = 0°-(-36.87°) = 36.87°$; $P = V_{rms}I_{rms}\cos\theta = 100\times5\times\cos 36.87° = 400$ W; $Q = 100\times5\times\sin 36.87° = 300$ VAR (inductive); S = 500 VA; pf = cos36.87° = 0.8 lagging

52. Compute complex power $S = V\cdot I^*$ where $V = 120\angle 30°$ V and $I = 4\angle -15°$ A (rms values).

Answer: $I^* = 4\angle +15°$; $S = 120\times4\angle(30°+15°) = 480\angle 45°$ VA $= 339.4 + j339.4$ VA; P = 339.4 W, Q = 339.4 VAR

53. A resistor R = 25 Ω carries Irms = 4 A. Find P, Q, and S.

Answer: $P = I^2R = 16\times25 = 400$ W; Q = 0 VAR (resistor); S = 400 VA; pf = 1.0

54. A pure inductor $X_L = 10\ \Omega$ has Irms = 3 A. Find P, Q, and S.

Answer: P = 0 W; $Q = I^2X_L = 9\times10 = 90$ VAR (inductive/positive); S = 90 VA; pf = 0 lagging

55. A pure capacitor $X_C = 8\ \Omega$ has Irms = 2 A. Find P, Q, and S.

Answer: P = 0 W; $Q = -I^2X_C = -32$ VAR (capacitive/negative); S = 32 VA; pf = 0 leading

56. A load $Z = 6 + j8\ \Omega$ is connected to $V_{rms} = 100\angle 0°$ V. Find P, Q, and power factor.

Answer: $I = 100/10\angle -53.13° = 10\angle -53.13°$ A; $P = I^2R = 100\times6 = 600$ W; $Q = I^2X = 100\times8 = 800$ VAR; pf = cos53.13° = 0.6 lagging

57. Two loads in parallel: Load 1: $P_1 = 500$ W, $pf_1 = 0.8$ lag; Load 2: $P_2 = 300$ W, $pf_2 = 1.0$. Find total P, Q, S.

Answer: $Q_1 = P_1\times\tan(\cos^{-1}0.8) = 500\times0.75 = 375$ VAR; $Q_2 = 0$; $P_T = 800$ W; $Q_T = 375$ VAR; $S_T = \sqrt{(800^2+375^2)} = 880.8$ VA

58. A factory load: S = 10 kVA at pf = 0.6 lagging from 240 V rms. Find P, Q, and line current.

Answer: $P = S\times pf = 6000$ W; $Q = S\times\sin(\cos^{-1}0.6) = 8000$ VAR; $I = S/V = 10000/240 = 41.67$ A

60. Three loads on a 120 V rms source: Load A: 200 W at pf=1; Load B: 150 W at pf=0.5 lag; Load C: 100 VAR capacitive. Find total S.

Answer: $Q_B = 150\times\tan(\cos^{-1}0.5) = 259.8$ VAR; $Q_C = -100$ VAR; $S_T = (200+150) + j(0+259.8-100) = 350 + j159.8 = 384.8\angle 24.6°$ VA