

Three-Phase Power Calculations

100 Questions with Answers: Real, Reactive, Complex, Apparent Power & Power Factor

1. What is the formula for real (active) power in a balanced three-phase system?

Answer: $P = \sqrt{3} \times V_L \times I_L \times \cos \phi$, or equivalently $P = 3 \times V_\phi \times I_\phi \times \cos \phi$, where V_L is line voltage, I_L is line current, V_ϕ is phase voltage, I_ϕ is phase current, and $\cos \phi$ is the power factor.

2. What is reactive power in a three-phase system and what is its unit?

Answer: Reactive power (Q) is the power stored and released by inductive or capacitive elements, measured in volt-amperes reactive (VAR). Formula: $Q = \sqrt{3} \times V_L \times I_L \times \sin \phi = 3 \times V_\phi \times I_\phi \times \sin \phi$.

3. Define apparent power in a three-phase system.

Answer: Apparent power (S) is the total power supplied by the source, combining real and reactive power. $S = \sqrt{3} \times V_L \times I_L = 3 \times V_\phi \times I_\phi$, measured in volt-amperes (VA).

4. What is the relationship between real, reactive, and apparent power?

Answer: $S^2 = P^2 + Q^2$, or $S = \sqrt{P^2 + Q^2}$. This forms the power triangle where S is the hypotenuse.

5. Define power factor in a three-phase system.

Answer: Power factor (PF) = $\cos \phi = P / S = \text{Real Power} / \text{Apparent Power}$. It ranges from 0 to 1 and indicates how effectively the load converts apparent power into useful real power.

6. What is the power factor of a purely resistive three-phase load?

Answer: PF = 1 (unity power factor), because $\phi = 0^\circ$, $\cos 0^\circ = 1$, meaning all apparent power is converted to real power and $Q = 0$.

7. What is the power factor of a purely inductive three-phase load?

Answer: PF = 0 (lagging), because $\phi = 90^\circ$, $\cos 90^\circ = 0$. All power is reactive ($Q = S$) and no real power is consumed.

8. What is the phase voltage relationship to line voltage in a balanced three-phase star (Y) system?

Answer: $V_L = \sqrt{3} \times V_\phi$, or $V_\phi = V_L / \sqrt{3}$. For a 400 V line voltage, the phase voltage = $400/\sqrt{3} \approx 230.9$ V.

9. What is the current relationship in a balanced three-phase star (Y) connected system?

Answer: In a Y-connected system, the line current equals the phase current: $I_L = I_\phi$.

10. What is the current relationship in a balanced three-phase delta (Δ) connected system?

Answer: In a Δ -connected system, $I_L = \sqrt{3} \times I_\phi$, or $I_\phi = I_L / \sqrt{3}$. Line current is $\sqrt{3}$ times the phase current.

11. A balanced Y-connected load has a phase impedance of $Z = 10 + j6 \Omega$. If $V_L = 400$ V, find the phase current.

Answer: $|Z| = \sqrt{(10^2 + 6^2)} = \sqrt{136} \approx 11.66 \Omega$. $V_\phi = 400/\sqrt{3} \approx 230.94$ V. $I_\phi = I_L = V_\phi/|Z| = 230.94/11.66 \approx 19.81$ A.

12. For the load in Q11, calculate the real power consumed.

Answer: $\cos \phi = R/|Z| = 10/11.66 \approx 0.858$ (lagging). $P = 3 \times V_\phi \times I_\phi \times \cos \phi = 3 \times 230.94 \times 19.81 \times 0.858 \approx 11,788$ W ≈ 11.79 kW.

13. For the load in Q11, calculate the reactive power.

Answer: $\sin \phi = X/|Z| = 6/11.66 \approx 0.515$. $Q = 3 \times V\phi \times I\phi \times \sin \phi = 3 \times 230.94 \times 19.81 \times 0.515 \approx 7,079 \text{ VAR} \approx 7.08 \text{ kVAR}$.

14. For the load in Q11, calculate the apparent power.

Answer: $S = 3 \times V\phi \times I\phi = 3 \times 230.94 \times 19.81 \approx 13,735 \text{ VA} \approx 13.74 \text{ kVA}$. Verify: $S = \sqrt{(P^2 + Q^2)} = \sqrt{(11788^2 + 7079^2)} \approx 13,736 \text{ VA} \checkmark$

15. A balanced Y-connected resistive load of 8Ω per phase is connected to a 415 V, 50 Hz supply. Find the line current.

Answer: $V\phi = 415/\sqrt{3} \approx 239.6 \text{ V}$. $I\phi = I_L = V\phi/R = 239.6/8 \approx 29.95 \text{ A}$.

16. What is the total real power for the load in Q15?

Answer: $P = 3 \times I\phi^2 \times R = 3 \times 29.95^2 \times 8 = 3 \times 897 \times 8 \approx 21,528 \text{ W} \approx 21.53 \text{ kW}$. Or $P = \sqrt{3} \times V_L \times I_L = \sqrt{3} \times 415 \times 29.95 \approx 21,530 \text{ W} \checkmark$

17. A Y-connected load draws 15 A per phase from a 380 V three-phase supply at a power factor of 0.8 lagging. Calculate P, Q, and S.

Answer: $V\phi = 380/\sqrt{3} \approx 219.4 \text{ V}$. $S = 3 \times 219.4 \times 15 \approx 9,873 \text{ VA}$. $P = S \times 0.8 \approx 7,898 \text{ W}$. $Q = S \times \sin(\cos^{-1}0.8) = 9,873 \times 0.6 \approx 5,924 \text{ VAR}$.

18. Express the complex power S for a Y-connected load with $Z = 5\angle 30^\circ \Omega$ per phase at $V_L = 200 \text{ V}$.

Answer: $V\phi = 200/\sqrt{3} \approx 115.5 \text{ V}$. $I\phi = 115.5/5 = 23.1 \text{ A}$. $S = 3 \times V\phi \times I\phi \angle \phi = 3 \times 115.5 \times 23.1 \angle 30^\circ \approx 8,012 \angle 30^\circ \text{ VA}$. So $P \approx 6,938 \text{ W}$, $Q \approx 4,006 \text{ VAR}$.

19. A balanced Y load has $R = 12 \Omega$ and $X_L = 9 \Omega$ per phase. $V_L = 440 \text{ V}$. Find the power factor angle.

Answer: $\phi = \tan^{-1}(X_L/R) = \tan^{-1}(9/12) = \tan^{-1}(0.75) \approx 36.87^\circ$. PF = $\cos(36.87^\circ) = 0.8$ lagging.

20. For the load in Q19, find the three-phase apparent power.

Answer: $|Z| = \sqrt{(12^2 + 9^2)} = \sqrt{225} = 15 \Omega$. $V\phi = 440/\sqrt{3} \approx 254.03 \text{ V}$. $I_L = I\phi = 254.03/15 \approx 16.94 \text{ A}$. $S = \sqrt{3} \times 440 \times 16.94 \approx 12,909 \text{ VA} \approx 12.91 \text{ kVA}$.

21. A balanced delta-connected load has $Z = 30\angle 45^\circ \Omega$ per phase. $V_L = 400 \text{ V}$. Find the phase current.

Answer: In delta, $V\phi = V_L = 400 \text{ V}$. $I\phi = V\phi/|Z| = 400/30 \approx 13.33 \text{ A}$.

22. For the load in Q21, find the line current.

Answer: $I_L = \sqrt{3} \times I\phi = \sqrt{3} \times 13.33 \approx 23.09 \text{ A}$.

23. For the load in Q21, calculate the total real power.

Answer: PF = $\cos(45^\circ) \approx 0.707$. $P = \sqrt{3} \times V_L \times I_L \times \cos \phi = \sqrt{3} \times 400 \times 23.09 \times 0.707 \approx 11,313 \text{ W} \approx 11.31 \text{ kW}$. Or $P = 3 \times I\phi^2 \times R = 3 \times 13.33^2 \times (30 \cos 45^\circ) = 3 \times 177.7 \times 21.21 \approx 11,310 \text{ W} \checkmark$

24. For the load in Q21, calculate the total reactive power.

Answer: $Q = \sqrt{3} \times V_L \times I_L \times \sin \phi = \sqrt{3} \times 400 \times 23.09 \times \sin(45^\circ) = \sqrt{3} \times 400 \times 23.09 \times 0.707 \approx 11,313 \text{ VAR} \approx 11.31 \text{ kVAR}$.

25. A delta-connected load of $20 + j15 \Omega$ per phase is supplied at 415 V. Find P and Q.

Answer: $|Z| = \sqrt{(20^2 + 15^2)} = 25 \Omega$. $I\phi = 415/25 = 16.6 \text{ A}$. $\cos \phi = 20/25 = 0.8$, $\sin \phi = 0.6$. $P = 3 \times 16.6^2 \times 20 = 3 \times 275.6 \times 20 \approx 16,536 \text{ W} \approx 16.54 \text{ kW}$. $Q = 3 \times 16.6^2 \times 15 \approx 12,402 \text{ VAR} \approx 12.40 \text{ kVAR}$.

26. How does the power consumed by a delta-connected load compare to an equivalent Y-connected load with the same line voltage?

Answer: $P_{\Delta} = 3 \times P_Y$. Because in delta, each phase sees the full line voltage ($V_{\phi_{\Delta}} = V_L$), while in Y each phase sees $V_L/\sqrt{3}$. The delta load consumes 3 times more power for the same impedance value.

27. A balanced delta load draws 10 kW at 0.75 power factor lagging from a 380 V supply. Find the line current.

Answer: $P = \sqrt{3} \times V_L \times I_L \times PF \rightarrow I_L = P / (\sqrt{3} \times V_L \times PF) = 10,000 / (\sqrt{3} \times 380 \times 0.75) = 10,000 / 494.0 \approx 20.24 \text{ A}$.

28. For the load in Q27, find the reactive power.

Answer: $\sin \phi = \sin(\cos^{-1}0.75) \approx 0.661$. $Q = P \times \tan \phi = 10,000 \times (0.661/0.75) = 10,000 \times 0.8819 \approx 8,819 \text{ VAR} \approx 8.82 \text{ kVAR}$.

29. For the load in Q27, find the apparent power.

Answer: $S = P / PF = 10,000 / 0.75 \approx 13,333 \text{ VA} \approx 13.33 \text{ kVA}$. Or $S = \sqrt{(P^2 + Q^2)} = \sqrt{(10000^2 + 8819^2)} \approx 13,333 \text{ VA} \checkmark$

30. A delta load has phase impedance of 60 Ω purely resistive. $V_L = 400 \text{ V}$. Find the total power.

Answer: $I_{\phi} = V_L/R = 400/60 \approx 6.667 \text{ A}$. $P = 3 \times I_{\phi}^2 \times R = 3 \times 44.44 \times 60 = 8,000 \text{ W} = 8 \text{ kW}$. Or $P = 3 \times V_L^2/R = 3 \times 160,000/60 = 8,000 \text{ W} \checkmark$

31. Define complex power S in phasor notation.

Answer: $S = P + jQ$ (VA), where P is real power (W) and Q is reactive power (VAR). For inductive loads $Q > 0$, for capacitive loads $Q < 0$. Also $S = V \times I^*$ where I^* is the complex conjugate of the current phasor.

32. A three-phase load has complex power $S = 12 + j9 \text{ kVA}$. Find P, Q, S (magnitude), and PF.

Answer: $P = 12 \text{ kW}$, $Q = 9 \text{ kVAR}$ (inductive). $|S| = \sqrt{(12^2 + 9^2)} = \sqrt{(144 + 81)} = \sqrt{225} = 15 \text{ kVA}$. $PF = P/|S| = 12/15 = 0.8$ lagging.

33. Two three-phase loads are connected in parallel: Load 1: $S_1 = 10 + j6 \text{ kVA}$; Load 2: $S_2 = 8 - j4 \text{ kVA}$. Find total S.

Answer: $S_{\text{total}} = S_1 + S_2 = (10+8) + j(6-4) = 18 + j2 \text{ kVA}$. $|S| = \sqrt{(18^2 + 2^2)} = \sqrt{328} \approx 18.11 \text{ kVA}$. $PF = 18/18.11 \approx 0.994$ lagging.

34. A generator supplies $S = 50 \angle 36.87^\circ \text{ kVA}$ to a three-phase load. Find P and Q.

Answer: $P = 50 \times \cos(36.87^\circ) = 50 \times 0.8 = 40 \text{ kW}$. $Q = 50 \times \sin(36.87^\circ) = 50 \times 0.6 = 30 \text{ kVAR}$ (inductive, lagging).

35. How is complex power computed from voltage and current phasors?

Answer: $S = V \times I^*$ per phase (single-phase equivalent). For three-phase: $S_{3\phi} = 3 \times V_{\phi} \times I_{\phi}^* = \sqrt{3} \times V_L \angle(\phi_v) \times I_L \angle(-\phi_i)$. The conjugate of I ensures the sign of Q is correct (positive for inductive loads).

36. A three-phase Y-load: $V_{\phi} = 120 \angle 0^\circ \text{ V}$, $I_{\phi} = 10 \angle -30^\circ \text{ A}$ per phase. Find S per phase and total S.

Answer: $S_{\text{phase}} = V_{\phi} \times I_{\phi}^* = 120 \angle 0^\circ \times 10 \angle +30^\circ = 1200 \angle 30^\circ \text{ VA} = 1039 + j600 \text{ VA}$. Total $S_{3\phi} = 3 \times 1200 \angle 30^\circ = 3600 \angle 30^\circ \text{ VA} = 3117 + j1800 \text{ VA}$.

37. What does a negative value of Q (reactive power) indicate?

Answer: $Q < 0$ indicates a capacitive load. The load is supplying reactive power to the system (leading power factor). Capacitive loads generate reactive power rather than absorbing it.

38. Three loads connected to a 480 V three-phase bus: L1 = 20 kW at PF 1.0; L2 = 30 kW at PF 0.9 lag; L3 = 15 kW at PF 0.8 lead. Find total P, Q, and |S|.

Answer: $P_{\text{total}} = 20+30+15 = 65 \text{ kW}$. $Q_1 = 0$; $Q_2 = 30 \times \tan(\cos^{-1}0.9) = 30 \times 0.4843 = 14.53 \text{ kVAR}$; $Q_3 = -15 \times \tan(\cos^{-1}0.8) = -15 \times 0.75 = -11.25 \text{ kVAR}$. $Q_{\text{total}} = 0+14.53-11.25 = 3.28 \text{ kVAR}$. $|S| = \sqrt{(65^2+3.28^2)} \approx 65.08 \text{ kVA}$.

39. Express the power triangle relationship mathematically.

Answer: The power triangle has sides P (adjacent), Q (opposite), and S (hypotenuse) with angle ϕ . Relations: $P = S \cos \phi$; $Q = S \sin \phi$; $\tan \phi = Q/P$; $S = P/\cos \phi = Q/\sin \phi = \sqrt{(P^2+Q^2)}$.

40. A three-phase motor draws 45 kVA at 0.85 PF lagging. Find P and Q.

Answer: $P = S \times \text{PF} = 45 \times 0.85 = 38.25 \text{ kW}$. $\sin \phi = \sqrt{(1-0.85^2)} = \sqrt{(1-0.7225)} = \sqrt{0.2775} \approx 0.5268$. $Q = S \times \sin \phi = 45 \times 0.5268 \approx 23.71 \text{ kVAR}$ (lagging/inductive).

41. A balanced three-phase load draws 60 kW at a power factor of 0.6 lagging. What is the apparent power?

Answer: $S = P / \text{PF} = 60,000 / 0.6 = 100,000 \text{ VA} = 100 \text{ kVA}$.

42. A three-phase load of 80 kVA operates at 0.9 power factor lagging. Determine real and reactive power.

Answer: $P = 80 \times 0.9 = 72 \text{ kW}$. $Q = 80 \times \sin(\cos^{-1}0.9) = 80 \times 0.4359 \approx 34.87 \text{ kVAR}$.

43. What is the power factor if a three-phase load has P = 30 kW and Q = 40 kVAR?

Answer: $|S| = \sqrt{(30^2 + 40^2)} = \sqrt{2500} = 50 \text{ kVA}$. $\text{PF} = P/S = 30/50 = 0.6$ lagging (since $Q > 0$, load is inductive).

44. What distinguishes lagging from leading power factor?

Answer: Lagging PF: current lags voltage (inductive load, $Q > 0$). Leading PF: current leads voltage (capacitive load, $Q < 0$). Unity PF: current in phase with voltage (resistive load, $Q = 0$).

45. A three-phase induction motor has P = 75 kW and operates at PF = 0.85 lag from a 415 V supply. Find the line current.

Answer: $S = P/\text{PF} = 75,000/0.85 \approx 88,235 \text{ VA}$. $I_L = S/(\sqrt{3} \times V_L) = 88,235/(\sqrt{3} \times 415) \approx 88,235/718.7 \approx 122.8 \text{ A}$.

46. A three-phase load draws $I_L = 50 \text{ A}$ from a 380 V supply. The real power is 25 kW. Find the power factor.

Answer: $S = \sqrt{3} \times V_L \times I_L = \sqrt{3} \times 380 \times 50 \approx 32,909 \text{ VA}$. $\text{PF} = P/S = 25,000/32,909 \approx 0.760$.

47. If $\phi = 53.13^\circ$, is the power factor leading or lagging, and what is its value?

Answer: $\text{PF} = \cos(53.13^\circ) = 0.6$. The sign depends on the load: if inductive (current lags), it is 0.6 lagging; if capacitive (current leads), it is 0.6 leading.

48. A three-phase system has P = 100 kW, Q = 0 kVAR. What is the power factor and what type of load is it?

Answer: $\text{PF} = P/S = 100/100 = 1.0$ (unity). $Q = 0$ means the load is purely resistive with no reactive component.

49. The power factor of a three-phase load is 0.707 lagging. The apparent power is 20 kVA. Find P and Q.

Answer: $P = 20 \times 0.707 = 14.14 \text{ kW}$. $\sin \phi = \sin(45^\circ) = 0.707$. $Q = 20 \times 0.707 = 14.14 \text{ kVAR}$ lagging.

50. A wattmeter reads 15 kW in a three-phase circuit where $V_L = 400$ V, $I_L = 30$ A. Find the power factor.

Answer: $S = \sqrt{3} \times 400 \times 30 \approx 20,785$ VA. $PF = P/S = 15,000/20,785 \approx 0.722$.

51. A balanced three-phase star-connected load consists of $Z = 8 + j6$ Ω per phase connected to a 415 V, 50 Hz system. Calculate I_L , P, Q, S, and PF.

Answer: $|Z| = 10$ Ω . $V_\phi = 415/\sqrt{3} \approx 239.6$ V. $I_L = I_\phi = 239.6/10 = 23.96$ A. $PF = 8/10 = 0.8$ lag. $P = 3 \times 239.6 \times 23.96 \times 0.8 \approx 13,806$ W ≈ 13.81 kW. $Q = 3 \times 239.6 \times 23.96 \times 0.6 \approx 10,354$ VAR ≈ 10.35 kVAR. $S = 3 \times 239.6 \times 23.96 \approx 17,208$ VA ≈ 17.21 kVA.

52. A delta load of $Z = 15 \angle -30^\circ$ Ω (capacitive) is connected to a 400 V supply. Find P, Q, and S.

Answer: $I_\phi = 400/15 \approx 26.67$ A. $I_L = \sqrt{3} \times 26.67 \approx 46.19$ A. $PF = \cos(30^\circ) \approx 0.866$ leading. $P = \sqrt{3} \times 400 \times 46.19 \times 0.866 \approx 27,711$ W ≈ 27.71 kW. $Q = \sqrt{3} \times 400 \times 46.19 \times 0.5 \approx 16,007$ VAR ≈ -16.0 kVAR (capacitive). $S = \sqrt{3} \times 400 \times 46.19 \approx 32,003$ VA ≈ 32.0 kVA.

53. Two balanced three-phase loads in parallel: Load A draws 50 kW at 0.8 PF lag; Load B draws 30 kW at 0.9 PF lag. Find total P, Q, S, and combined PF.

Answer: $Q_A = 50 \times (0.6/0.8) = 37.5$ kVAR. $Q_B = 30 \times (\sqrt{(1-0.81)}/0.9) = 30 \times 0.4843/0.9 = 14.53$ kVAR. $P = 80$ kW. $Q = 52.03$ kVAR. $S = \sqrt{(80^2 + 52.03^2)} \approx 95.53$ kVA. $PF = 80/95.53 \approx 0.838$ lag.

54. A 415 V three-phase motor develops 20 hp mechanical output with 90% efficiency and 0.85 PF lag. Find the line current. (1 hp = 746 W)

Answer: $P_{out} = 20 \times 746 = 14,920$ W. $P_{in} = 14,920/0.90 = 16,578$ W. $S = P_{in}/PF = 16,578/0.85 = 19,503$ VA. $I_L = S/(\sqrt{3} \times 415) = 19,503/718.7 \approx 27.1$ A.

55. In a balanced three-phase system, $V_L = 11$ kV, $I_L = 100$ A, $PF = 0.95$ lag. Calculate P, Q, and S.

Answer: $S = \sqrt{3} \times 11,000 \times 100 \approx 1,904,941$ VA ≈ 1.905 MVA. $P = 1.905 \times 0.95 \approx 1.810$ MW. $\sin \phi = \sqrt{(1-0.9025)} \approx 0.3122$. $Q = 1.905 \times 0.3122 \approx 0.595$ MVAR.

56. A three-phase transformer supplies a load of $S = 500$ kVA at 0.8 PF lagging, 11 kV. Find P, Q, and I_L .

Answer: $P = 500 \times 0.8 = 400$ kW. $Q = 500 \times 0.6 = 300$ kVAR. $I_L = S/(\sqrt{3} \times V_L) = 500,000/(\sqrt{3} \times 11,000) = 500,000/19,053 \approx 26.25$ A.

57. Find the complex power per phase for a Y-connected load: $Z = 10 \angle 60^\circ$ Ω , $V_L = 200$ V.

Answer: $V_\phi = 200/\sqrt{3} \approx 115.47$ V. $I_\phi = 115.47/10 = 11.547$ A. $S_{phase} = V_\phi \times I_\phi \angle 60^\circ$ (since V leads I by $\phi = 60^\circ$) = $115.47 \times 11.547 \angle 60^\circ = 1333 \angle 60^\circ$ VA = $666.5 + j1154.7$ VA.

58. A balanced three-phase system supplies a load where each phase absorbs 2 kW at 0.7 PF lagging. Find total P, Q, and S.

Answer: $P = 3 \times 2 = 6$ kW. Per phase: $Q_{phase} = P_{phase} \times \tan(\cos^{-1}0.7) = 2000 \times 1.0202 \approx 2040$ VAR. $Q_{total} = 3 \times 2040 = 6,122$ VAR ≈ 6.12 kVAR. $S_{total} = 6000/0.7 \approx 8,571$ VA ≈ 8.57 kVA.

59. A Y-connected capacitive load: each phase has $R = 10$ Ω , $X_C = 10$ Ω . $V_L = 346$ V. Find P, Q, and PF.

Answer: $|Z| = \sqrt{(10^2 + 10^2)} = \sqrt{200} \approx 14.14$ Ω . $V_\phi = 346/\sqrt{3} = 200$ V. $I_\phi = 200/14.14 \approx 14.14$ A. $P = 3 \times 14.14^2 \times 10 = 6,000$ W = 6 kW. $Q = 3 \times 14.14^2 \times (-10) = -6,000$ VAR = -6 kVAR (capacitive). $PF = \cos(45^\circ) = 0.707$ leading.

60. What is the per-phase equivalent circuit approach for analyzing balanced three-phase loads?

Answer: In a balanced three-phase system, we can analyze one phase (usually phase A) and multiply by 3 for total 3-phase power. For Y loads, use $V_\phi = V_L/\sqrt{3}$ and $I_\phi = I_L$. For delta loads, convert to equivalent Y ($Z_Y = Z_\Delta/3$), solve, then convert back if needed.

61. A 3-phase generator rated 10 MVA, 11 kV, 0.8 PF lag delivers full load power. Find P, Q, and S.

Answer: $S = 10$ MVA. $P = 10 \times 0.8 = 8$ MW. $Q = 10 \times \sin(\cos^{-1}0.8) = 10 \times 0.6 = 6$ MVAR. These represent the maximum rated outputs at rated power factor.

62. What is the three-phase power formula in terms of line quantities only?

Answer: $P = \sqrt{3} \times V_L \times I_L \times \cos \phi$. $Q = \sqrt{3} \times V_L \times I_L \times \sin \phi$. $S = \sqrt{3} \times V_L \times I_L$. These formulas apply to both Y and delta connections when using line voltage and line current.

63. A three-phase system has $V_L = 6.6$ kV and feeds a load of 3 MW at 0.85 PF. Find Q and S.

Answer: $S = 3/0.85 \approx 3.529$ MVA. $\sin \phi = \sqrt{(1-0.7225)} \approx 0.5268$. $Q = 3.529 \times 0.5268 \approx 1.858$ MVAR.

64. Define the power angle ϕ and its significance in three-phase power calculations.

Answer: ϕ is the phase angle between the voltage and current phasors. $\cos \phi = \text{PF}$ determines how much of apparent power is useful real power. $\tan \phi = Q/P$ gives the ratio of reactive to real power. The sign of ϕ determines leading (capacitive) or lagging (inductive) behavior.

65. A factory's three-phase supply is 440 V, 50 Hz. It has three loads: L1 = 10 kW (PF 1.0); L2 = 15 kW (PF 0.8 lag); L3 = 8 kW (PF 0.6 lag). Find total line current.

Answer: $Q_1 = 0$; $Q_2 = 15 \times 0.75 = 11.25$ kVAR; $Q_3 = 8 \times (0.8/0.6) = 10.67$ kVAR. $P = 33$ kW, $Q = 21.92$ kVAR. $S = \sqrt{(33^2 + 21.92^2)} \approx 39.55$ kVA. $I_L = 39,550/(\sqrt{3} \times 440) \approx 51.91$ A.

66. Two three-phase loads: Load 1 is 100 kVA, 0.7 PF lag; Load 2 is 50 kW, 0.8 PF lead. Find combined PF.

Answer: $P_1 = 70$ kW, $Q_1 = 100 \times \sin(\cos^{-1}0.7) = 71.41$ kVAR. $P_2 = 50$ kW, $Q_2 = -50 \times (0.6/0.8) = -37.5$ kVAR. $P = 120$ kW, $Q = 33.91$ kVAR. $S = \sqrt{(120^2 + 33.91^2)} \approx 124.69$ kVA. $\text{PF} = 120/124.69 \approx 0.962$ lag.

67. How is power measured using the two-wattmeter method in a three-phase system?

Answer: Total $P = W_1 + W_2$ (sum of both wattmeter readings). Reactive power: $Q = \sqrt{3} \times (W_1 - W_2)$. Power factor: $\text{PF} = \cos[\tan^{-1}(\sqrt{3} \times (W_1 - W_2)/(W_1 + W_2))]$. If one wattmeter reads negative, it must be reversed and subtracted.

68. In the two-wattmeter method, $W_1 = 8$ kW and $W_2 = 4$ kW. Find P, Q, and PF.

Answer: $P = W_1 + W_2 = 12$ kW. $Q = \sqrt{3} \times (W_1 - W_2) = \sqrt{3} \times 4 = 6.928$ kVAR. $\phi = \tan^{-1}(6.928/12) = \tan^{-1}(0.577) = 30^\circ$. $\text{PF} = \cos(30^\circ) \approx 0.866$ lagging.

69. In the two-wattmeter method, $W_1 = 5$ kW and $W_2 = -2$ kW. Find P and Q.

Answer: $P = 5 + (-2) = 3$ kW (the negative reading means reverse the wattmeter connections). $Q = \sqrt{3} \times (5 - (-2)) = \sqrt{3} \times 7 \approx 12.12$ kVAR. This indicates a highly inductive load.

70. What is the condition in the two-wattmeter method when $\text{PF} = 0.5$?

Answer: When $\text{PF} = 0.5$ ($\phi = 60^\circ$), one wattmeter reads zero ($W_2 = 0$). When $\text{PF} < 0.5$, one wattmeter reads negative. When $\text{PF} = 1.0$, both wattmeters read equal positive values ($W_1 = W_2$).

71. A balanced three-phase Y load: $R = 6 \Omega$, $L = 25.5$ mH per phase, $f = 50$ Hz, $V_L = 415$ V. Find P, Q, S.

Answer: $X_L = 2\pi \times 50 \times 0.0255 \approx 8 \Omega$. $|Z| = \sqrt{(36 + 64)} = 10 \Omega$. $V_\phi = 415/\sqrt{3} \approx 239.6 \text{ V}$. $I_\phi = 23.96 \text{ A}$. $P = 3 \times 23.96^2 \times 6 \approx 10,375 \text{ W}$. $Q = 3 \times 23.96^2 \times 8 \approx 13,833 \text{ VAR}$. $S = 3 \times 239.6 \times 23.96 \approx 17,225 \text{ VA}$.

72. A three-phase delta load: $Z = 24 - j18 \Omega$ per phase (capacitive). $V_L = 400 \text{ V}$. Find P , Q , PF .

Answer: $|Z| = \sqrt{(576 + 324)} = 30 \Omega$. $\cos \phi = 24/30 = 0.8$, $\sin \phi = 0.6$. $I_\phi = 400/30 \approx 13.33 \text{ A}$. $P = 3 \times 13.33^2 \times 24 \approx 12,800 \text{ W} = 12.8 \text{ kW}$. $Q = -3 \times 13.33^2 \times 18 \approx -9,600 \text{ VAR} = -9.6 \text{ kVAR}$ (capacitive). $PF = 0.8$ leading.

73. Calculate the line current for a 3 ϕ load: $S = 200 \text{ kVA}$, $V_L = 11 \text{ kV}$.

Answer: $I_L = S / (\sqrt{3} \times V_L) = 200,000 / (\sqrt{3} \times 11,000) = 200,000 / 19,053 \approx 10.5 \text{ A}$.

74. A balanced 3 ϕ Y load dissipates 36 kW total. Each phase has $R = 16 \Omega$. Find V_ϕ and V_L .

Answer: $P_{\text{phase}} = 36/3 = 12 \text{ kW}$. $I_\phi^2 \times R = 12,000 \rightarrow I_\phi = \sqrt{(12000/16)} = \sqrt{750} \approx 27.39 \text{ A}$. $V_\phi = I_\phi \times R = 27.39 \times 16 \approx 438.2 \text{ V}$. $V_L = \sqrt{3} \times V_\phi \approx 759 \text{ V}$.

75. A three-phase motor load: $P = 22 \text{ kW}$, $Q = 16.5 \text{ kVAR}$. Find S , PF , and ϕ .

Answer: $S = \sqrt{(22^2 + 16.5^2)} = \sqrt{(484 + 272.25)} = \sqrt{756.25} = 27.5 \text{ kVA}$. $PF = 22/27.5 = 0.8$ lagging. $\phi = \cos^{-1}(0.8) = 36.87^\circ$.

76. Three single-phase loads, each 5 kVA at $PF = 0.9$ lag, are connected in a balanced three-phase system. Find total 3 ϕ P and Q .

Answer: Total $S = 3 \times 5 = 15 \text{ kVA}$. $P = 15 \times 0.9 = 13.5 \text{ kW}$. $\sin \phi = \sqrt{(1-0.81)} \approx 0.4359$. $Q = 15 \times 0.4359 \approx 6.54 \text{ kVAR}$.

77. A 3 ϕ system: $V_L = 230 \text{ V}$ (low voltage), load takes 10 A per line at 0.75 PF. Find P and Q .

Answer: $P = \sqrt{3} \times 230 \times 10 \times 0.75 \approx 2,990 \text{ W} \approx 2.99 \text{ kW}$. $\sin \phi = \sqrt{(1-0.5625)} \approx 0.6614$. $Q = \sqrt{3} \times 230 \times 10 \times 0.6614 \approx 2,638 \text{ VAR} \approx 2.64 \text{ kVAR}$.

78. If the line voltage is 400 V and the load is purely capacitive ($Z = -j20 \Omega$ per phase, delta), find P , Q , and S .

Answer: $I_\phi = 400/20 = 20 \text{ A}$. $I_L = \sqrt{3} \times 20 \approx 34.64 \text{ A}$. $P = 0 \text{ W}$ (purely reactive). $S = \sqrt{3} \times 400 \times 34.64 = 24,000 \text{ VA} = 24 \text{ kVA}$. $Q = -24 \text{ kVAR}$ (capacitive, leading).

79. A power system bus has $P = 500 \text{ MW}$, $Q = 200 \text{ MVAR}$. What is PF and $|S|$?

Answer: $|S| = \sqrt{(500^2 + 200^2)} = \sqrt{(250,000 + 40,000)} = \sqrt{290,000} \approx 538.5 \text{ MVA}$. $PF = 500/538.5 \approx 0.929$ lagging.

80. A Y-connected source with $V_\phi = 120\angle 0^\circ \text{ V}$ feeds a Y-load $Z = 5 + j5 \Omega$. Find $S_{3\phi}$.

Answer: $I_\phi = 120/(5+j5) = 120/(7.071\angle 45^\circ) = 16.97\angle -45^\circ \text{ A}$. $S_{3\phi} = 3 \times V_\phi \times I_\phi^* = 3 \times 120\angle 0^\circ \times 16.97\angle +45^\circ = 3 \times 2036\angle 45^\circ = 6108\angle 45^\circ \text{ VA} = 4320 + j4320 \text{ VA}$. $P = 4.32 \text{ kW}$, $Q = 4.32 \text{ kVAR}$.

81. A three-phase system feeds parallel loads: L1: 20 kVA, PF 0.8 lag; L2: 10 kVA, PF 0.6 lag; L3: 15 kVA, PF = 1.0. Find the total complex power.

Answer: $S_1 = 20\angle 36.87^\circ = 16 + j12 \text{ kVA}$. $S_2 = 10\angle 53.13^\circ = 6 + j8 \text{ kVA}$. $S_3 = 15 + j0 \text{ kVA}$. $S_{\text{total}} = (16+6+15) + j(12+8+0) = 37 + j20 \text{ kVA}$. $|S| = \sqrt{(37^2 + 20^2)} \approx 42.10 \text{ kVA}$. $PF = 37/42.10 \approx 0.879$ lag.

82. For a balanced three-phase system, if P increases while S remains constant, what happens to PF and Q ?

Answer: $PF = P/S$ increases (improves). Since $S^2 = P^2 + Q^2$, and S is constant, $Q = \sqrt{(S^2 - P^2)}$ decreases. A higher P for the same S means less reactive power and better utilization of the supply.

83. What is the significance of unity power factor in three-phase systems?

Answer: At unity PF: $P = S$ (all apparent power is real), $Q = 0$, minimum line current for given real power, minimum conductor losses (I^2R), and maximum efficiency. Generators and lines are utilized to their maximum capacity for useful work.

84. A Y-connected load: $Z_1 = 10\angle 0^\circ$, $Z_2 = 10\angle 30^\circ$, $Z_3 = 10\angle -30^\circ \Omega$ in each of phases A, B, C respectively connected to a 400 V balanced source. Is this a balanced load?

Answer: No, this is an unbalanced load because the three phase impedances are different. The phase angles and magnitudes may differ, creating unequal currents and requiring full three-phase analysis (cannot use single-phase equivalent and multiply by 3).

85. A three-phase induction motor: rated 15 kW, $\eta = 85\%$, PF = 0.8 lag, $V_L = 415$ V. Find the apparent power input and line current.

Answer: $P_{\text{input}} = 15,000/0.85 \approx 17,647$ W. $S = P_{\text{input}}/\text{PF} = 17,647/0.8 \approx 22,059$ VA. $I_L = S/(\sqrt{3} \times 415) = 22,059/718.7 \approx 30.7$ A.

86. What is the formula for three-phase power in terms of per-phase impedance for a Y-connected load?

Answer: $P_{3\phi} = 3 \times V_{\phi}^2/R$ (where R is per-phase resistance). Or more generally: $P = 3 \times |I_{\phi}|^2 \times R = 3 \times (V_{\phi}/|Z|)^2 \times R$. For delta: $P = 3 \times V_L^2/R_{\Delta}$ where R_{Δ} is the delta phase resistance.

87. A 33 kV, three-phase feeder carries 50 A at 0.9 PF. Calculate the power transmitted.

Answer: $P = \sqrt{3} \times 33,000 \times 50 \times 0.9 = \sqrt{3} \times 33,000 \times 45 \approx 2,572,274$ W ≈ 2.57 MW.

88. How is total reactive power of a three-phase system calculated from per-phase data?

Answer: $Q_{\text{total}} = 3 \times Q_{\text{phase}} = 3 \times V_{\phi} \times I_{\phi} \times \sin \phi = \sqrt{3} \times V_L \times I_L \times \sin \phi$. For Y: $Q = 3 \times |I_{\phi}|^2 \times X_{\text{phase}}$. For delta: $Q = 3 \times V_L^2/X_{\Delta}$, where X_{Δ} is per-phase reactance.

89. Calculate the per-phase real and reactive power: $V_{\phi} = 240$ V, $I_{\phi} = 15$ A, $\phi = 40^\circ$.

Answer: $P_{\text{phase}} = V_{\phi} \times I_{\phi} \times \cos \phi = 240 \times 15 \times \cos(40^\circ) = 3,600 \times 0.766 \approx 2,758$ W. $Q_{\text{phase}} = 240 \times 15 \times \sin(40^\circ) = 3,600 \times 0.643 \approx 2,314$ VAR. Total 3 ϕ : $P = 8,274$ W, $Q = 6,942$ VAR.

90. A 400 V, 3 ϕ system: $Z_Y = 4 + j3 \Omega$. Find the equivalent delta impedance and verify power is the same.

Answer: $Z_{\Delta} = 3 \times Z_Y = 3 \times (4 + j3) = 12 + j9 \Omega$. Y: $V_{\phi} = 231$ V, $I_{\phi} = 231/5 = 46.2$ A, $P = 3 \times 46.2^2 \times 4 \approx 25,663$ W. Delta: $I_{\phi} = 400/15 = 26.67$ A, $P = 3 \times 26.67^2 \times 12 \approx 25,603$ W \approx same \checkmark (small difference due to rounding).

91. An industrial plant has the following 3 ϕ loads on a 415 V bus: Motor A: 30 kW, 0.85 PF lag; Lighting: 10 kW, PF 1.0; Heaters: 20 kW, PF 1.0; Motor B: 25 kVA, 0.9 PF lag. Find total P, Q, S, and I_L .

Answer: $P_A = 30$ kW, $Q_A = 30 \times \tan(31.79^\circ) \approx 18.6$ kVAR. $P_{\text{light}} = 10$ kW, $Q_{\text{light}} = 0$. $P_{\text{heat}} = 20$ kW, $Q_{\text{heat}} = 0$. $P_B = 22.5$ kW, $Q_B = 25 \times 0.4359 \approx 10.9$ kVAR. $P = 82.5$ kW, $Q = 29.5$ kVAR. $S = \sqrt{(82.5^2 + 29.5^2)} \approx 87.65$ kVA. $I_L = 87,650/(\sqrt{3} \times 415) \approx 121.9$ A.

92. Show that for a Y-connected load: $S_{3\phi} = 3 \times |V_{\phi}|^2 / Z^*$

Answer: Per phase: $S_{\text{phase}} = V_{\phi} \times I_{\phi}^* = V_{\phi} \times (V_{\phi}/Z)^* = V_{\phi} \times V_{\phi}^*/Z^* = |V_{\phi}|^2/Z^*$. Total $S_{3\phi} = 3 \times S_{\text{phase}} = 3|V_{\phi}|^2/Z^*$. Example: $V_{\phi} = 120$ V, $Z = 4 + j3 \rightarrow S = 3 \times 14400/(4 - j3) = 43200/(4 - j3) = 6912 + j5184$ VA.

93. A 3 ϕ load consumes 100 kVA. At PF 0.6 lag, how much real and reactive power is consumed?

Answer: $P = 100 \times 0.6 = 60$ kW. $\sin(\cos^{-1}0.6) = 0.8$. $Q = 100 \times 0.8 = 80$ kVAR lagging. This shows that at low PF, a large portion of the apparent power is reactive and 'wasted'.

94. Compare the line currents for Y and delta connections of the same load impedance at the same line voltage.

Answer: Y connection: $I_L = V_L / (\sqrt{3} \times |Z|)$. Delta connection: $I_L = \sqrt{3} \times V_L / |Z|$. Ratio: $I_{L_delta} / I_{L_Y} = 3$. The delta connection draws 3 times more line current than the Y connection for the same impedance at the same line voltage.

95. A three-phase 480 V system powers a 60 kW resistive load. Find the per-phase resistance for both Y and delta connections.

Answer: For Y: $V_\phi = 480/\sqrt{3} = 277.1$ V, $I_\phi = P/(3V_\phi) = 60,000/(3 \times 277.1) = 72.17$ A, $R_{Y} = 277.1/72.17 \approx 3.84$ Ω . For delta: $V_\phi = 480$ V, $I_\phi = P/(3 \times 480) = 41.67$ A, $R_{\Delta} = 480/41.67 \approx 11.52$ Ω . Note $R_{\Delta} = 3 \times R_Y = 11.52 \approx 3 \times 3.84$ \checkmark

96. What happens to P, Q, and S if the line voltage in a balanced three-phase system is doubled?

Answer: Since $P = 3|V_\phi|^2 R/|Z|^2$ and $V_\phi \propto V_L$, doubling V_L doubles V_ϕ , quadrupling ($\times 4$) all power quantities: P, Q, and S all become 4 times their original values. Line current also doubles: $I_L = V_\phi/|Z|$ doubles.

97. Express reactive power Q in terms of real power P and power factor angle ϕ .

Answer: $Q = P \times \tan \phi$. Since $\tan \phi = \sin \phi / \cos \phi = (Q/S)/(P/S) = Q/P$. Therefore $Q = P \tan \phi = P \times \sqrt{(1 - PF^2)} / PF$. Example: $P = 100$ kW, $PF = 0.8$ ($\phi = 36.87^\circ$) $\rightarrow Q = 100 \times \tan(36.87^\circ) = 100 \times 0.75 = 75$ kVAR.

98. A balanced Y-connected source has $V_\phi = 200\angle 0^\circ$ V. It feeds a Y-load of $8 + j6$ Ω per phase. Write the complex power in rectangular form.

Answer: $Z = 8 + j6$, $|Z| = 10$. $I_\phi = 200/10\angle -36.87^\circ = 20\angle -36.87^\circ$ A. $S_{\text{phase}} = V_\phi \times I_\phi^* = 200\angle 0^\circ \times 20\angle +36.87^\circ = 4000\angle 36.87^\circ = 3200 + j2400$ VA. $S_{3\phi} = 9600 + j7200$ VA. So $P = 9.6$ kW, $Q = 7.2$ kVAR lag.

99. A substation bus has $P = 2$ MW, $Q = 1.5$ MVAR on a 6.6 kV three-phase system. Find |S|, PF, and I_L .

Answer: $|S| = \sqrt{(2^2 + 1.5^2)} = \sqrt{(4 + 2.25)} = \sqrt{6.25} = 2.5$ MVA. $PF = 2/2.5 = 0.8$ lagging. $I_L = S/(\sqrt{3} \times V_L) = 2,500,000/(\sqrt{3} \times 6,600) \approx 218.7$ A.

100. Summarize the key power relationships for a balanced three-phase system.

Answer: Real power: $P = \sqrt{3} V_L I_L \cos \phi = 3V_\phi I_\phi \cos \phi$ (W). Reactive power: $Q = \sqrt{3} V_L I_L \sin \phi = 3V_\phi I_\phi \sin \phi$ (VAR, +inductive/-capacitive). Apparent power: $S = \sqrt{3} V_L I_L = 3V_\phi I_\phi$ (VA). Complex power: $S = P + jQ$. Power factor: $PF = P/S = \cos \phi$. Power triangle: $S^2 = P^2 + Q^2$. Y: $V_\phi = V_L/\sqrt{3}$, $I_\phi = I_L$. Δ : $V_\phi = V_L$, $I_L = \sqrt{3} I_\phi$.