

Complex Numbers

Study Tutorial — 100 Practice Problems

Rectangular & Polar Forms • Four Fundamental Operations • Squares & Roots • De Moivre's Theorem

Instructions: Show all work. Express final answers in simplified form. For polar form results, express angles in degrees between 0° and 360° unless otherwise stated. Use exact values where possible.

Part I: Rectangular Form — Addition & Subtraction

Perform the indicated operations and express results in rectangular form $a + bi$.

1. Add: $(3 + 4i) + (1 - 2i)$

Answer: _____

2. Add: $(-5 + 3i) + (7 + i)$

Answer: _____

3. Subtract: $(6 + 2i) - (4 - 5i)$

Answer: _____

4. Subtract: $(-3 - i) - (2 + 4i)$

Answer: _____

5. Add: $(\frac{1}{2} + 3i) + (\frac{3}{2} - i)$

Answer: _____

6. Subtract: $(\sqrt{2} + i\sqrt{3}) - (\sqrt{2} - i\sqrt{3})$

Answer: _____

7. Add: $(-2 + 5i) + (2 - 5i)$

Answer: _____

8. Subtract: $(0 + 4i) - (3 + 0i)$

Answer: _____

9. Add: $(1 + i) + (-1 - i) + (2 + 3i)$

Answer: _____

10. Simplify: $(3 - 2i) + (4 + i) - (1 - 3i)$

Answer: _____

Part II: Rectangular Form — Multiplication

Multiply the following complex numbers and simplify. Remember $i^2 = -1$.

11. Multiply: $(2 + 3i)(4 - i)$

Answer: _____

12. Multiply: $(1 + i)(1 - i)$

Answer: _____

13. Multiply: $(3 - 2i)(3 - 2i)$

Answer: _____

14. Multiply: $(-1 + 2i)(3 + i)$

Answer: _____

15. Multiply: $(2 + i)^2$

Answer: _____

16. Multiply: $(\sqrt{3} + i)(\sqrt{3} - i)$

Answer: _____

17. Multiply: $(5i)(3 - 2i)$

Answer: _____

18. Multiply: $(4 + i)(4 + i)(4 - i)$

Answer: _____

19. Multiply: $(1 + 2i)(2 + 3i)(1 - i)$

Answer: _____

20. Multiply: $(3 - i)^3$

Answer: _____

Part III: Rectangular Form — Division

Divide the following complex numbers. Rationalize the denominator and express in $a + bi$ form.

21. Divide: $(3 + 4i) \div (1 + 2i)$

Answer: _____

22. Divide: $(5 - i) \div (2 + 3i)$

Answer: _____

23. Divide: $(6 + 0i) \div (1 - i)$

Answer: _____

24. Divide: $(2 + 3i) \div (2 - 3i)$

Answer: _____

25. Divide: $(-4 + 2i) \div (1 + i)$

Answer: _____

26. Divide: $(1 + i) \div i$

Answer: _____

27. Divide: $(7 + i) \div (3 + 2i)$

Answer: _____

28. Divide: $(3 - 4i) \div (3 + 4i)$

Answer: _____

29. Divide: $(10) \div (3 - i)$

Answer: _____

30. Simplify: $(2 + i)^2 \div (1 + 2i)$

Answer: _____

Part IV: Rectangular Form — Squares & Square Roots

Find the square and square roots of the following complex numbers.

31. Find z^2 if $z = 3 - 4i$

Answer: _____

32. Find z^2 if $z = -1 + 2i$
 Answer: _____
33. Find $\sqrt{-16}$
 Answer: _____
34. Find $\sqrt{-25}$
 Answer: _____
35. Find both square roots of $z = 5 + 12i$
 Answer: _____
36. Find both square roots of $z = -3 + 4i$
 Answer: _____
37. Find both square roots of $z = 7 - 24i$
 Answer: _____
38. Find both square roots of $z = -5 - 12i$
 Answer: _____
39. Find z^2 if $z = 2 + i\sqrt{5}$
 Answer: _____
40. Verify that $z = 2 + i$ is a square root of $z^2 = 3 + 4i$ by squaring z
 Answer: _____

Part V: Conversion — Rectangular to Polar Form

Convert to polar form $r(\cos \theta + i \sin \theta)$ or $r \operatorname{cis} \theta$. Express angles in degrees ($0^\circ \leq \theta < 360^\circ$).

41. Convert to polar form: $z = 1 + i\sqrt{3}$
 Answer: _____
42. Convert to polar form: $z = -3 + 3i$
 Answer: _____
43. Convert to polar form: $z = -2 - 2i$
 Answer: _____
44. Convert to polar form: $z = 4 - 4i\sqrt{3}$
 Answer: _____
45. Convert to polar form: $z = -5$
 Answer: _____
46. Convert to polar form: $z = 3i$
 Answer: _____
47. Convert to polar form: $z = 1 + i$
 Answer: _____
48. Convert to polar form: $z = -\sqrt{3} - i$
 Answer: _____
49. Convert to polar form: $z = 3 + 4i$ (use exact r , approximate θ to 2 decimals)
 Answer: _____
50. Convert to polar form: $z = -2 + 5i$ (approximate θ to 2 decimals)
 Answer: _____

Part VI: Conversion — Polar to Rectangular Form

Convert to rectangular form $a + bi$. Use exact trigonometric values where possible.

51. Convert: $z = 6(\cos 120^\circ + i \sin 120^\circ)$

Answer: _____

52. Convert: $z = 2(\cos 315^\circ + i \sin 315^\circ)$

Answer: _____

53. Convert: $z = 4 \operatorname{cis} 0^\circ$

Answer: _____

54. Convert: $z = 5 \operatorname{cis} 270^\circ$

Answer: _____

55. Convert: $z = 8(\cos 150^\circ + i \sin 150^\circ)$

Answer: _____

56. Convert: $z = 3(\cos 225^\circ + i \sin 225^\circ)$

Answer: _____

57. Convert: $z = 2(\cos 60^\circ + i \sin 60^\circ)$

Answer: _____

58. Convert: $z = \sqrt{2} \operatorname{cis} 45^\circ$

Answer: _____

59. Convert: $z = 10(\cos 300^\circ + i \sin 300^\circ)$

Answer: _____

60. Convert: $z = 1(\cos 180^\circ + i \sin 180^\circ)$

Answer: _____

Part VII: Polar Form — Multiplication & Division

Use polar rules: multiply/divide moduli and add/subtract arguments. Give results in polar and rectangular form.

61. Multiply: $z_1 = 3(\cos 40^\circ + i \sin 40^\circ)$, $z_2 = 2(\cos 80^\circ + i \sin 80^\circ)$

Answer: _____

62. Multiply: $z_1 = 4(\cos 150^\circ + i \sin 150^\circ)$, $z_2 = 2(\cos 30^\circ + i \sin 30^\circ)$

Answer: _____

63. Multiply: $z_1 = \sqrt{2} \operatorname{cis} 45^\circ$, $z_2 = \sqrt{2} \operatorname{cis} 45^\circ$

Answer: _____

64. Multiply: $z_1 = 5 \operatorname{cis} 100^\circ$, $z_2 = 2 \operatorname{cis} 200^\circ$

Answer: _____

65. Divide: $z_1 = 8(\cos 210^\circ + i \sin 210^\circ)$ by $z_2 = 2(\cos 60^\circ + i \sin 60^\circ)$

Answer: _____

66. Divide: $z_1 = 6 \operatorname{cis} 300^\circ$ by $z_2 = 3 \operatorname{cis} 120^\circ$

Answer: _____

67. Divide: $z_1 = 10(\cos 270^\circ + i \sin 270^\circ)$ by $z_2 = 5(\cos 90^\circ + i \sin 90^\circ)$

Answer: _____

68. Given $z_1 = 2 + 2i$ and $z_2 = 1 + i\sqrt{3}$, compute $z_1 \cdot z_2$ using polar multiplication

Answer: _____

69. Compute z_1/z_2 where $z_1 = 3(\cos 330^\circ + i \sin 330^\circ)$ and $z_2 = 3(\cos 30^\circ + i \sin 30^\circ)$

Answer: _____

70. Multiply three numbers: $z_1 = 2 \operatorname{cis} 30^\circ$, $z_2 = 3 \operatorname{cis} 60^\circ$, $z_3 = \operatorname{cis} 90^\circ$

Answer: _____

Part VIII: Powers — De Moivre's Theorem

Apply De Moivre's Theorem: $z^n = r^n(\cos n\theta + i \sin n\theta)$. Express results in $a + bi$ form.

71. Compute $(1 + i)^8$

Answer: _____

72. Compute $(\sqrt{3} + i)^6$

Answer: _____

73. Compute $(1 - i\sqrt{3})^4$

Answer: _____

74. Compute $[2(\cos 30^\circ + i \sin 30^\circ)]^5$

Answer: _____

75. Compute $(-1 + i)^3$ and verify algebraically

Answer: _____

76. Compute $(1 + i\sqrt{3})^9$

Answer: _____

77. Compute $(2 \operatorname{cis} 45^\circ)^4$

Answer: _____

78. Compute $(\sqrt{2} \operatorname{cis} 135^\circ)^6$

Answer: _____

79. Compute $(-1 - i)^{10}$

Answer: _____

80. Compute $(\cos 18^\circ + i \sin 18^\circ)^{10}$

Answer: _____

Part IX: Roots of Complex Numbers

Find all n th roots using: $z_k = r^{1/n}[\cos((\theta + 360^\circ k)/n) + i \sin((\theta + 360^\circ k)/n)]$, $k = 0, 1, \dots, n-1$

81. Find the 2 square roots of $z = 4(\cos 80^\circ + i \sin 80^\circ)$

Answer: _____

82. Find the 2 square roots of $z = 9(\cos 240^\circ + i \sin 240^\circ)$

Answer: _____

83. Find the 3 cube roots of $z = 8(\cos 0^\circ + i \sin 0^\circ)$

Answer: _____

84. Find the 4th roots of $z = 16(\cos 180^\circ + i \sin 180^\circ)$

Answer: _____

85. Find the 2 square roots of $z = 25 \operatorname{cis} 90^\circ$

Answer: _____

86. Find the 3 cube roots of $z = 27 \operatorname{cis} 270^\circ$

Answer: _____

87. Find the 4th roots of $z = 81(\cos 0^\circ + i \sin 0^\circ)$

Answer: _____

88. Find the cube roots of $z = -8$ (convert first)

Answer: _____

89. Find all square roots of $z = i$

Answer: _____

90. Find all cube roots of $z = 1$ (the three cube roots of unity)

Answer: _____

Part X: Mixed and Challenge Problems

These problems combine multiple operations. Show all steps. Express final answers in $a + bi$ form unless stated otherwise.

91. Simplify: $[(2 + 2i)/(1 - i)]^2$

Answer: _____

92. If $z = \cos \theta + i \sin \theta$, show that $z + 1/z = 2 \cos \theta$

Answer: _____

93. Express $(1 + i)^{10}$ in the form $a + bi$ using polar conversion

Answer: _____

94. Let $z_1 = 2 \operatorname{cis} 60^\circ$ and $z_2 = 3 \operatorname{cis} 150^\circ$. Compute $(z_1^2 \cdot z_2) / \bar{z}_1$

Answer: _____

95. Find all z such that $z^4 = -16$

Answer: _____

96. Compute $(3 + 4i)/(3 - 4i) + (3 - 4i)/(3 + 4i)$ and show the result is real

Answer: _____

97. If $z = 2(\cos 40^\circ + i \sin 40^\circ)$, find z^3 and z^{-2} in polar form

Answer: _____

98. Find z if $z^2 = 2i$. Express in both rectangular and polar forms.

Answer: _____

99. Compute $(1+i)^6/(1-i)^4$ and simplify to $a + bi$

Answer: _____

100. Given $z_1 = 1 + i\sqrt{3}$ and $z_2 = \sqrt{3} + i$, find $|z_1 z_2|$, $\arg(z_1 z_2)$, and the rectangular form

Answer: _____