1)

a) How many minima does $f(x, y) = x^2 + y$, subject to $x > y \geq 1$, have?
b) How many inflection points does $x^3 - 10$ have?

2) Compute the gradient of

$$f(x) = \ln x + e^x + \frac{1}{x}$$

for a point x close to zero. What term dominates in the expression?

3) Suppose we have $f(x) = x^2/2 - x$. Apply the bisection method to find an interval containing the minimizer of f starting with the interval [0, 1000]. Execute three steps of the algorithm.

4) The first Wolfe condition requires

$$f\left(x^{(k)} + \alpha d^{(k)}\right) \leq f\left(x^{(k)}\right) + \beta\alpha\nabla_{d^{(k)}} f\left(x^{(k)}\right)$$

What is the maximum step length α that satisfies this condition, given that

$$f(x) = 5 + x_1^2 + x_2^2,\ x^{(k)} = [-1,-1],\ d = [1,\ 0],\ and\ \beta = 10^{-4}$$

5) Apply gradient descent with a unit step size to $f(x) = x^4$ from a starting point of your choice. Compute two iterations.

6) In conjugate gradient descent, what is the normalized descent direction at the first iteration for the function $f(x, y) = x^2 + xy + y^2 + 5$ when initialized at (x, y) = (1, 1)? What is the resulting point after two steps of the conjugate gradient method?

7) Suppose we have a function $f(x) = (x_1 + 1)^2 + (x_2 + 3)^2 + 4$. If we start at the origin, what is the resulting point after one step of Newton's method?

8) Suppose we want to minimize $x_1^3 + x_2^2 + x_3$ subject to the constraint that $x_1 + 2x_2 + 3x_3 = 6$. How might we transform this into an unconstrained problem with the same minimizer?