

SOLUTION EX.2.10

Given

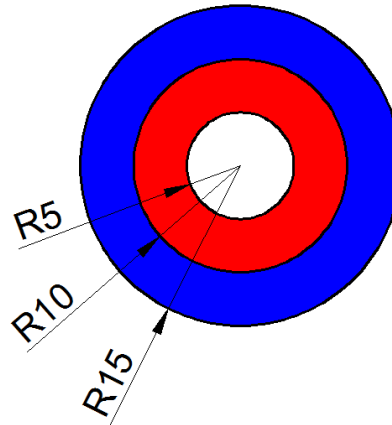
$$\sigma_{\max} = 70\% S_y, \quad \sigma_{\max} = 0.7 \cdot 530 = 371 \text{ MPa}$$

materials are the same

$$S_y = 530 \text{ MPa}$$

$$E = 207 \text{ GPa}$$

$$\mu = 0.292$$



Remember (Lame Equations)

$$P_o = 0;$$

$$P_i = 0$$

$$\sigma_r = P_i \frac{r_i^2}{r_o^2 - r_i^2} \left(1 - \frac{r_o^2}{r^2} \right) \quad \sigma_r = -P_o \frac{r_o^2}{r_o^2 - r_i^2} \left(1 - \frac{r_i^2}{r^2} \right)$$

$$\sigma_t = P_i \frac{r_i^2}{r_o^2 - r_i^2} \left(1 + \frac{r_o^2}{r^2} \right) \quad \sigma_t = -P_o \frac{r_o^2}{r_o^2 - r_i^2} \left(1 + \frac{r_i^2}{r^2} \right)$$

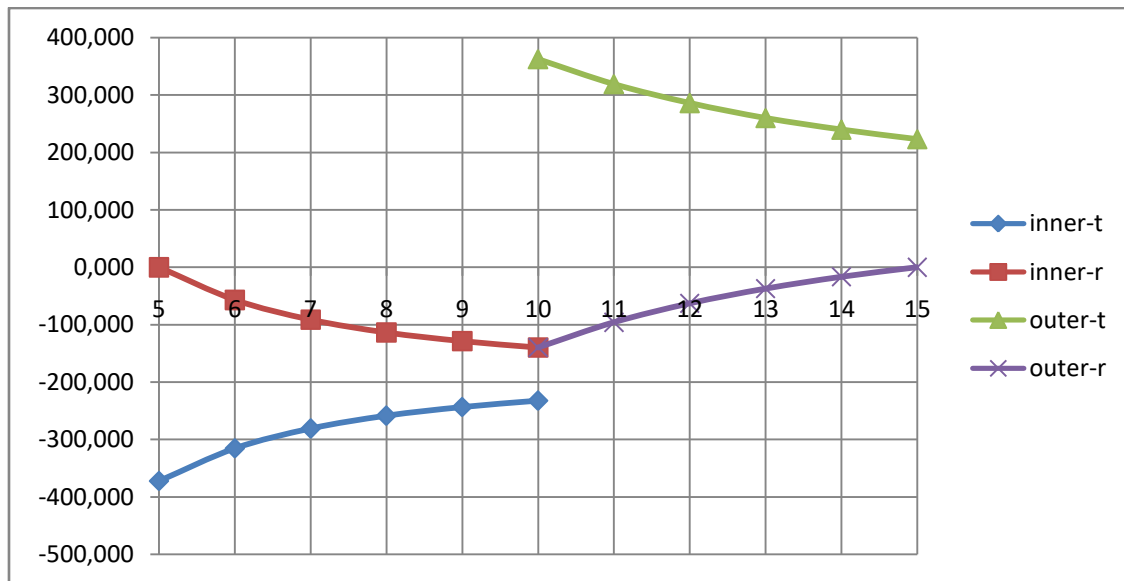
$\sigma_{\max}$ (critical stress) may occur at;

σ_t – inner radius of inner member

σ_t – outer radius of inner member

σ_t – inner radius of outer member

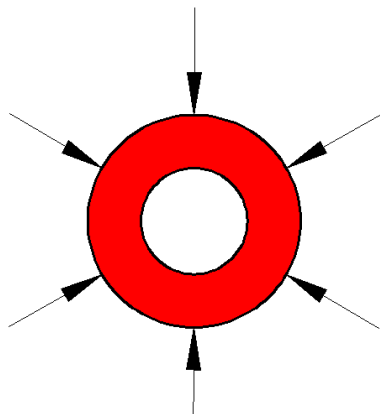
σ_r – outer radius of inner member or inner radius of outer member.



First we need to find where the maximum stress is

In order to find it, we must calculate all the stresses on the critical surfaces by taking the interface pressure as “p”.

Inner Member



$$r_i = a = 5 \text{ mm}$$

$$r_o = b = 10 \text{ mm}$$

σ_t – inner radius of inner member (for $r=a=5 \text{ mm}$)

$$\sigma_t = -p \frac{r_o^2}{r_o^2 - r_i^2} \left(1 + \frac{r_i^2}{r^2} \right)$$

$$\sigma_t = -p \frac{b^2}{b^2 - a^2} \left(1 + \frac{a^2}{a^2} \right)$$

$$\sigma_t = -p \frac{10^2}{10^2 - 5^2} (2)$$

$$\sigma_t = -2.66p$$

σ_t – outer radius of inner member ($r=b=10 \text{ mm}$)

$$\sigma_t = -p \frac{r_o^2}{r_o^2 - r_i^2} \left(1 + \frac{r_i^2}{r^2}\right)$$

$$\sigma_t = -p \frac{b^2}{b^2 - a^2} \left(1 + \frac{a^2}{b^2}\right)$$

$$\sigma_t = -p \frac{10^2}{10^2 - 5^2} \left(1 + \frac{5^2}{10^2}\right)$$

$$\sigma_t = -1.66p$$

σ_r – outer radius of inner member

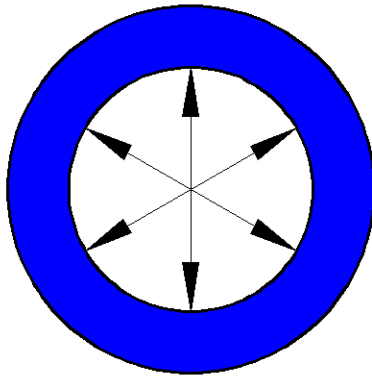
$$\sigma_r = -p \frac{r_o^2}{r_o^2 - r_i^2} \left(1 - \frac{r_i^2}{r^2}\right)$$

$$\sigma_r = -p \frac{b^2}{b^2 - a^2} \left(1 - \frac{a^2}{b^2}\right)$$

$$\sigma_r = -p \frac{10^2}{10^2 - 5^2} \left(1 - \frac{5^2}{10^2}\right)$$

$$\sigma_r = -p$$

Outer Member



$$\begin{aligned} r_i &= b = 10 \text{ mm} \\ r_o &= c = 15 \text{ mm} \end{aligned}$$

σ_t – inner radius of outer member (for $r=b=10$ mm)

$$\sigma_t = -p \frac{r_i^2}{r_o^2 - r_i^2} \left(1 + \frac{r_o^2}{r^2}\right)$$

$$\sigma_t = -p \frac{b^2}{c^2 - b^2} \left(1 + \frac{c^2}{b^2}\right)$$

$$\sigma_t = -p \frac{10^2}{15^2 - 10^2} \left(1 + \frac{15^2}{10^2}\right)$$

$$\sigma_t = -2.60p$$

Comparing the stresses, maximum principle stress is $\sigma_t = -2.66$ (at inner radius of inner member) is the maximum.

$$\sigma_{max} = 2.66p = 371 \text{ MPa} \quad , \quad p = 139.47 \text{ MPa}$$

Amount of total interference,

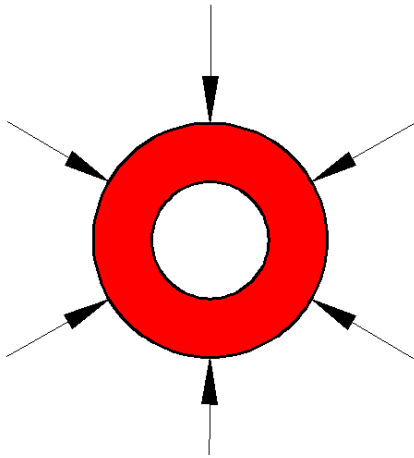
$$\delta = \frac{bp}{E} \left(\frac{2b^2(c^2 - a^2)}{(c^2 - b^2)(b^2 - a^2)} \right)$$

Where, a=5mm, b=10mm, c=15mm, p=139.47MPa

$$\delta = 0.029 \text{ mm}$$

Stress Distributions

Stress distribution for inner member



$$\sigma_r = -P_o \frac{r_o^2}{r_o^2 - r_i^2} \left(1 - \frac{r_i^2}{r^2} \right)$$

$$\sigma_t = -P_o \frac{r_o^2}{r_o^2 - r_i^2} \left(1 + \frac{r_i^2}{r^2} \right)$$

Where,

$r_i = a = 5 \text{ mm}$

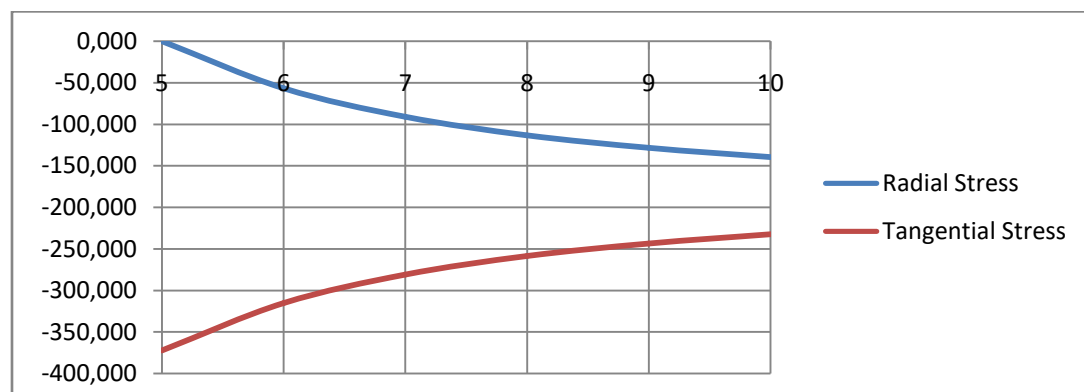
$r_o = b = 10 \text{ mm}$

and for different values of r

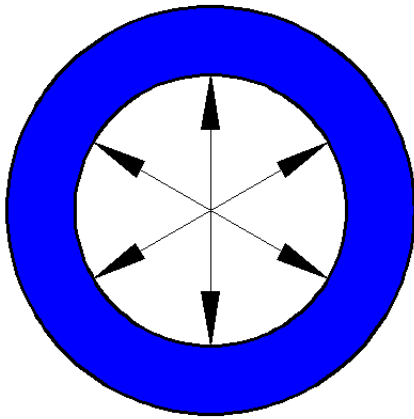
$$\sigma_{ir} = -190.26 - \frac{4756.5}{r^2}$$

$$\sigma_{it} = -190.26 + \frac{4756.5}{r^2}$$

p	a	b	r	σ_r	σ_t
139.47	5	10	5	0.000	-371.920
139.47	5	10	6	-56.821	-315.099
139.47	5	10	7	-91.082	-280.838
139.47	5	10	8	-113.319	-258.601
139.47	5	10	9	-128.565	-243.355
139.47	5	10	10	-139.470	-232.450



Stress Distribution for outer member



$$\sigma_r = P_i \frac{r_i^2}{r_o^2 - r_i^2} \left(1 - \frac{r_o^2}{r^2} \right)$$

$$\sigma_t = P_i \frac{r_i^2}{r_o^2 - r_i^2} \left(1 + \frac{r_o^2}{r^2} \right)$$

Where,

$r_i = b = 10 \text{ mm}$

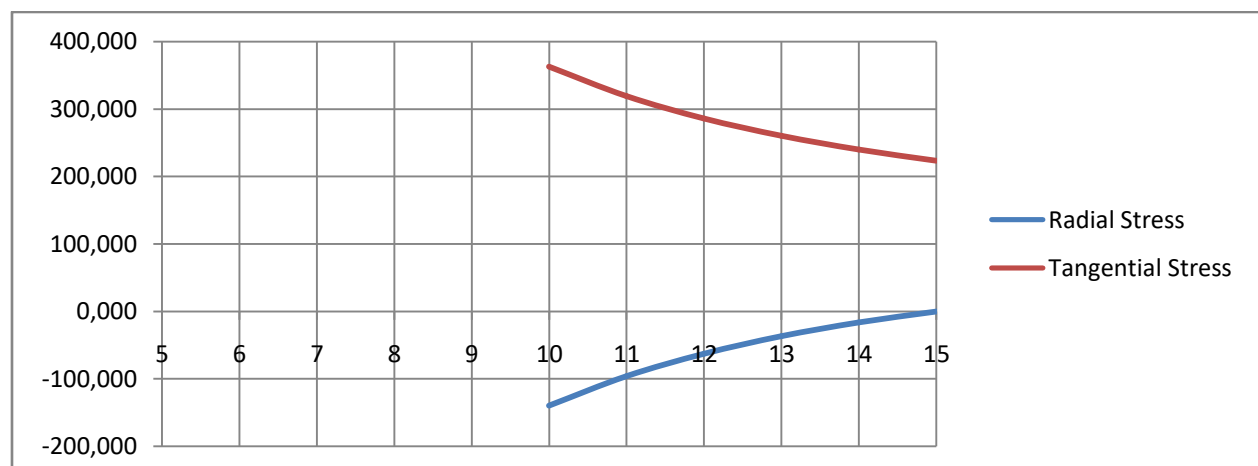
$r_o = c = 15 \text{ mm}$

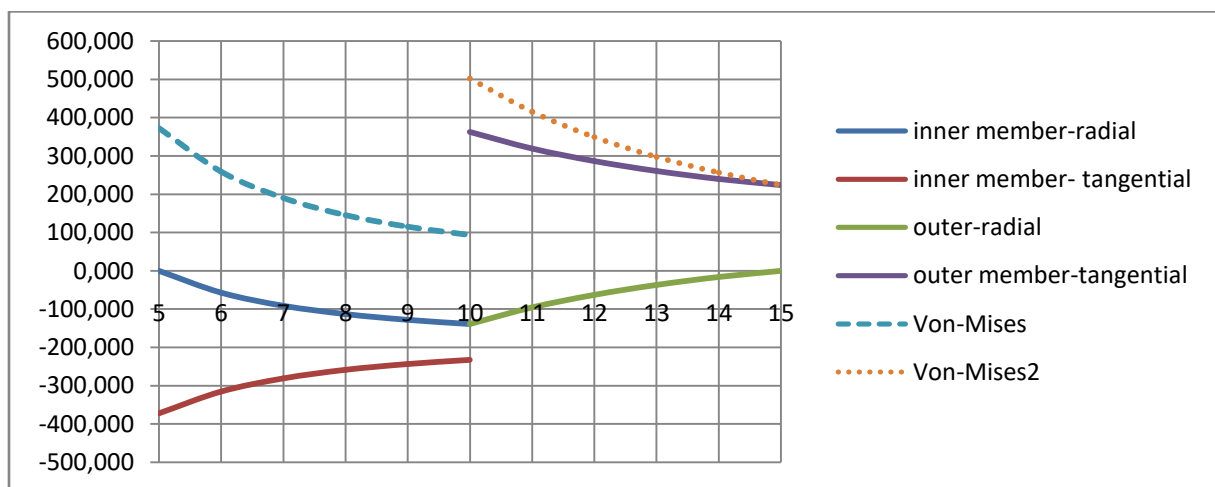
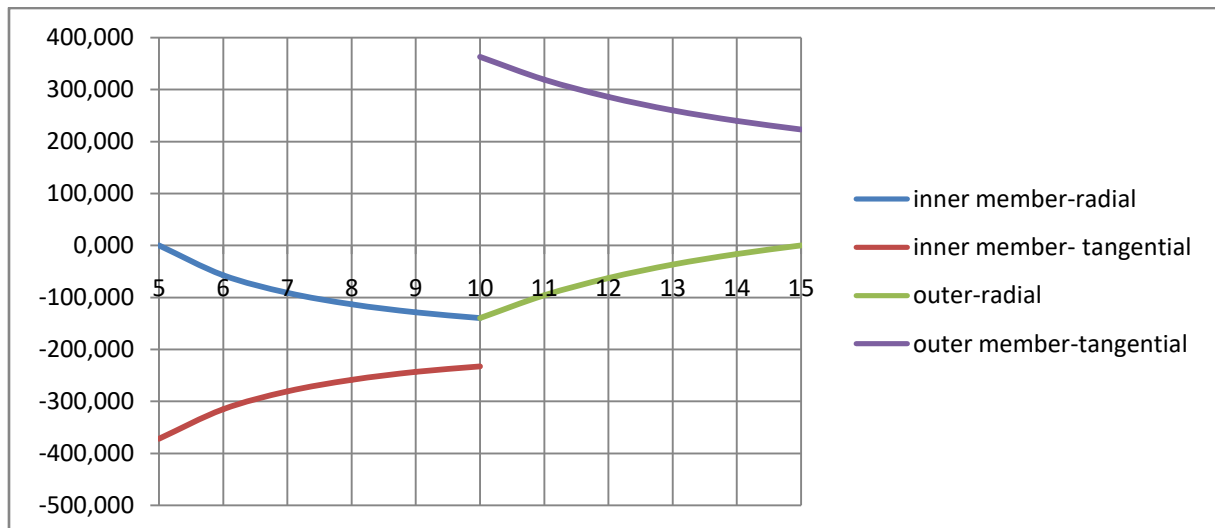
and for different values of r

$$\sigma_{ir} = 114.16 - \frac{25686}{r^2}$$

$$\sigma_{it} = 114.16 + \frac{25686}{r^2}$$

p	b	c	r	σ_r	σ_t
139.47	10	15	10	-139.470	362.622
139.47	10	15	11	-95.900	319.052
139.47	10	15	12	-62.762	285.914
139.47	10	15	13	-36.972	260.124
139.47	10	15	14	-16.509	239.661
139.47	10	15	15	0.000	223.152





When the gun fire;

If there is no interference fit and the cylinder is made of a single cylinder, max stress occurs at the inner radius as 350MPa (tension)

single						
Pi	a	c	r	σ_r	σ_t	σ_i
280	5	15	5	-280.0	350.0	630.0
280	5	15	6	-183.8	253.8	437.5
280	5	15	7	-125.7	195.7	321.4
280	5	15	8	-88.0	158.0	246.1
280	5	15	9	-62.2	132.2	194.4
280	5	15	10	-43.8	113.8	157.5
280	5	15	11	-30.1	100.1	130.2
280	5	15	12	-19.7	89.7	109.4
280	5	15	13	-11.6	81.6	93.2
280	5	15	14	-5.2	75.2	80.4
280	5	15	15	0.0	70.0	70.0

After interference fit, when the gun fire, again the cylinders behave as a single cylinder and stresses are superposed,

	single		residual		superposed		
	σ_r	σ_t	σ_r	σ_t	σ_r	σ_t	σ_i
5	-280	350.000	0.000	-371.92	-280.000	-21.920	258.08
6	-183.75	253.750	-56.821	-315.099	-240.571	-61.349	179.2222
7	-125.714	195.714	-91.082	-280.838	-216.797	-85.123	131.6735
8	-88.0469	158.047	-113.319	-258.601	-201.366	-100.554	100.8125
9	-62.2222	132.222	-128.565	-243.355	-190.787	-111.133	79.65432
10	-43.75	113.750	-139.470	-232.45	-183.220	-118.700	64.52
10	-43.75	113.750	-139.470	362.622	-183.220	476.372	659.592
11	-30.0826	100.083	-95.900	319.052	-125.983	419.135	545.1174
12	-19.6875	89.688	-62.762	285.9135	-82.449	375.601	458.05
13	-11.5976	81.598	-36.972	260.1239	-48.570	341.722	390.2911
14	-5.17857	75.179	-16.509	239.6607	-21.687	314.839	336.5265
15	0	70.000	0.000	223.152	0.000	293.152	293.152