ME 307-Machine Elements-I

Pressurized Cylinders

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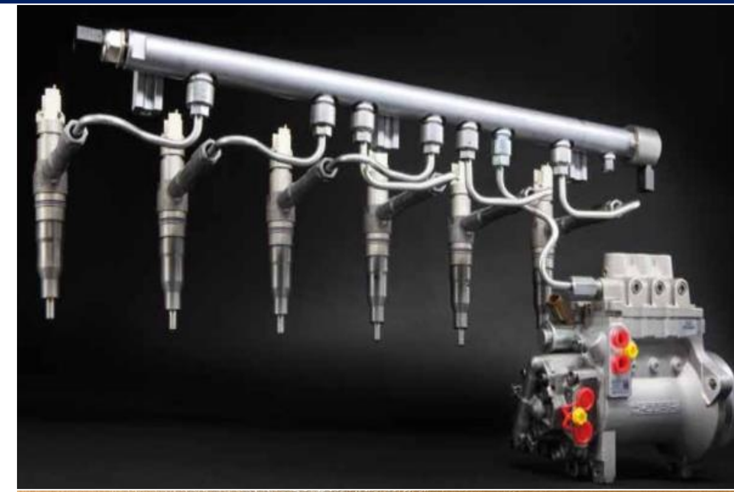
OUTLINE

- INTRODUCTION
- STRESSES IN A THICK WALLED CYLINDER
- INTERFERENCE OR SHRINK FITS
- EXAMPLE QUESTION

INTRODUCTION

- **THICK-WALLED CYLINDERS**
  - **Thick-walled cylinders are used in many industrial applications such as;**
    - Military (gun, cannon barrels)
    - Automobile (fuel rails, common rail systems)
    - Waterjet (intensifier, steel pipes)
    - Nuclear (nuclear reactors)

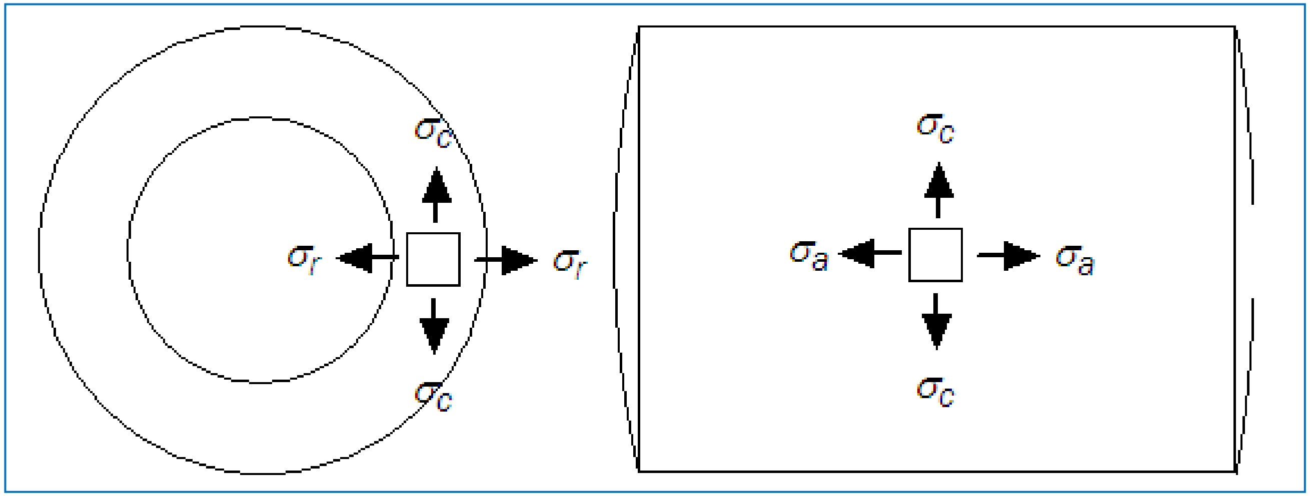


STRESSES IN A THICK WALLED CYLINDER

– Radial stress

– Tangential (hoop) stress

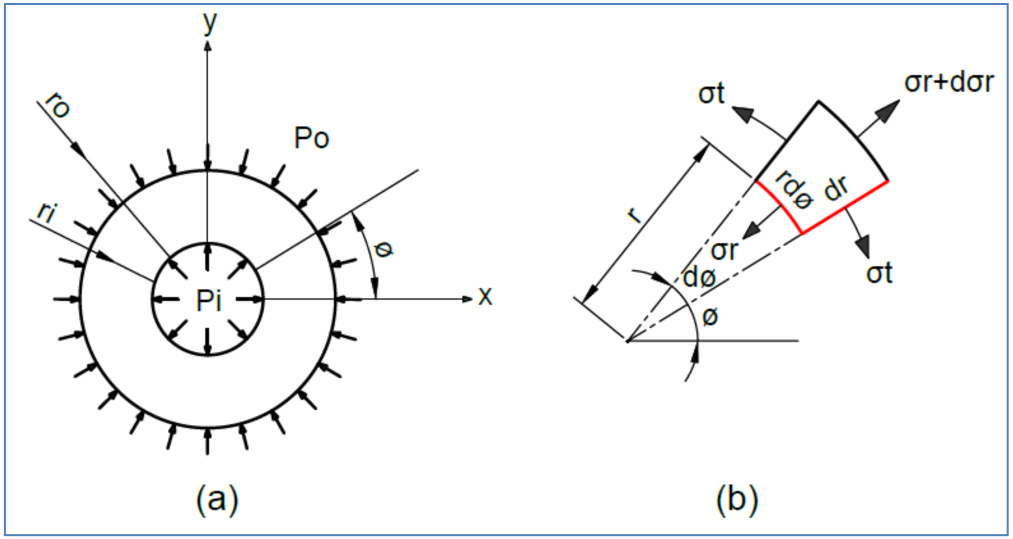
– Axial (longutidunal) stress (*if both ends are closed*)



- **Lâme equations;**

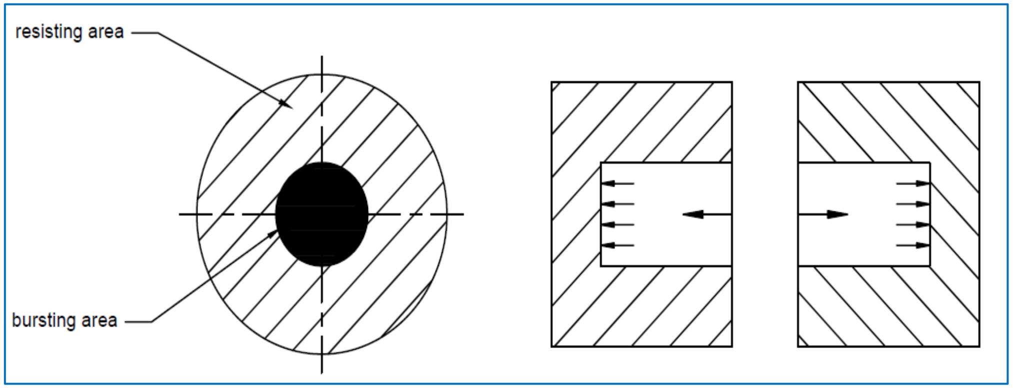
$$\sigma_r = \frac{p_i {r_i}^2 - p_o {r_o}^2 + {r_i}^2 {r_o}^2 (p_o - p_i)/r^2}{{r_o}^2 - {r_i}^2}$$

$$\sigma_t = \frac{p_i {r_i}^2 - p_o {r_o}^2 - {r_i}^2 {r_o}^2 (p_o - p_i)/r^2}{{r_o}^2 - {r_i}^2}$$



If the ends are closed;

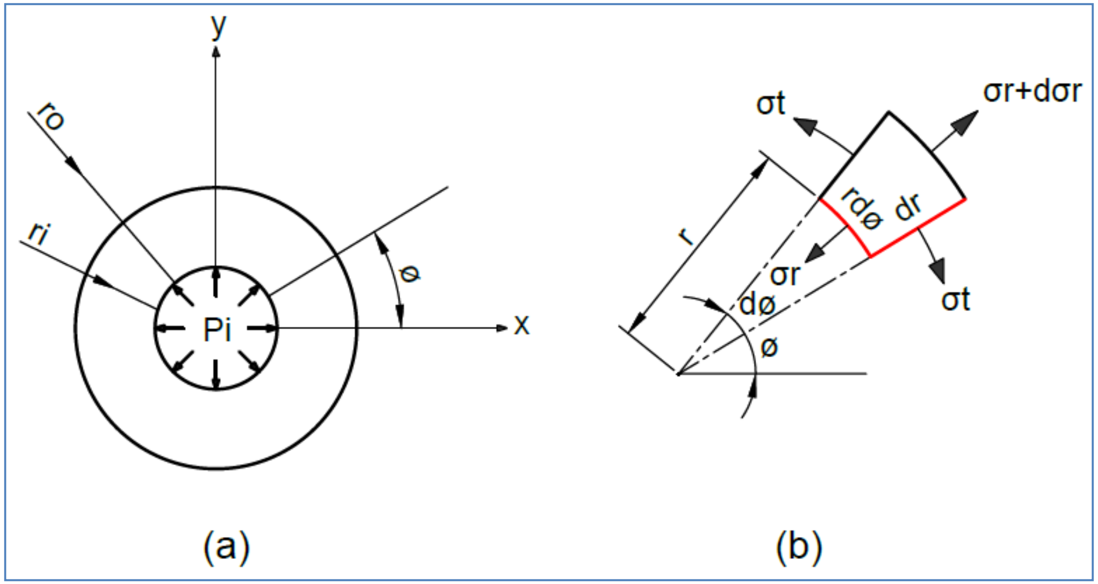
$$\sigma_l = P_i \frac{r_i^2}{r_o^2 - r_i^2}$$



- **Lâme equations (*P_o=0 MPa*);**

$$\sigma_r = P_i \frac{r_i^2}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right)$$

$$\sigma_t = P_i \frac{r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

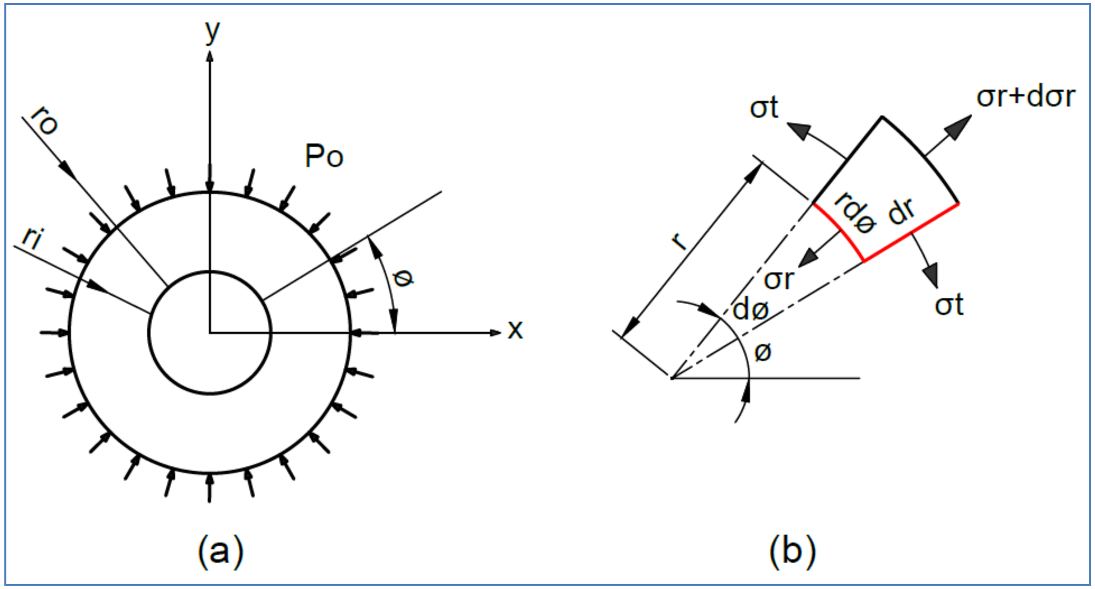


Where; σ_r **is compression,**
σ_t **is tension**

- **Lâme equations (*P_i=0 MPa*);**

$$\sigma_r = -P_o \frac{r_o^2}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r^2}\right)$$

$$\sigma_t = -P_o \frac{r_o^2}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$



Where; σ_t is compression,
σ_r is compression

- **Lâme equations (*Ends are closed, P_o=0 MPa*);**

$$\sigma_r = P_i \frac{r_i^2}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right)$$

$$\sigma_t = P_i \frac{r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

$$\sigma_l = P_i \frac{r_i^2}{r_o^2 - r_i^2}$$

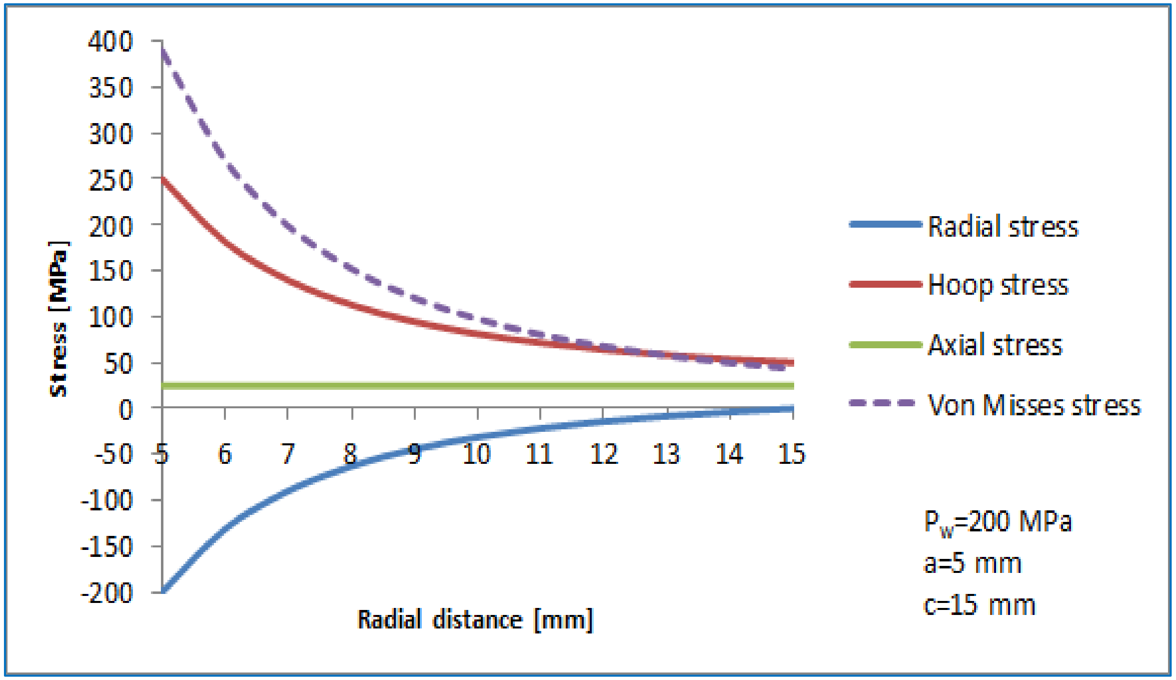


Figure 1 *Stress distributions of a thick walled cylinder subjected to P_i=200 MPa internal pressure*

$$\sigma' = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}$$

STRESSES IN A THICK WALLED CYLINDER

Figure 2 *Failure on a high pressure cylinder of an intensifier pump*

– *Interference fit*

– *Autofrettage*

Compressive type of residual stress at the inner wall must be created!!!

- **Introduction**
  - Interference fit is one of the common mechanical methods used to join two cylindrical components with a certain amount of interference;
    - to transmit torque (i.e. shaft gear connection)
    - to create compressive type of residual stress in thick walled cylinders.
  - Interference fit can be obtained by heating and/or cooling the members or by applying a certain amount of force in axial direction.

Figure 3 *Press fitted cannon barrel*

– After fitting, a certain amount of interface pressure (p) develops between the members as illustrated in Figure 3.6.
– Interface pressure acts as an external pressure for the inner member and as an internal pressure for the outer member.

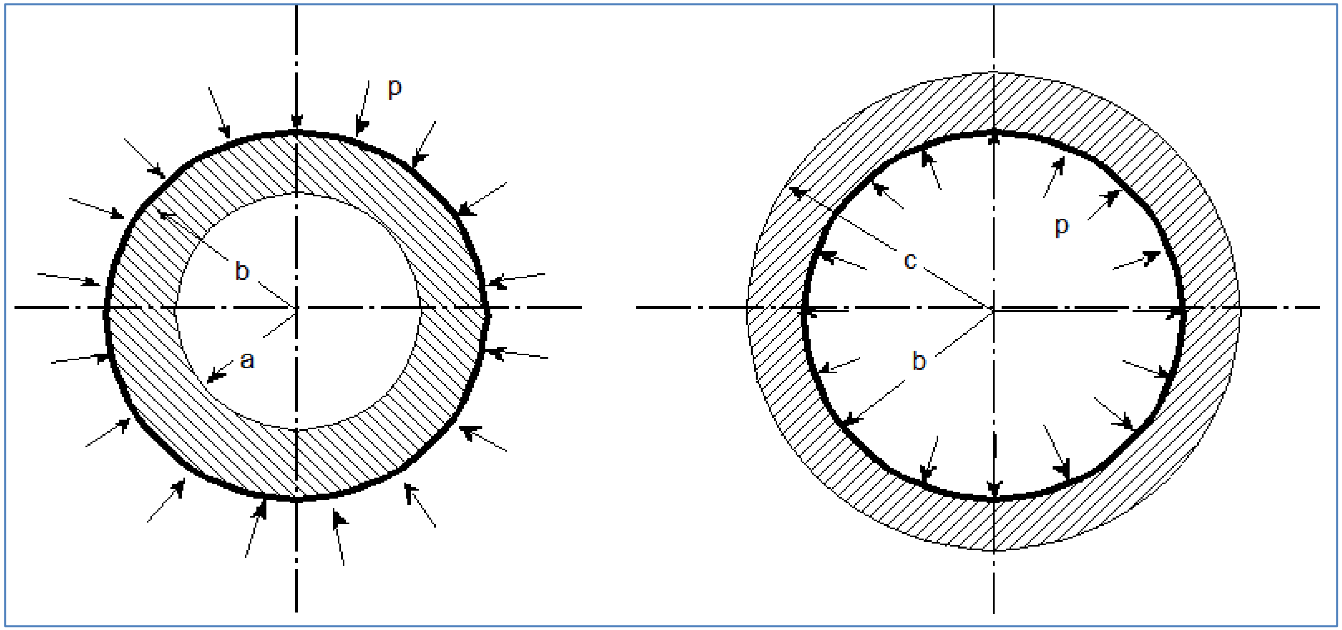


Figure 4 *Representation of interference pressure for inner and outer member*

- Lâme equations for each member;

-For inner member;

$$\sigma_{ir} = -p\frac{b^2}{b^2-a^2}\left(1-\frac{a^2}{r^2}\right)$$

$$\sigma_{it} = -p\frac{b^2}{b^2-a^2}\left(1+\frac{a^2}{r^2}\right)$$

-For outer member

$$\sigma_{or} = p\frac{b^2}{c^2-b^2}\left(1-\frac{c^2}{r^2}\right)$$

$$\sigma_{ot} = p\frac{b^2}{c^2-b^2}\left(1+\frac{c^2}{r^2}\right)$$

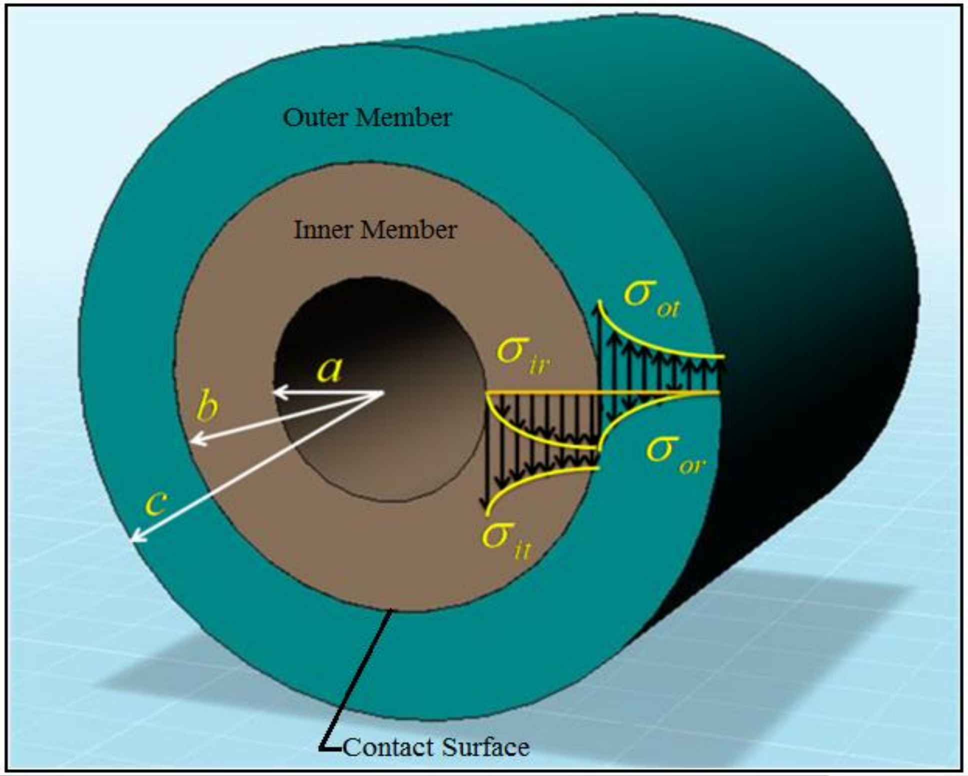


Figure 5 *Stress distribution in shrink-fitted cylinder*

Note: The assembly has 14.25mm inner, 24.09mm interface and 38.6mm outer radius which are denoted by "a", "b" and "c" respectively

- Lâme equations for each member when the working pressure is acting;

-For inner member;

$$\sigma_{ir} = P_w \frac{a^2}{c^2 - a^2}\left(1 - \frac{c^2}{r^2}\right) - p\frac{b^2}{b^2 - a^2}\left(1 - \frac{a^2}{r^2}\right)$$

$$\sigma_{it} = P_w \frac{a^2}{c^2 - a^2}\left(1 + \frac{c^2}{r^2}\right) - p\frac{b^2}{b^2 - a^2}\left(1 + \frac{a^2}{r^2}\right)$$

-For outer member

$$\sigma_{or} = P_w \frac{a^2}{c^2 - a^2}\left(1 - \frac{c^2}{r^2}\right) + p\frac{b^2}{c^2 - b^2}\left(1 - \frac{c^2}{r^2}\right)$$

$$\sigma_{ot} = P_w \frac{a^2}{c^2 - a^2}\left(1 + \frac{c^2}{r^2}\right) + p\frac{b^2}{c^2 - b^2}\left(1 + \frac{c^2}{r^2}\right)$$

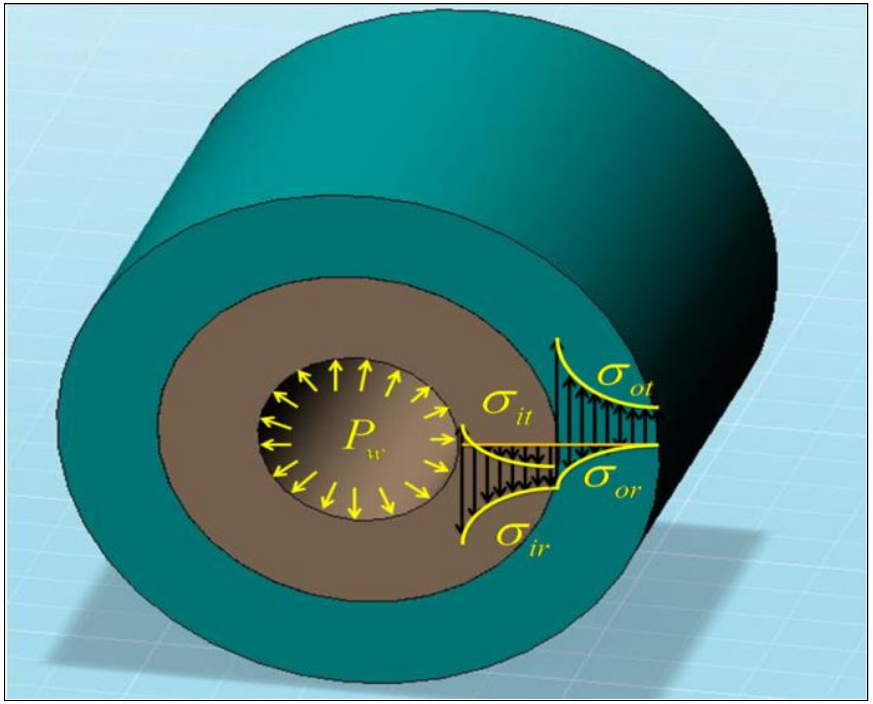


Figure 6 *Stress distribution in shrink-fitted cylinder under working pressure of 410 MPa*

Note:*Two cylinders behave as a single cylinder*

INTERFERENCE OR SHRINK FITS

– In interference fit applications, the problem is how to determine the amount of interference between inner and outer members.

– From strength requirement, interface pressure is determined first and then the amount of interference will be calculated.

– The interface pressure (p) depends on the total amount of interference (δ) which is the sum of deformations experienced by the inner and the outer member ($\delta = \delta_i + \delta_o$).

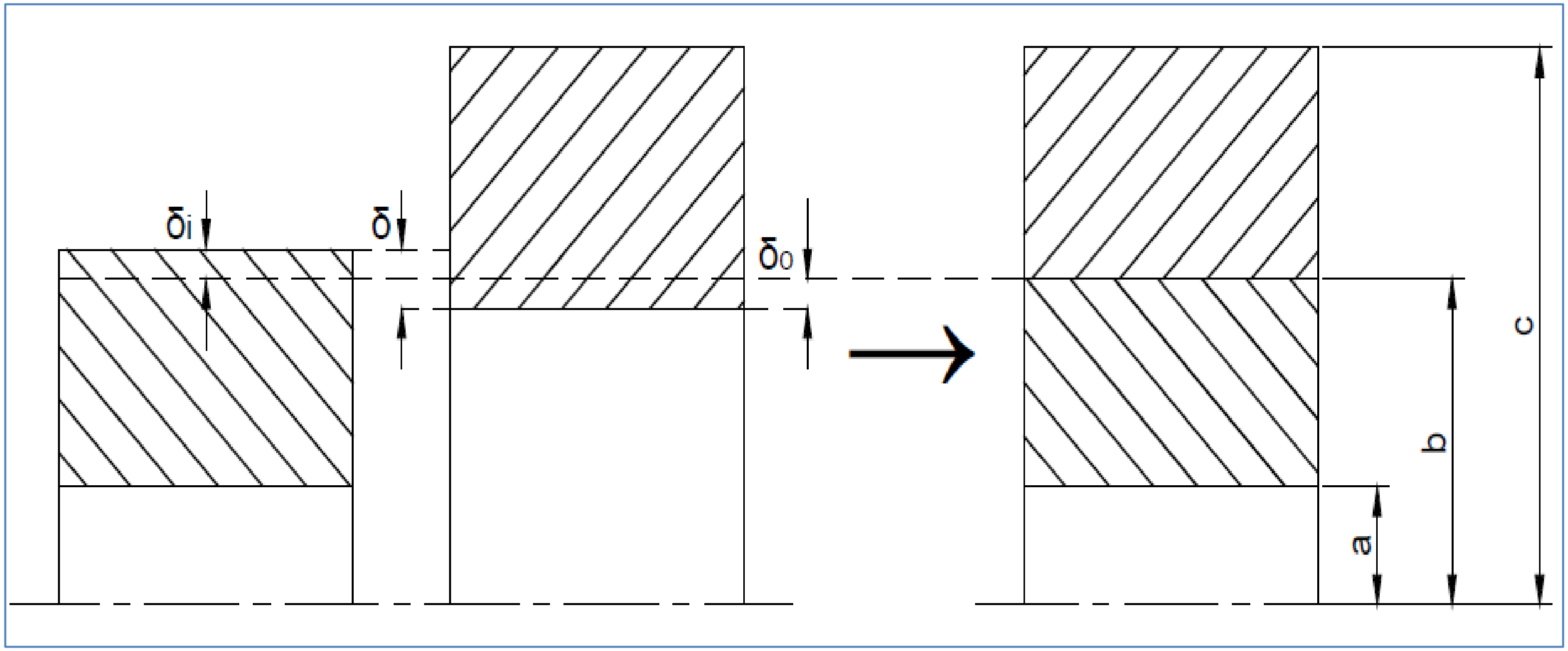


Figure 7 *Half sectional view of shrink fit assembly*

– The relationship between the interface pressure and the amount of interference is derived as follows:

- From the circumferential change in the outer member, equation for tangential strain is;

$$\varepsilon_t = \frac{2\pi(b+\delta_o) - 2\pi b}{2\pi b} = \frac{\delta_o}{b} \qquad (1)$$

- Tangential strain also equals to;

$$\varepsilon_t = \frac{\sigma_t}{E_o} - \mu_o \frac{\sigma_r}{E_o} \qquad (2)$$

- Combining the equations yields;

$$\delta_o = \frac{b}{E_o}(\sigma_t - \mu_o \sigma_r) \qquad (3)$$

- Lâme equations are arranged by giving “b”, “c” and “p” instead of “ri”, “ro” and Pi respectively and for r=b as;

$$\sigma_r = -p \qquad (4)$$

$$\sigma_t = p\frac{b^2+c^2}{c^2-b^2} \qquad (5)$$

INTERFERENCE OR SHRINK FITS

- Substituting Eqs. (4) and (5) into Eq. (3), deformation experienced by the outer member is found as:

$$\delta_o = \frac{bp}{E_o}\left(\frac{b^2 + c^2}{c^2 - b^2} + \nu_o\right)$$

- By using similar expressions for the inner member, deformation experienced by the inner member is found as:

$$\delta_i = \frac{bp}{E_i}\left(\frac{b^2 + a^2}{b^2 - a^2} - \nu_i\right)$$

- And the total amount of interference is found as:

$$\delta = \frac{bp}{E_o}\left(\frac{b^2 + c^2}{c^2 - b^2} + \nu_o\right) + \frac{bp}{E_i}\left(\frac{b^2 + a^2}{b^2 - a^2} - \nu_i\right)$$

- If the materials are the same $E_o=E_i=E$, δ is written as:

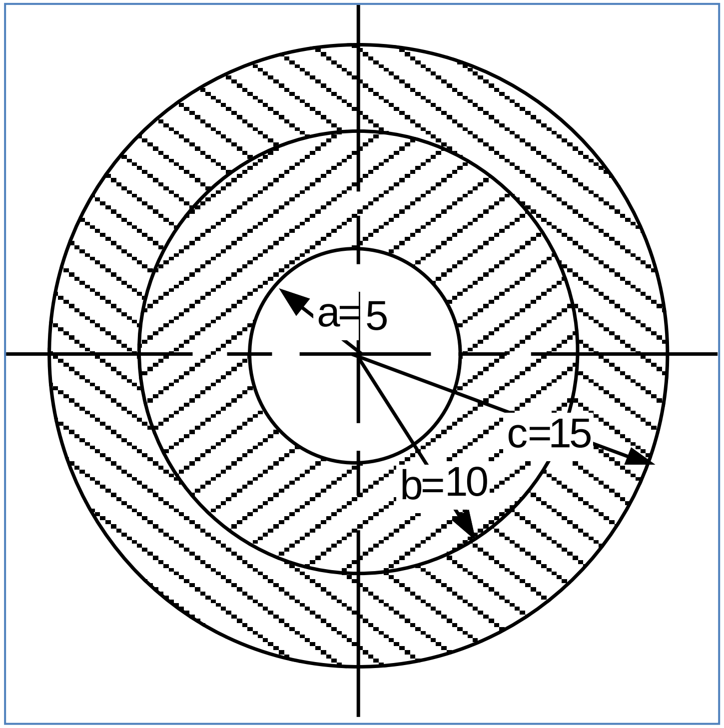
$$\delta = \frac{pb}{E}\left[\frac{2b^2(c^2 - a^2)}{(c^2 - b^2)(b^2 - a^2)}\right]$$

- **Example 2.10**

A gun barrel is assembled by shrinking an outer barrel over an inner barrel so that the maximum principal stress equal to 70% of the yield strength of the material of the barrels. Both barrels are made from steel and have properties Sy=530MPa, E=207 GPa and μ (poisson's ratio)=0.292. Inner and outer diameters of the inner barrel are 10mm and 20mm respectively. Outer diameter of the outer barrel is 30mm.

a. What value of radial interference should be used in the assembly ? Plot the stress distribution on the inner and outer barrel.

b. When the gun is fired, an internal pressure of 280 MPa is created. Plot the stress distribution on the inner and the outer barrels.



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