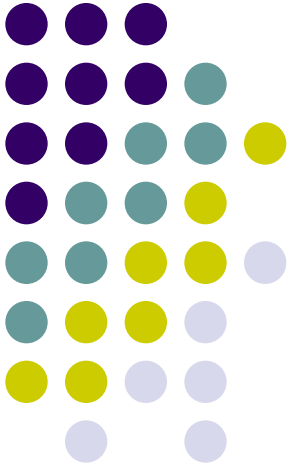
ME 307 – Machine Elements I

Chapter 5

Design for Fatigue Strength (Part I)



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Introduction to Fatigue

- **In the static design**, the system is **motionless**, and loading condition on the system **does not change with respect to time**.
  - *For a ductile material*, if the **equivalent stress** of the element is greater than the **yield strength** of the material, **failure occurs**. Because, when the stress exceeds the yield strength than the plastic deformation occurs which means that the material experiences a permanent deformation.
  - *For brittle materials*, when the **equivalent stress** is greater than **ultimate strength** of the material, than there is going to be a **sudden fracture**.
- Although the stress developed on the component less than yield strength of the material, there can be a **sudden fracture** of the component **if the loading on that component is of dynamic type**. This failure is caused by **"fatigue".**
- The name **"fatigue"** is based on the concept that a material becomes **"tired"** and fails at a stress level below nominal strength of the material.

Simple Examples of Fatigue

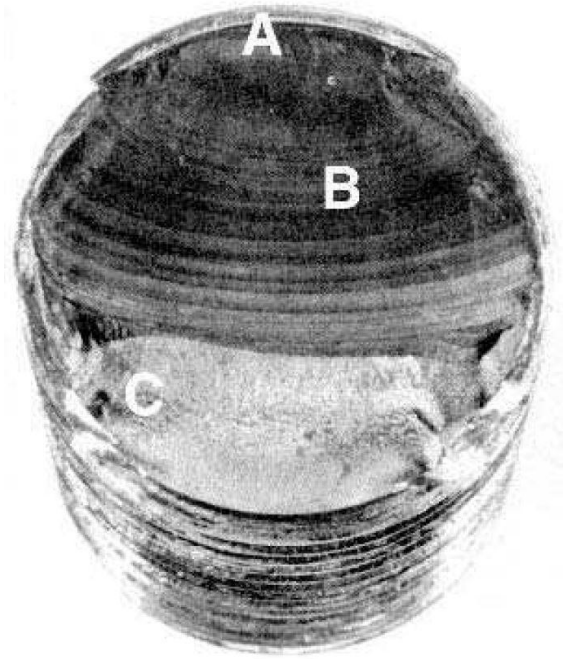
- You can easily lift 1 kg weight up. However, you may not be successful when you try to lift that weight up and down **a hundred times**. The reason for this unsuccess is that you are **“tired”**.
- If you apply **a bending moment** to a thin wire by your hand, it may not **break at once**, however if you continue to apply **a reversed bending moment repeatedly**, you will notice that the wire is going to be broken **after a while**.





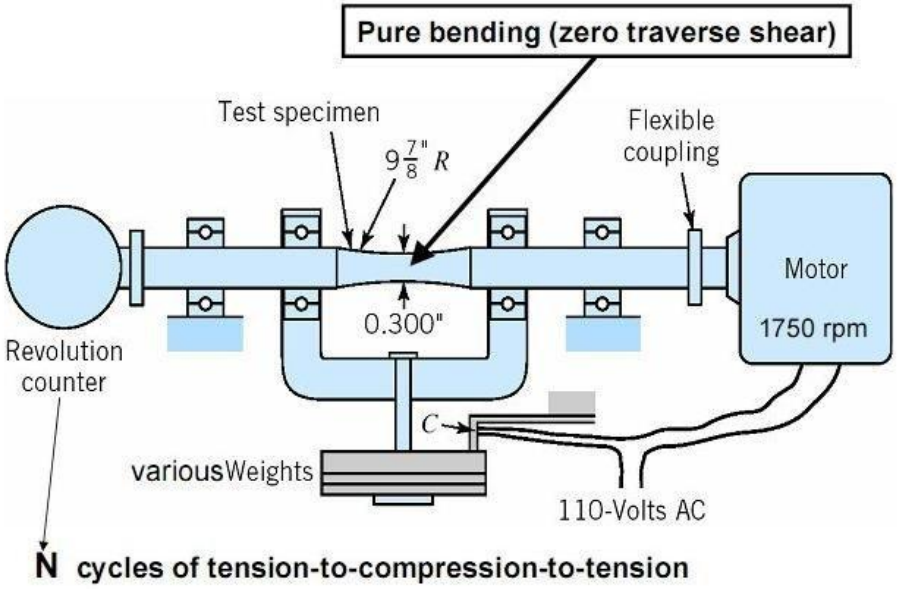
Fatigue Failure

- A fatigue failure has an appearance similar to a brittle fracture, as the fracture surfaces are flat and perpendicular to the stress axis with the absence of necking. However, it is quite different from a static brittle fracture arising from three stages of development.
- **Stage I** is the initiation of one or more **microcracks** due to cyclic plastic deformation on the surface where stress concentrations exist *(step section of shaft, keyway, hole, marks on the surface)*. It is not normally discernible to the naked eye.
- **Stage II** progresses from microcracks to **macrocracks** forming parallel plateau-like fracture surfaces separated by longitudinal ridges.
- **Stage III** occurs during the final stress cycle when the remaining material cannot support the loads, resulting in **a sudden, fast fracture**.



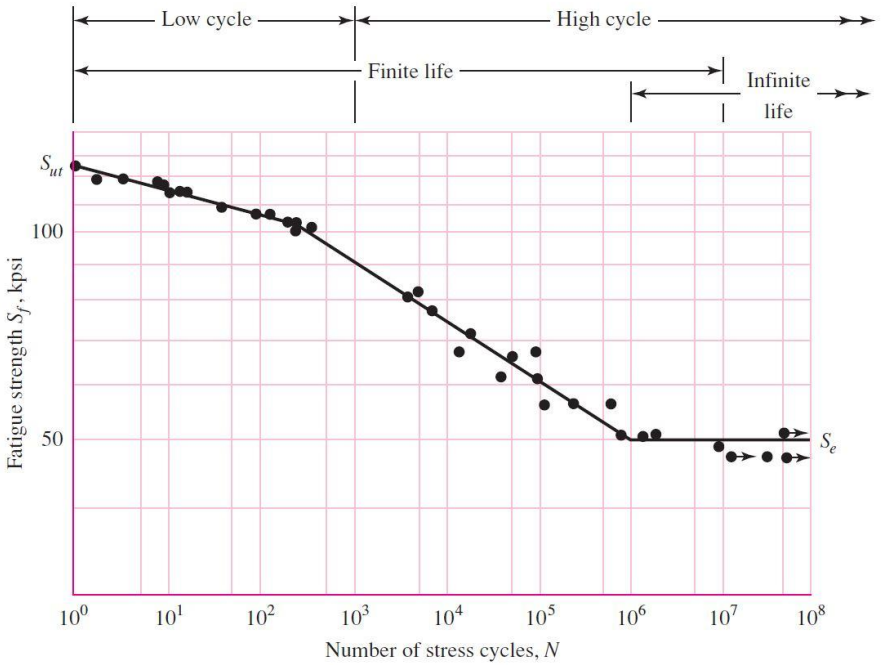
Fatigue Life

- To determine the strength of materials under the action of fatigue loads, specimens are subjected to repeated or varying forces of specified magnitudes while the **cycles or stress reversals are counted to destruction**.
- The most widely used fatigue-testing device is the high-speed rotating-beam machine. This machine subjects the specimen to **pure bending** *(no transverse shear)* by means of weights. The specimen is very carefully machined and polished.



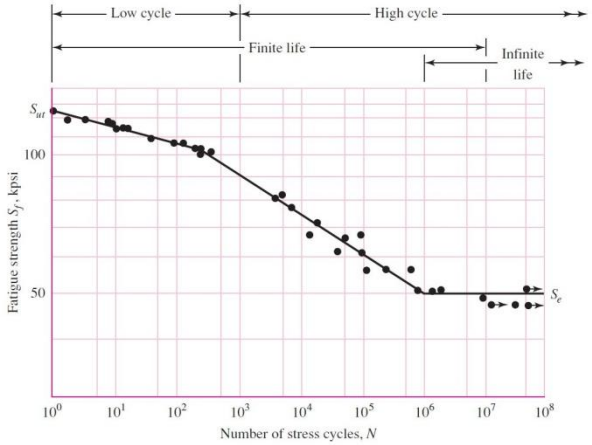
Fatigue Life

- To establish the fatigue strength of a material, **quite a number of tests are necessary** because of the statistical nature of fatigue.
- For the rotating-beam test, **a constant bending load is applied**, and the number of revolutions of the beam required for failure is recorded.
- The first test is made at a stress that is somewhat under the ultimate strength of the material. The second test is made at a stress that is less than that used in the first. This process is continued, and the results are plotted as an **S-N diagram)**.
- In the case of ferrous metals and alloys, the graph becomes **horizontal** at a certain point (knee point).
- The strength corresponding to this point is named as **"endurance limit".**

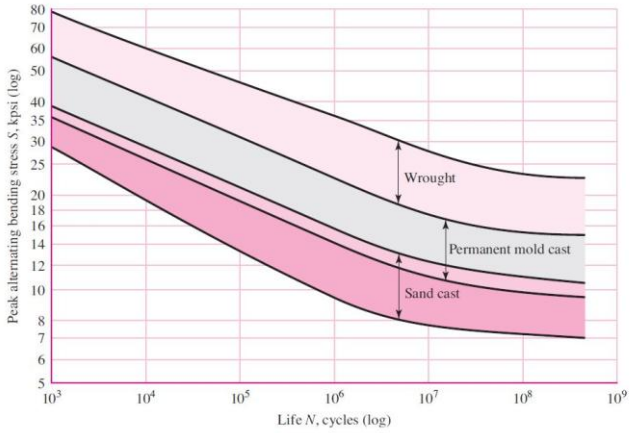


Endurance Limit

- **Endurance limit** is the fatigue strength for an element which has infinite life.
- For ferrous metals and alloys, **infinite life** is assumed as ***(10)^6*** **cycles**.
- For non ferrous materials, there is no knee point but strength value which corresponds to ***(10)^8*** or ***5(10)^8*** may be considered as the **endurance limit**.
- In the design of many machine components, the elements must be designed as to work for **infinite cycles**. Because fracture of this elements may cause serious problems.



S-N curve for steel



S-N curve for aluminum alloys

Endurance Limit

- Endurance limit for test specimen ***(S_e')*** is expressed in terms of ultimate strength ***(S_{ut})***.
- **For ductile materials:**

  $S_e' = 0.5\ Sut$ if $S_{ut} < 1400MPa$

  $S_e' = 700\ MPa$ if $S_{ut} \geq 1400MPa$

  For irons:

  $S_e' = 0.4\ Sut$ if $S_{ut} < 400MPa$

  $S_e' = 160\ MPa$ if $S_{ut} \geq 400MPa$

  For aliminums:

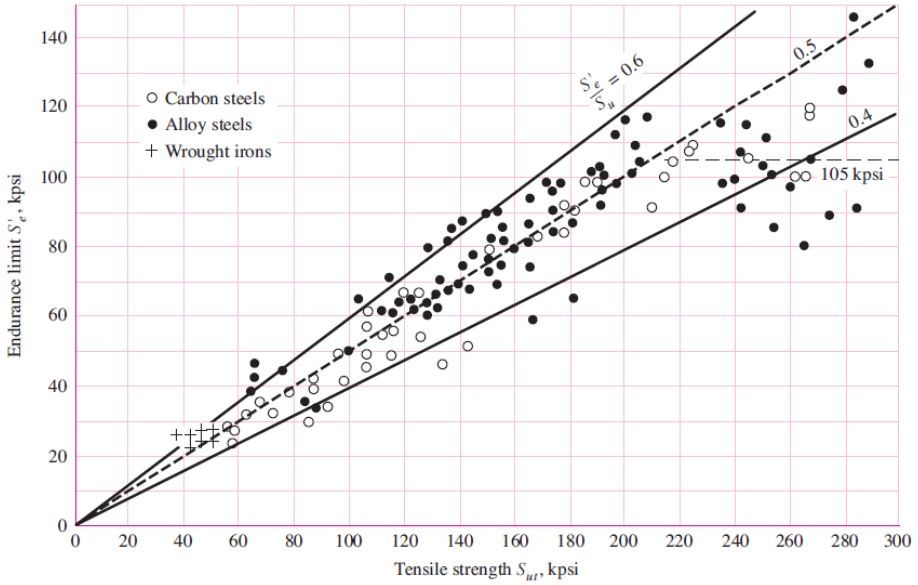
  $S_e' = 0.4\ Sut$ if $S_{ut} < 330MPa$

  $S_e' = 130\ MPa$ if $S_{ut} \geq 330MPa$

  For copper alloys:

  $S_e' \approx 0.4\ Sut$ if $S_{ut} < 280MPa$

  $S_e' \approx 100\ MPa$ if $S_{ut} \geq 280MPa$



Endurance limit of the actual machine element is different from these values. ***Some factors have detrimental effect*** *on the life of the machine element.*

$$S_e = k_a k_b k_c k_d k_e S_e'$$

Fatigue Strength

- Machine components are generally designed for infinite life. However, in some special cases, the element may need to be designed for **a finite life**.
- In this case, **fatigue strength** for expected finite life must be calculated from following formula.

$$S_f = 10^C N^b$$ where *C and b are constants*

$$b = -\frac{1}{3}\log\left[\frac{0.8S_u}{S_e}\right] \quad \text{and} \quad c = \log\left[\frac{(0.8S_u)^2}{S_e}\right]$$

- Fatigue strength ***(S_f)*** will be greater than Endurance limit **(S_e)**

Factors Influencing Fatigue Failure

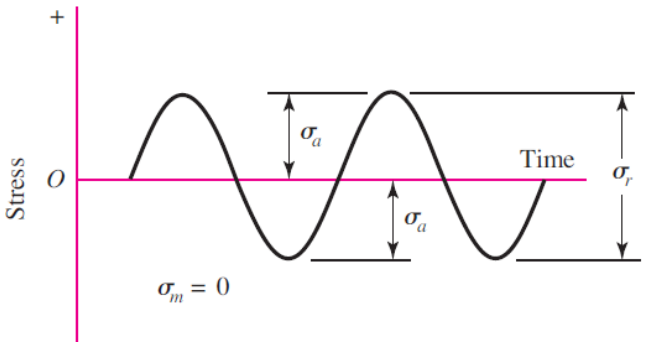


- **Loading:** Axial Tension, Bending, Torsion and Combined
- **Nature and type of loading:** Reversible, Repeated, Fluctuating and Alternating, Mean and Variable components, Frequency of loading and rest periods
- **Geometry:** size effects and stress concentration
- **Material:** composition, structure, directional properties and notch sensitivity
- **Manufacturing:** surface finish, heat treatment and residual stresses Surface residual stress has a significant effect on fatigue life. Compressive residual stresses from machining, cold working, heat treating will oppose a tensile load and thus lower the amplitude of cyclic loading which will improve fatigue life.
- **Environment:** corrosion, high temperature, radiation

Nature and Type of Loading

Reversed Type

When a rotating machine element is subjected to **a pure bending load**, any element on the surface experiences both tension and compression stresses in one full rotation *(cycle)*.

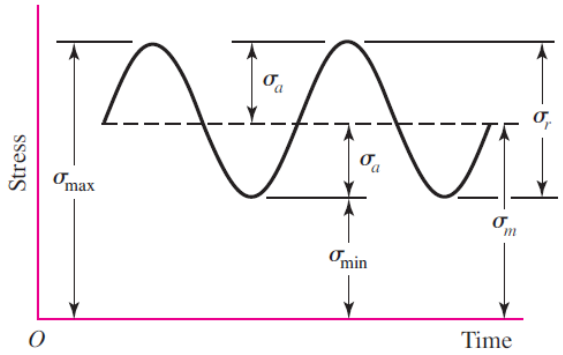


$$\sigma_a = \sigma_{\max} = -\sigma_{\min}$$

$$\sigma_m = 0$$

Fluctuating Type

In addition to **bending load**, there may be **an axial load** acting on the machine element *(forces developed on helical gear)* then the stress will be fluctuating between two limits where we are going to have alternating and mean components of stresses

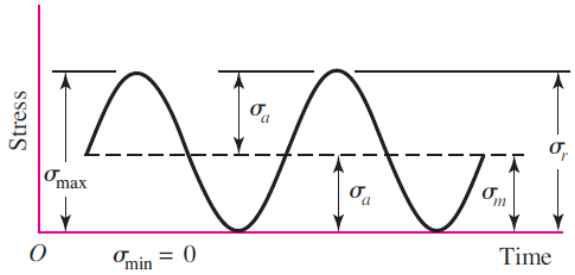


$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

Repeated Type

It is a secial case of fluctuating type stress which fluctuating between zero and a maximum value.



$$\sigma_a = \sigma_m = \frac{\sigma_{\max}}{2}$$

Surface Factor (k_a)

- The elements may be produced by **forging, hot rolling, cold drawn** and **machining** methods or they may be subjected to some surface finishing operations like **grinding** and **polishing**. Surface effect is calculated with following formula:

$$k_a = aS_{ut}^{\ b}$$ *where a and b are constants for different surface finish*

ground surface	*a=1.58*	*b=-0.085*
machined	*a=4.51*	*b=-0.265*
cold drawn	*a=4.10*	*b=-0.265*
hot rolled	*a=57.7*	*b=-0.718*
forged	*a=272*	*b=-0.995*

- As it is seen, there is a big difference between **ground** and **forged surfaces**. Thus, surface finish is very important factor which **can not be neglected**.

Size Factor (k_b)

- **Size factor** is dependent on the size of the element and also on the type of loading.

For bending and torsional loading

For rotating parts having circular cross sections

k_b=1 when $d \leq 8mm$

$k_b = 1.189 d^{-0.097}$ when $8 < d \leq 250mm$

k_b=0.6 for larger diameters

For rotating parts having non-circular cross sections, equivalent diameter $\boldsymbol{d_{eq}}$ must be calculated for determination of size factor. In the determination of equivalent diameter, areas at or above of 95% of maximum stress are equated for rotating and non-rotating parts.

For rotating circular cross section $$A_{95} = \frac{\pi}{4}\left[d^2 - \left(0.95 d^2\right)\right] = 0.0766 d^2$$

For rotating rectangular cross section $$A_{95} = \left[hb - \left(0.95 hb\right)\right] = 0.05 hb$$

For rotating rectangular cross section $$d_{eq} = \sqrt{\frac{0.05 hb}{0.0766}} = 0.808\sqrt{hb}$$

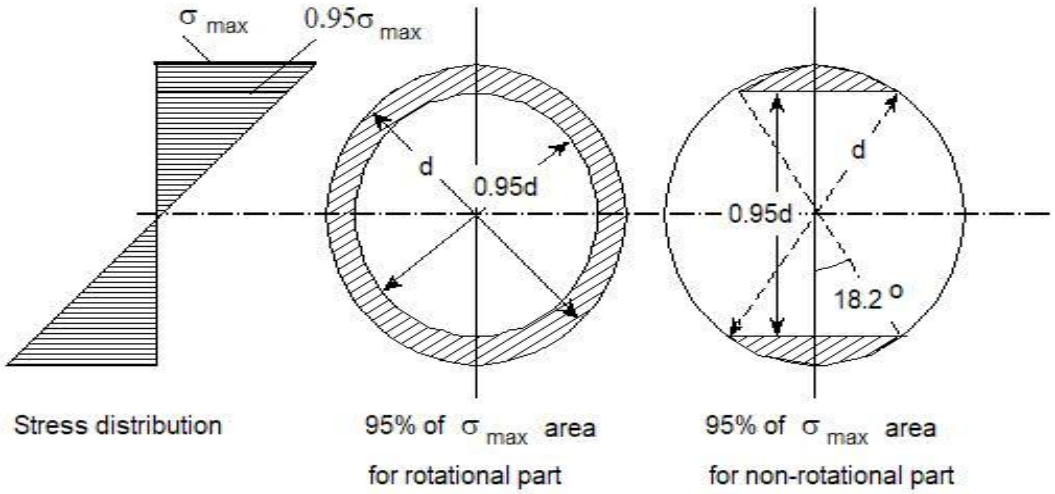
Size Factor (k_b)

- **Size factor** is dependent on the size of the element and also on the type of loading.

For bending and torsional loading

For non rotating cases, the same approach is used. For circular cross section (non rotating), 95% of maximum stress area is:

$$A_{95} = 4\left[\frac{18.2}{180}\left(\frac{\pi d^2}{4}\right)^2 - \frac{0.156d}{2}\left(\frac{0.95d}{2}\right)\right] = 0.0107d^2$$



Equivalent diameter for circular

$$d_{eq} = \sqrt{\frac{0.0107d^2}{0.0766}} = 0.370d$$

Equivalent diameter for rectangular

$$d_{eq} = 0.370\left(0.808\sqrt{hb}\right) = 0.3\sqrt{hb}$$

Size Factor (k_b)

- **Size factor** is dependent on the size of the element and also on the type of loading.

For axial loading

For pure axial loading

k_b=1 and S_e' =0.45 Sut

For combined loading

Size factor is calculated for bending and then alternating axial stress component is multiplied by axial correction factor "**α**".

α=1.11 for S_{ut} <1520 MPa

α=1.00 for S_{ut} >1520 MPa

Reliability Factor (k_c)

- **Reliability factor** is the result of statistical study on overlapping stress distribution and strength distribution. It may be defined as the probability of not yielding (failure). The following list gives you an idea about reliability factors:

Reliability (R)	***Reliability factor (k_c)***
0,50	*1,000*
0,90	*0,897*
0,95	*0,868*
0,99	*0,814*
0,999	*0,753*
0,999 9	*0,702*
0,999 99	*0,659*
0,999 999	*0,620*
0,999 999 9	*0,584*

Temperature Factor (k_d)

- When operating temperatures are below room temperature, **brittle fracture** is a strong possibility and should be investigated first. When the operating temperatures are higher than room temperature, **yielding** should be investigated first because the **yield strength drops off so rapidly with temperature**.

where S_T is tensile strength at operating temperature and S_{RT} is tensile strength at room temperature

Assumed as

$k_d = 1.0$ *if* $T < 350\ ^oC$

$k_d = 0.5$ *if* $350\ ^oC < T < 500\ ^oC$

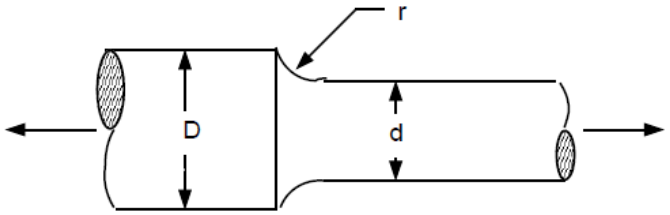
Temperature, °C	S_T/S_{RT}
20	1.000
50	1.010
100	1.020
150	1.025
200	1.020
250	1.000
300	0.975
350	0.943
400	0.900
450	0.843
500	0.768
550	0.672
600	0.549

Fatigue Stress Concentration Factor (k_e)

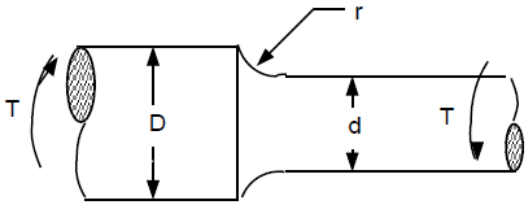
- **Stress concentration factor (k_e)** is equal to the reciprocal of the fatigue strength reduction factor which is described as:

$$k_e = \frac{1}{K_f} \quad \text{and} \quad K_f = 1 + q(K_t - 1)$$

where *where **q** is the notch sensitivity factor and **K_t** is the geometric stress concentration factor.*



	D/d =1,02	D/d =1,05	D/d =1,1	D/d=1,5
r/d	Kt	Kt	Kt	Kt
0,025	1,800	-	-	-
0,028	1,728	-	2,200	-
0,031	1,678	2,000	2,125	-
0,037	1,610	1,868	2,020	-
0,044	1,550	1,778	1,938	2,522
0,050	1,508	1,714	1,866	2,400
0,062	1,452	1,626	1,766	2,235
0,075	1,408	1,550	1,684	2,086
0,088	1,370	1,502	1,624	1,970
0,100	1,336	1,457	1,568	1,893
0,125	1,286	1,400	1,496	1,760
0,150	1,254	1,364	1,452	1,662
0,175	1,230	1,340	1,400	1,600
0,200	1,220	1,314	1,372	1,546
0,250	1,216	1,292	1,342	1,508
0,275	1,200	1,270	1,325	1,480
0,300	1,200	1,250	1,296	1,452



	D/d =1,09	D/d =1,20	D/d =1,33	D/d =2,0
r/d	Kt	Kt	Kt	Kt
0,009	-	-	-	-
0,012	1,800	2,300	-	2,600
0,030	1,566	2,040	2,144	2,288
0,025	1,472	1,894	2,020	2,122
0,033	1,384	1,761	1,878	1,966
0,042	1,322	1,644	1,755	1,828
0,050	1,283	1,576	1,677	1,750
0,062	1,244	1,500	1,600	1,644
0,075	1,206	1,434	1,516	1,572
0,087	1,184	1,378	1,458	1,510
0,100	1,166	1,342	1,412	1,466
0,125	1,144	1,275	1,344	1,400
0,150	1,122	1,220	1,294	1,344
0,200	1,110	1,160	1,220	1,266
0,250	1,100	1,130	1,178	1,222
0,300	1,100	1,120	1,160	1,200

Fatigue Stress Concentration Factor (k_e)

$$k_e = \frac{1}{K_f} \quad \text{and} \quad K_f = 1 + q(K_t - 1)$$

- If bending stress, shear stress and axial stress have **alternating components**, K_f values for each one of these differ from each other and as a result, **a unique value of k_e** does not exist.
- If this is the case, **use $k_e=1$** and **increase** various alternating stress components by their respective **strength reduction factors K_f**.
- That is; $K_{fa}\sigma_a \quad K_{fb}\sigma_b \quad K_{ft}\tau_t$
- Increasing the value of stress or decreasing the strength has the same effect on the fatigue life.

Design for Fatigue Strength

- If the loading on the element creates **only alternating stress components**, Von-Mises stress is calculated. For biaxial stress state, **Von-Mises stress is:**

$$\sigma'_a = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2}$$

- It is compared with fatigue strength to determine whether the element will fail or not.

For infinite life requirement

$$n = \frac{S_e}{\sigma'_a}$$

For finite life requirement

$$n = \frac{S_f}{\sigma'_a}$$

- If we have **steady (mean) components** of the stresses in addition alternating components, one of the **Fatigue Theories** will be employed in the design or in analyzing the machine elements for fatigue.

Design for Fatigue Strength

- **Modified Goodman** and **Soderberg** approaches are linear theories and commonly used in the design for fatigue strength.
- Soderberg approach which is based on the material yield point, gives **more conservative** results compared to Modified Goodman approach which is based on the material ultimate strength.

Modified Goodman

For infinite life

$$n = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_u}}$$

For finite life

$$n = \frac{1}{\frac{\sigma_a}{S_f} + \frac{\sigma_m}{S_u}}$$

Soderberg

For infinite life

$$n = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y}}$$

For finite life

$$n = \frac{1}{\frac{\sigma_a}{S_f} + \frac{\sigma_m}{S_y}}$$

