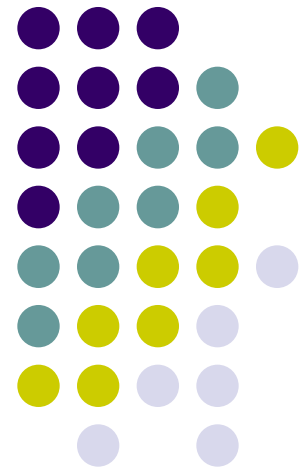


ME 307 – Machine Elements I

Chapter 2

Stress Analysis (Part I)



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- What is **stress**?
- What is the difference between **stress** and **pressure**?

$$\sigma = \text{stress}$$

$$P = \text{pressure}$$

$$\sigma = \frac{F}{A}$$

$$P = \frac{F}{A}$$

$$\sigma = \frac{N}{mm^2} = MPa$$

$$P = \frac{N}{mm^2} = MPa$$

- Can we say **$\sigma = P$** ?

Absolutely No!



- When a load / loads are applied to an element, the element **tries to resist** for any deformation. **This internal resistance to the external load is stress.**
- Being an internal resistance, **stress can not be measured**, only can be calculated by
 - **Analytically** (by using stress formulations)
 - **FEM analysis** (ANSYS, DEFORM, Abaqus)
 - **Strain gauges** (by measuring strain)
- On the other hand, **pressure is an external effect**, and **can be directly measured by pressure gauges.**
- Therefore, the main difference between stress and pressure is, **stress is an internal resistance** and can not be directly measured, but the **pressure is an external effect** and can be measured.



Stress

Normal Stress

- Normal stress is the **stress acting normal to the stress element**.
- Changes both;
 - **Volume**
 - **Shapes** of the stress element

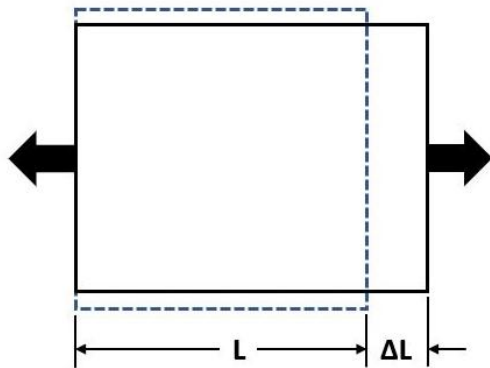
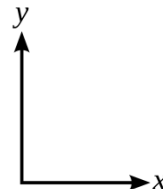


Fig 1a. Normal strain, ϵ
 $\epsilon = \Delta L / L$ (mm/mm)

Poisson's ratio

$$\nu = \frac{-\epsilon_y}{\epsilon_x}$$



Shear Stress

- Shear stress is the **stress acting parallel to the stress element**.
- Changes only the **shape** of the stress elements
- **Volume is constant**.

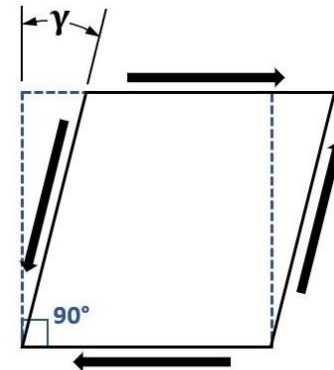
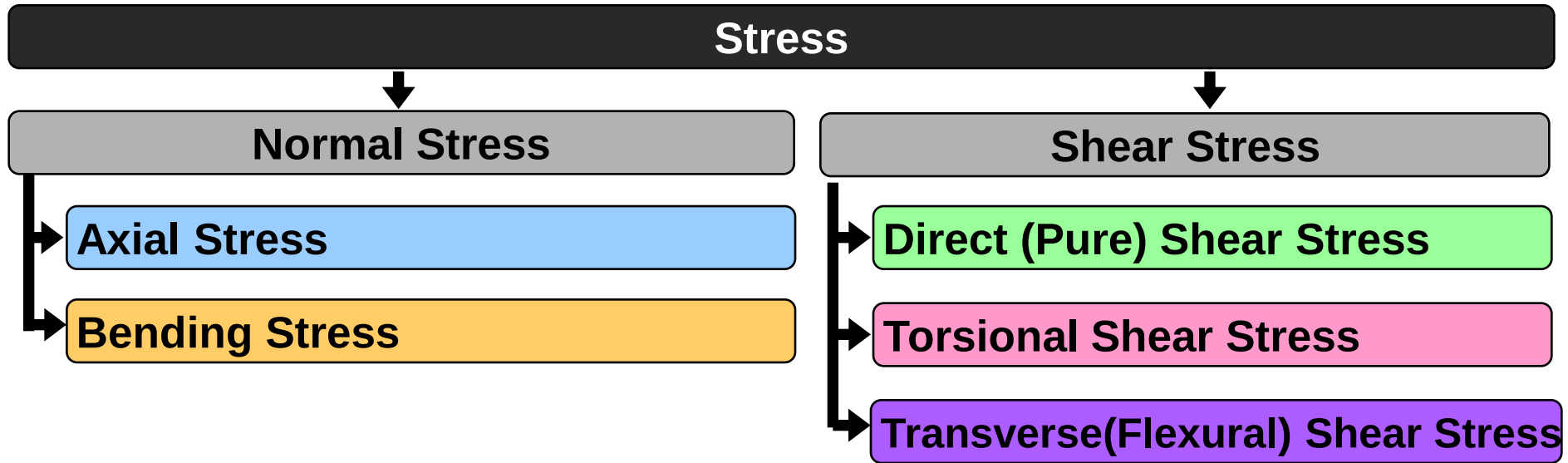


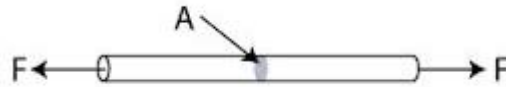
Fig 1b. Shear strain, γ
 γ = Change in 90° angle (radians)





Axial Stress

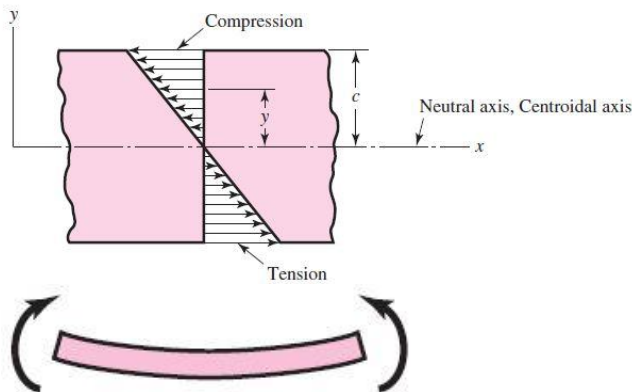
- Normal stress due to **axial load**.
- Considered to be **uniformly distributed** on the **cross section** of an element.



$$\sigma = \frac{F}{A}$$

Bending Stress

- Normal stress due to **bending moment**.
- Maximum & minimum at the surface and zero at the neutral axis.
- Neutral axis is the **axis coincides with the axis of symmetry** of the parts.



$$\sigma = \frac{My}{I}$$

$$\sigma_{\max} = \frac{Mc}{I}$$

M = bending moment

y = distance of the element from neutral axis.

I = area moment of inertia

(ability of beam to resist against bending)

For circular solid beam

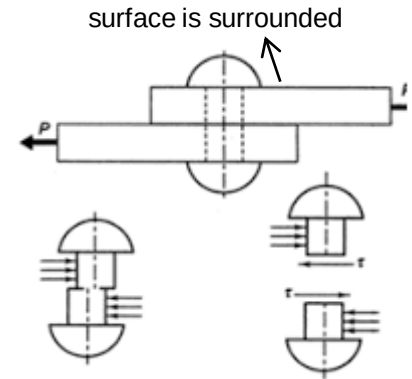
$$I = \frac{\pi d^4}{64} \quad \sigma_b = \frac{32M}{\pi d^3}$$



Direct (Pure) Shear Stress

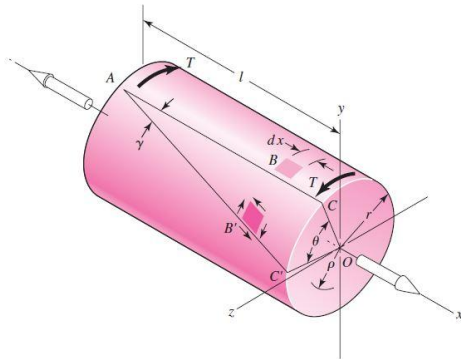
- Occurs when there is a **shearing action with no bending**.
- Bolts, rivets, pins etc.

$$\tau = \frac{V}{A}$$

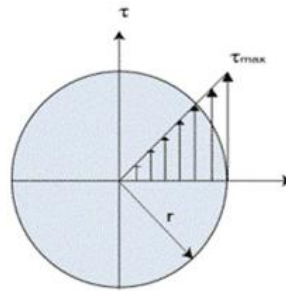


Torsional Shear Stress

- **The internal resistance to twisting** is the torsional shear stress.
- Some power transmission elements (**gears, pulleys, chains etc.**) creates torque on the shaft and the **shaft will twist** as a result.
- **Zero at the neutral axis** and **maximum at the surface of the element**



$$\tau = \frac{T \rho}{J}$$



$$\tau_{\max} = \frac{T r}{J}$$

T = torque

ρ = distance of the element from neutral axis.

J = polar moment of inertia

(ability of beam to resist torsion)

For circular solid beam

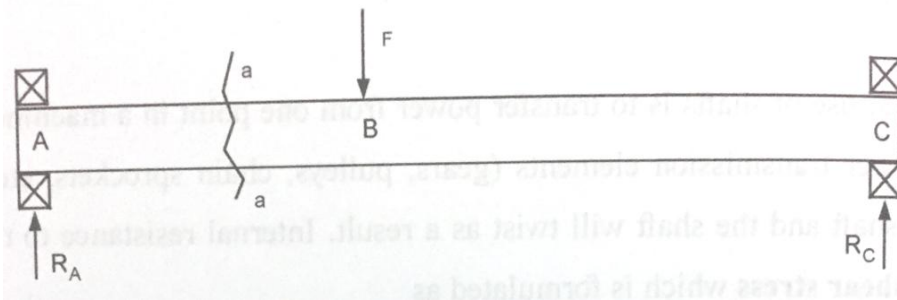
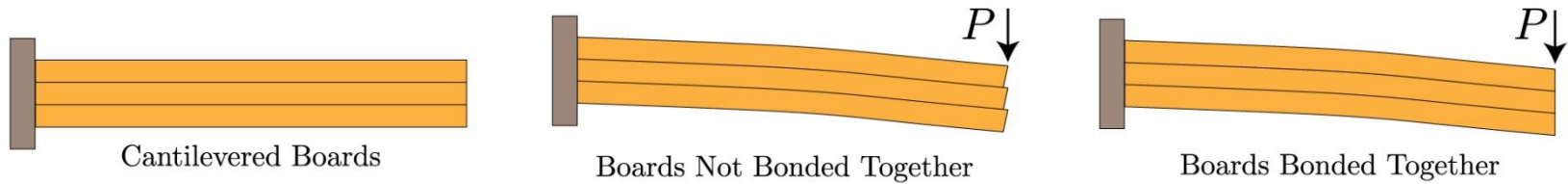
$$J = \frac{\pi d^4}{32}$$

$$\tau = \frac{16T}{\pi d^3}$$



Transverse (Flexural) Shear Stress

- In addition to normal stresses induced by bending; **transverse shear stresses** are induced between the elements **providing that bending moment varies along the length of the beam.**



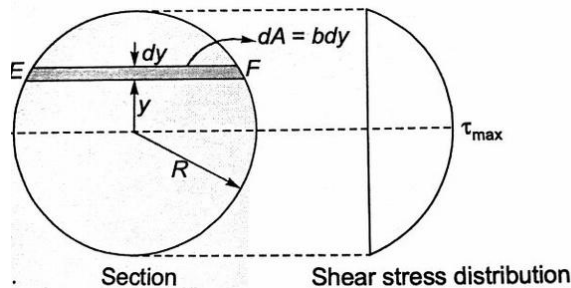
$$\tau = \frac{QV}{Ib_y}$$

V = shear force

Q = first moment of the area.

I = area moment of inertia

b_y = width of the section at the particular distance from the neutral axis.



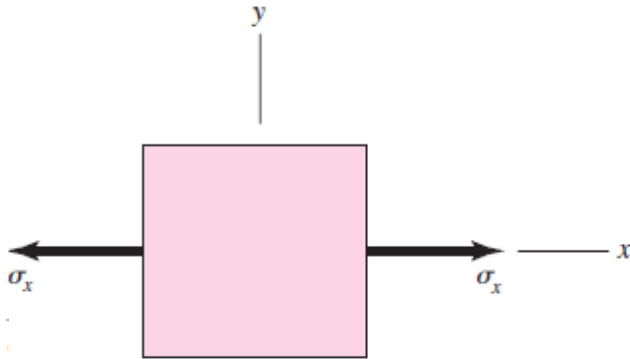
$$\tau_{\max} = 4V / 3A \quad \text{for solid circular beam}$$

$$\tau_{\max} = 2V / A \quad \text{for hollow circular section}$$

$$\tau_{\max} = 3V / 2A \quad \text{for rectangular section}$$

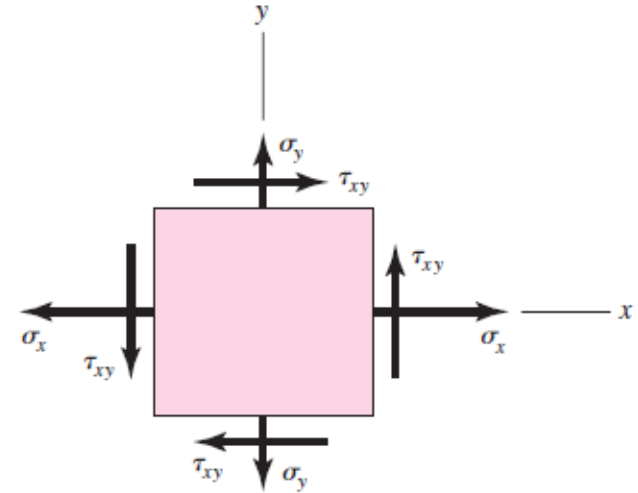


- Loading conditions may result different stress states on the element within the body of a mechanical element.



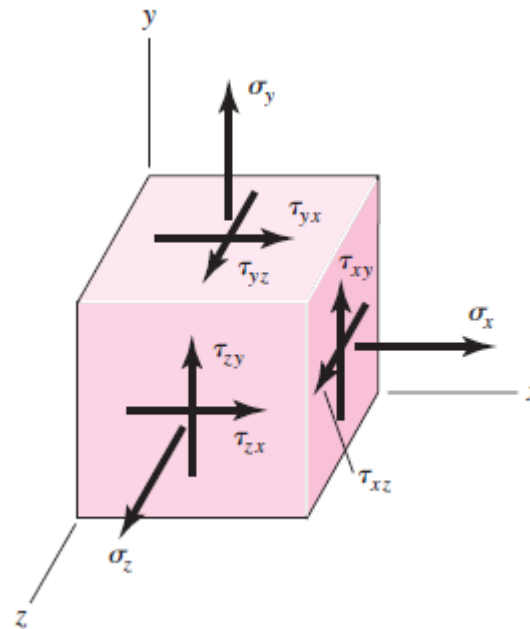
Uniaxial stress state

(a bar loaded in **simple tension or compression**)



Biaxial stress state

(a shaft loaded by **bending and torsion**)

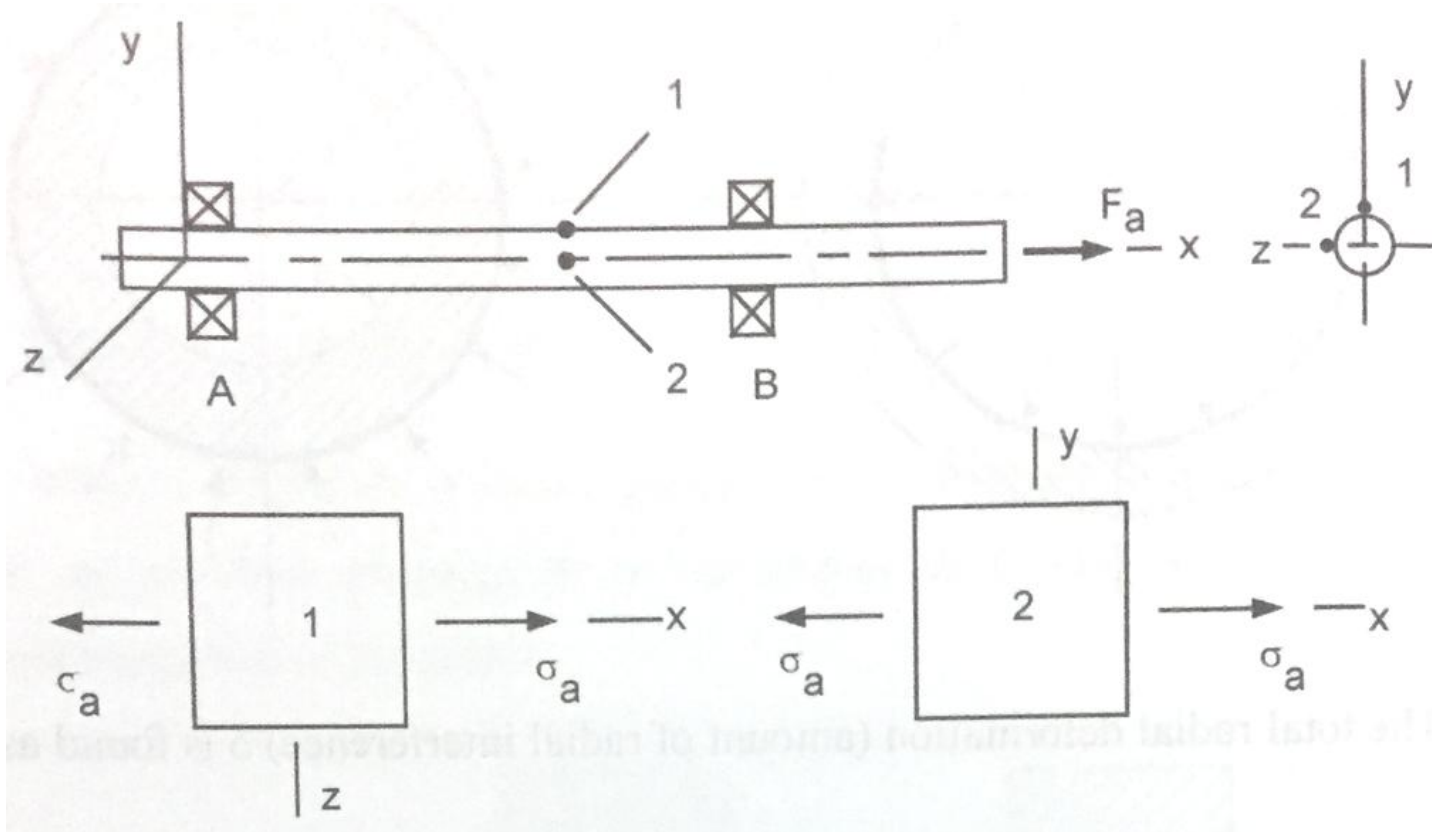


Triaxial stress state

(**pressure cylinder with closed ends or Hertz contact stresses on gears**)



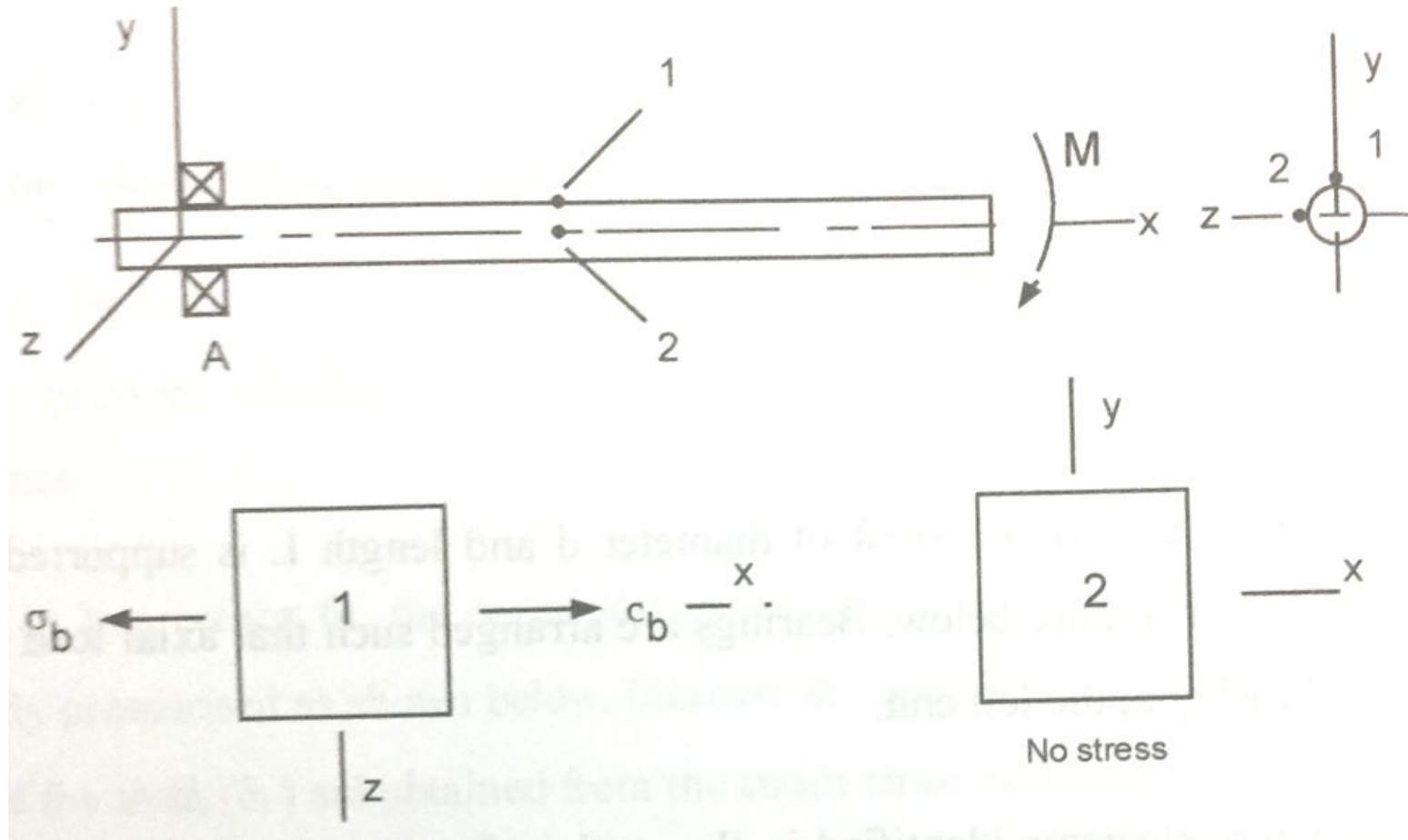
- **Axial load is acting overhang** Axial load is carried by left hand support (bearing at A)



If the right bearing (bearing at B) were to carry the axial (thrust) load, then there will be **no stress on the elements between the bearings.*

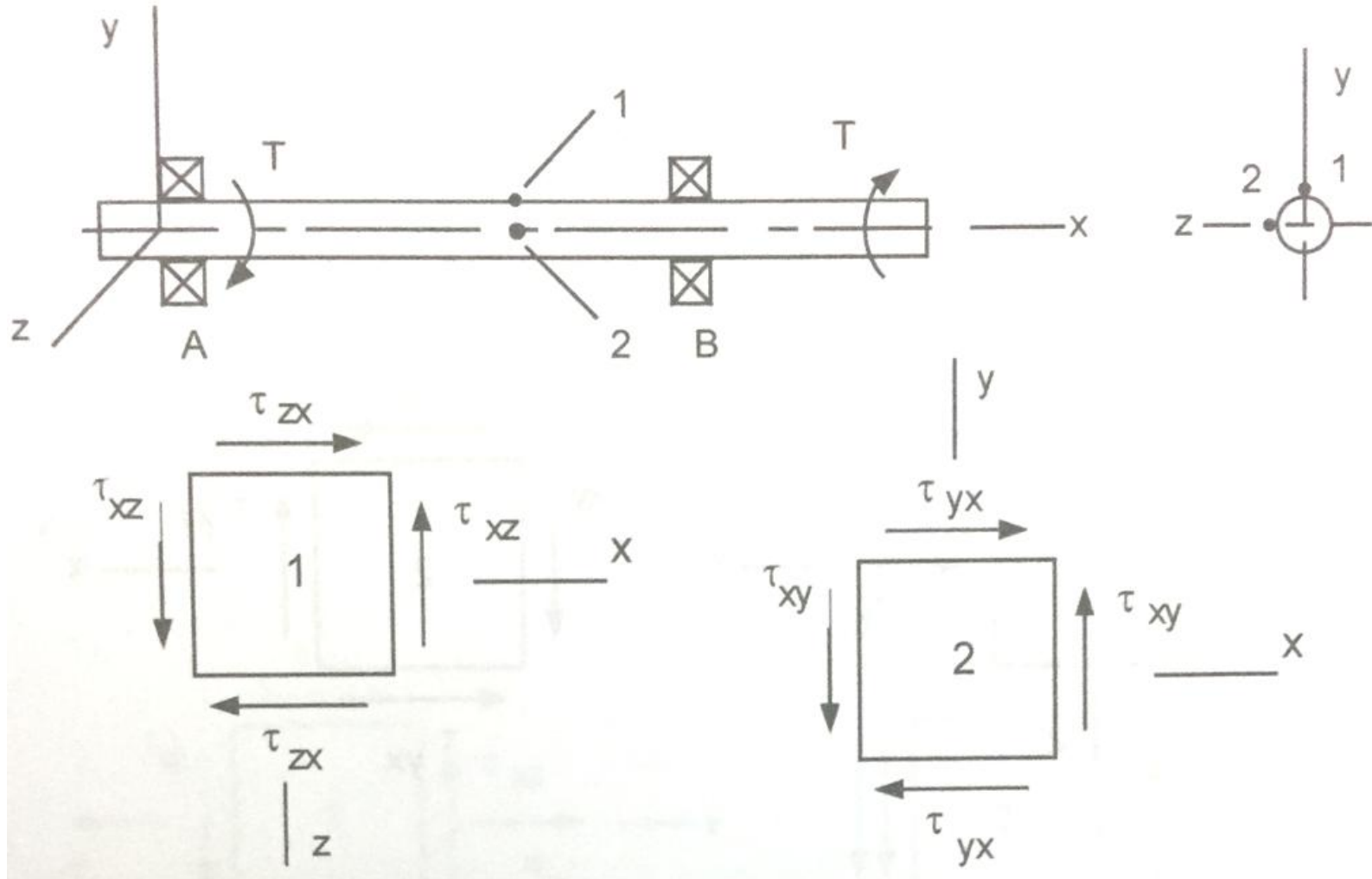


► Bending moment is acting only



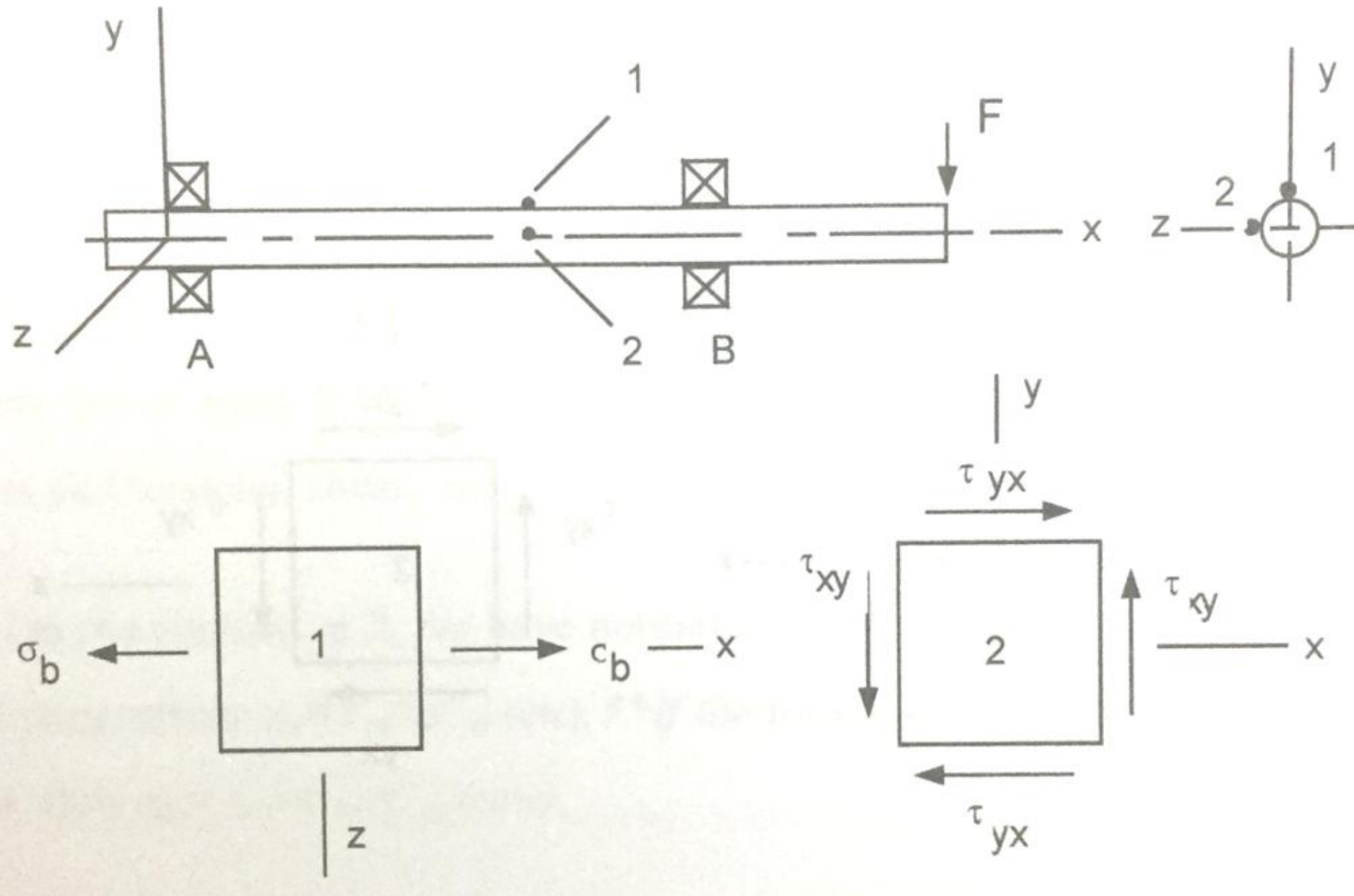


► Torsional load is acting only



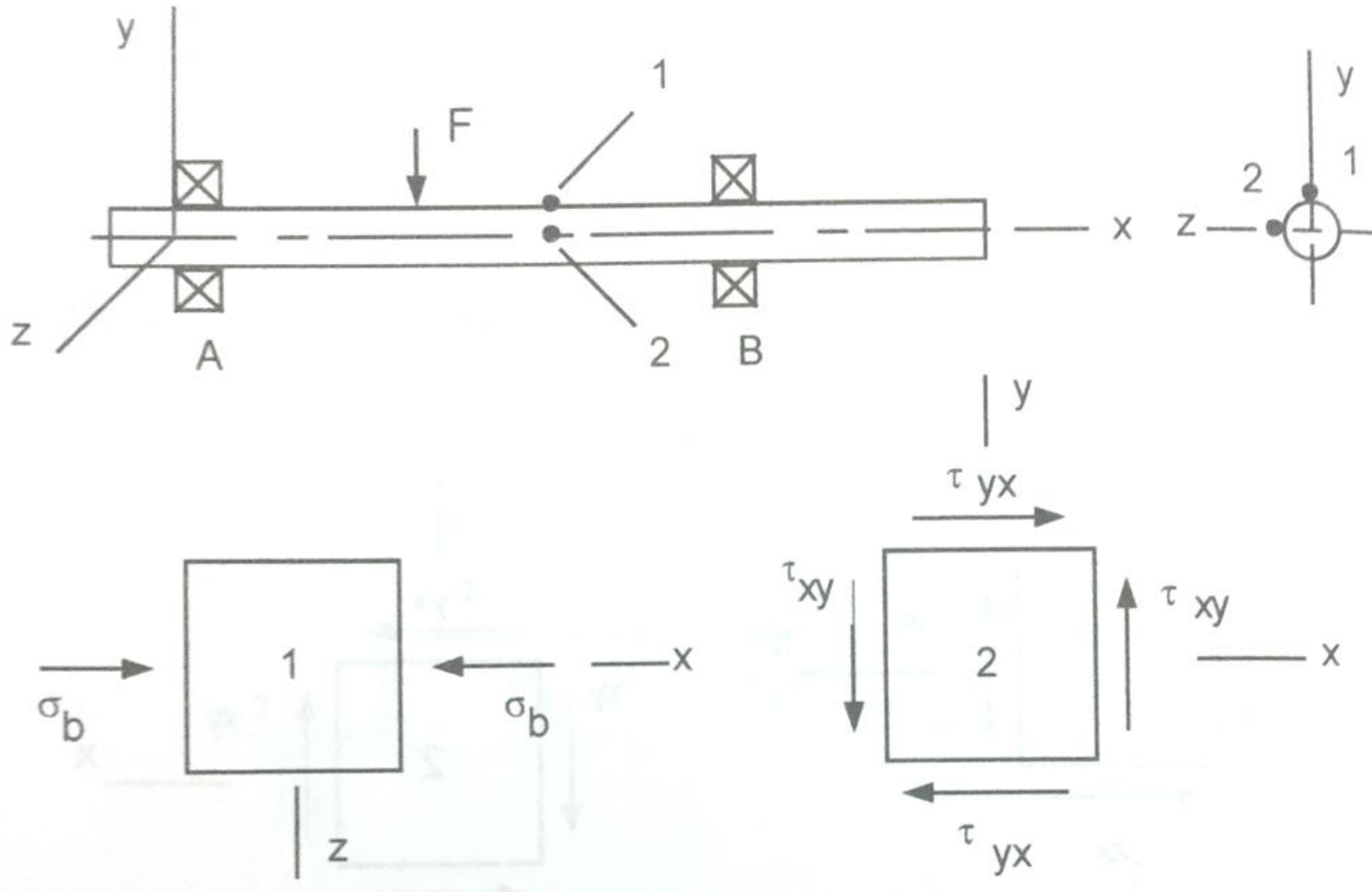


- Bending load is acting only (*F is acting on overhang*)



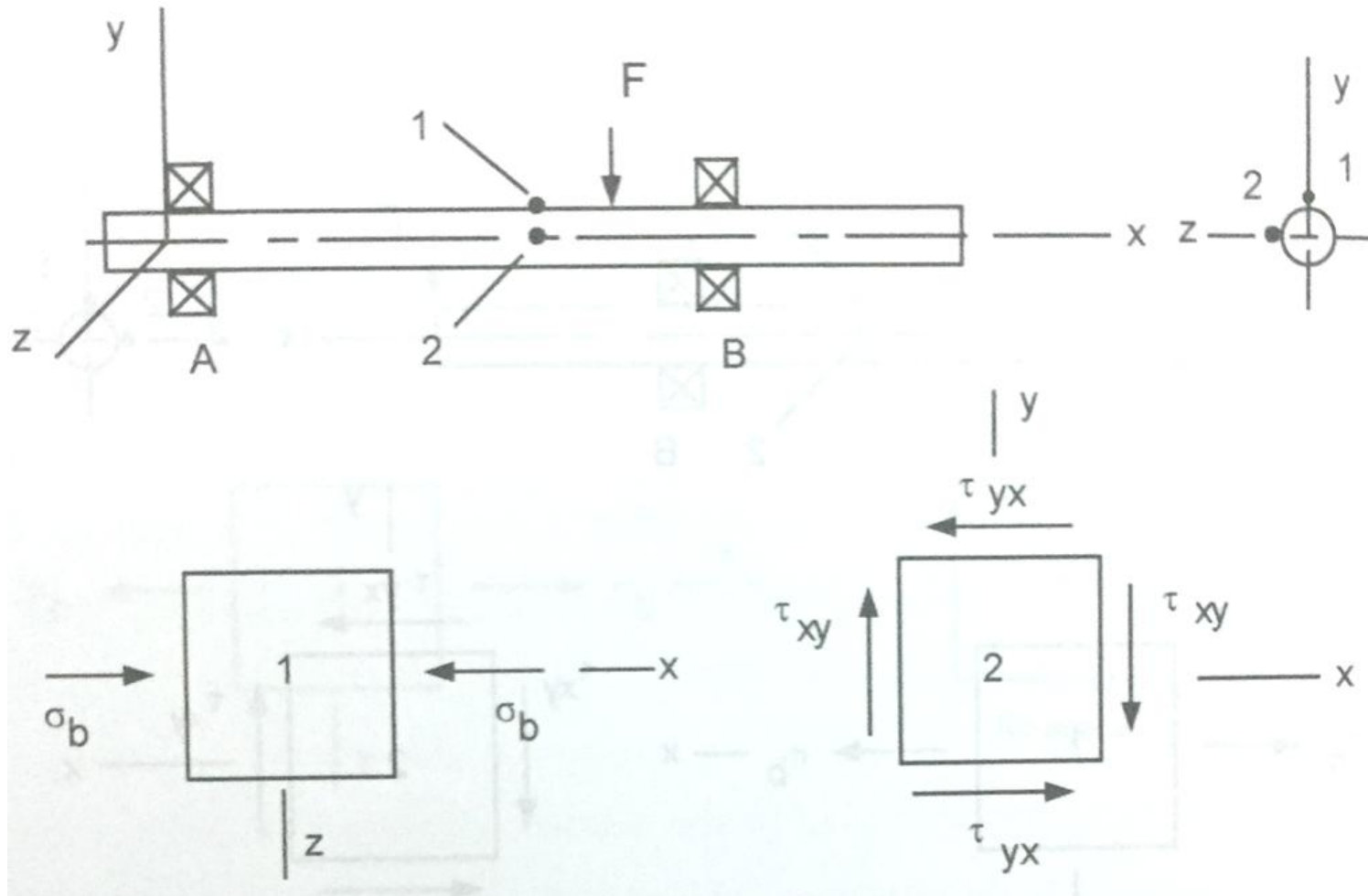


- Bending load is acting only (*F is acting between bearings and to the left of stress elements*)



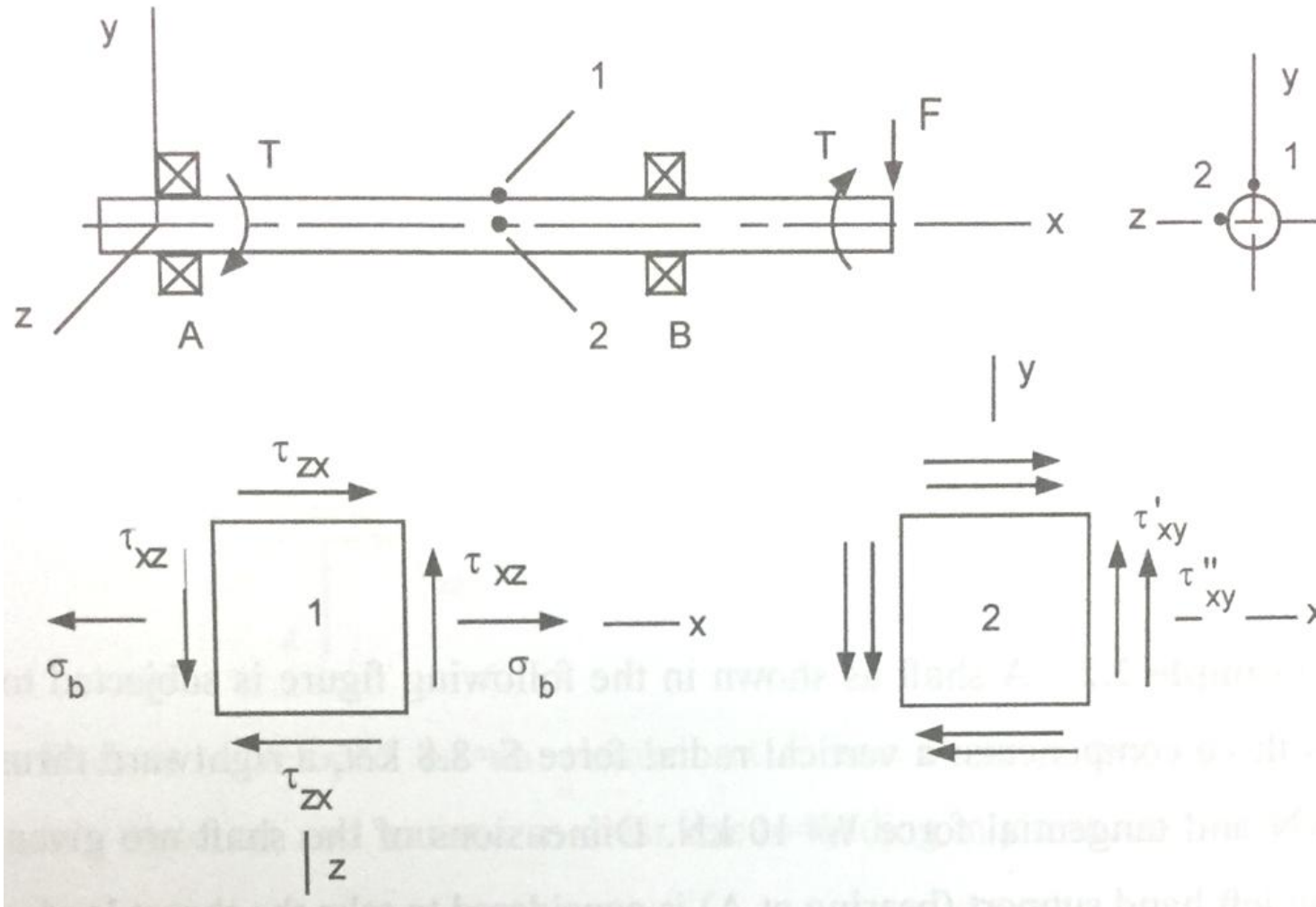


- Bending load is acting only (*F is acting between bearings and to the right of stress elements*)





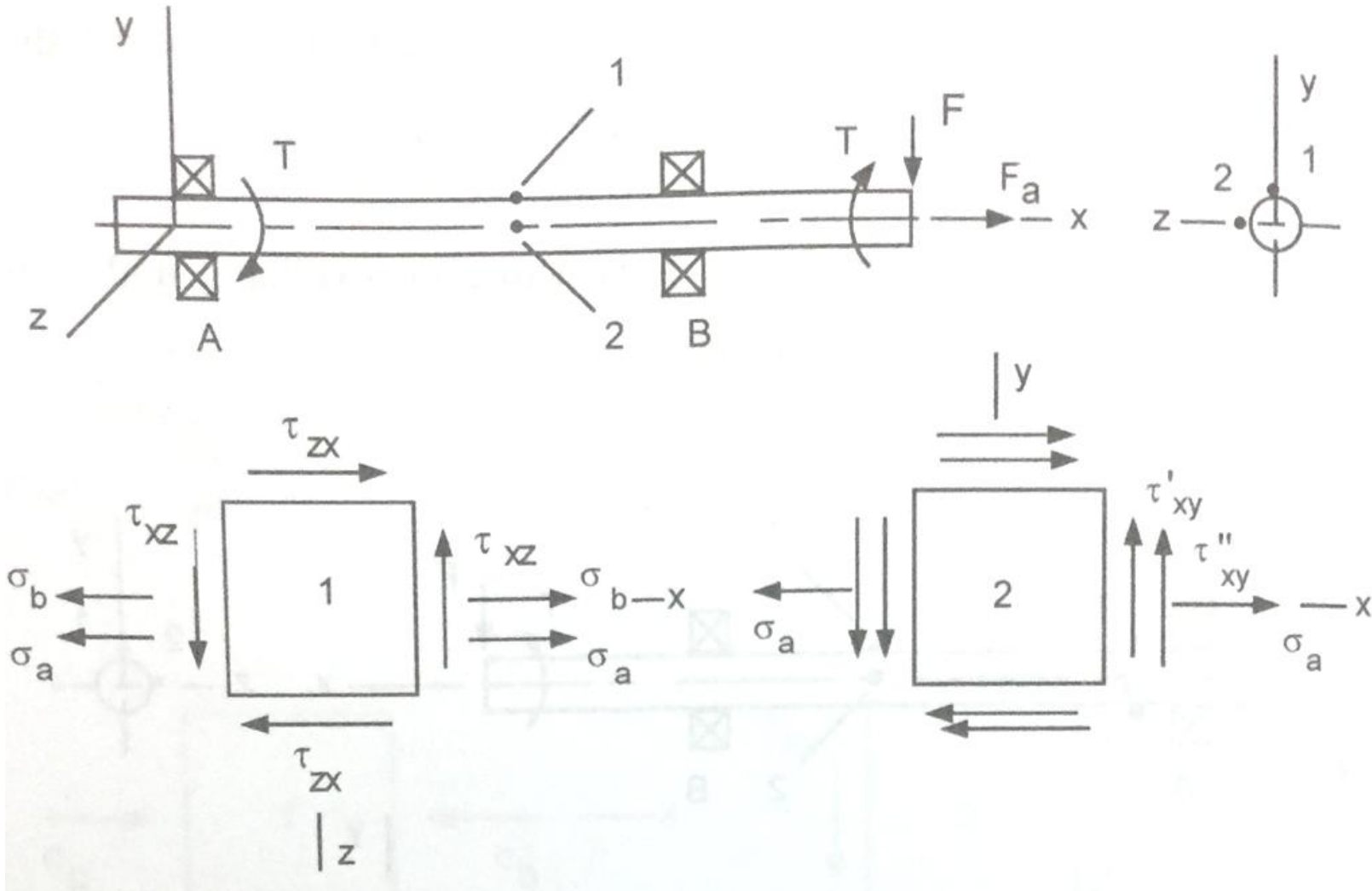
► Bending plus torsional loads





► Bending plus torsional plus axial loads

Axial load is carried by left hand support (bearing at A)





- Whatever the stress state is, **designers' job is to determine whether the machine element will fail or not under given loading condition.**
- Materials exhibit only **a unique value of the strength** (*yield strength “ S_y ”, ultimate strength “ S_u ”*).
- Therefore, there must be some means **to combine stress components** in such a way that **the resulting stresses on a machine component will be compared with the strength of the material to determine safeness of the design.**
- **Mohr's circle** is the special mean for the determination of these parameters.

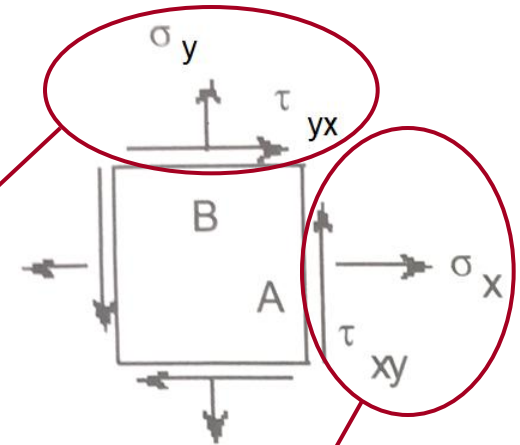


- By using **Mohr's circle**, it is possible to determine **maximum normal stresses** and **the orientation of the stress element** such that **there exist only these maximum normal stresses acting on the faces**.
- That special orientation is named as **principal direction** and the stresses are called as **principal stresses**.
- There is another orientation of the stress element such that **shear stresses reach to extreme values**.
- **Principal stresses** are used in the **determination of the safeness of the elements** according to different failure criteria (*will be discussed in next chapters*)

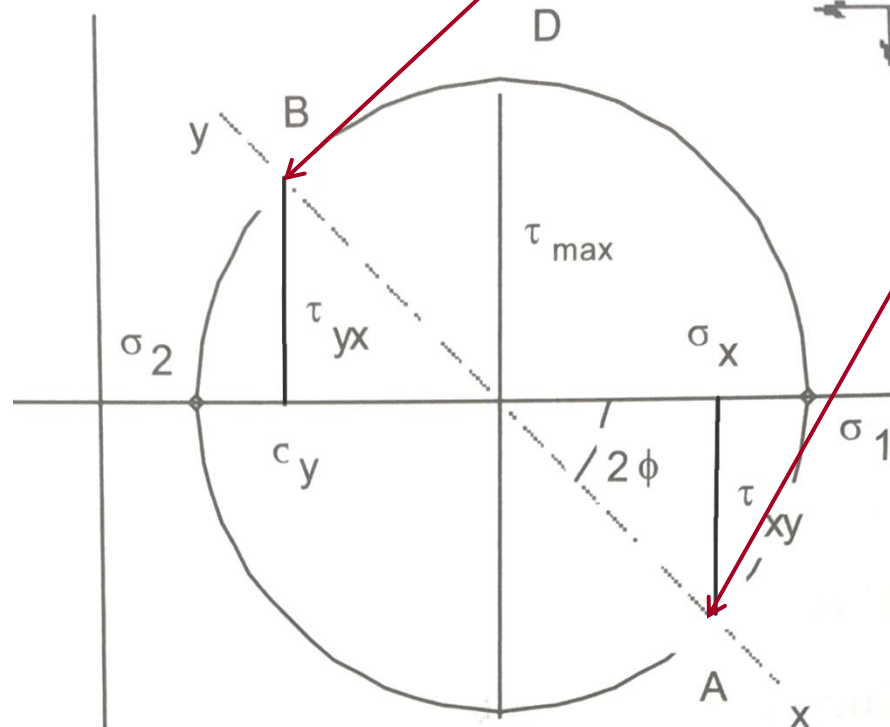


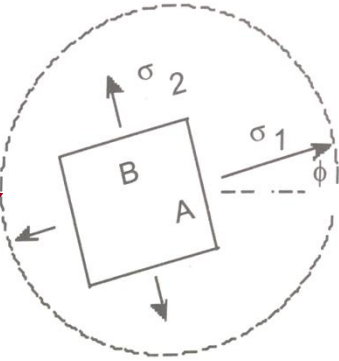
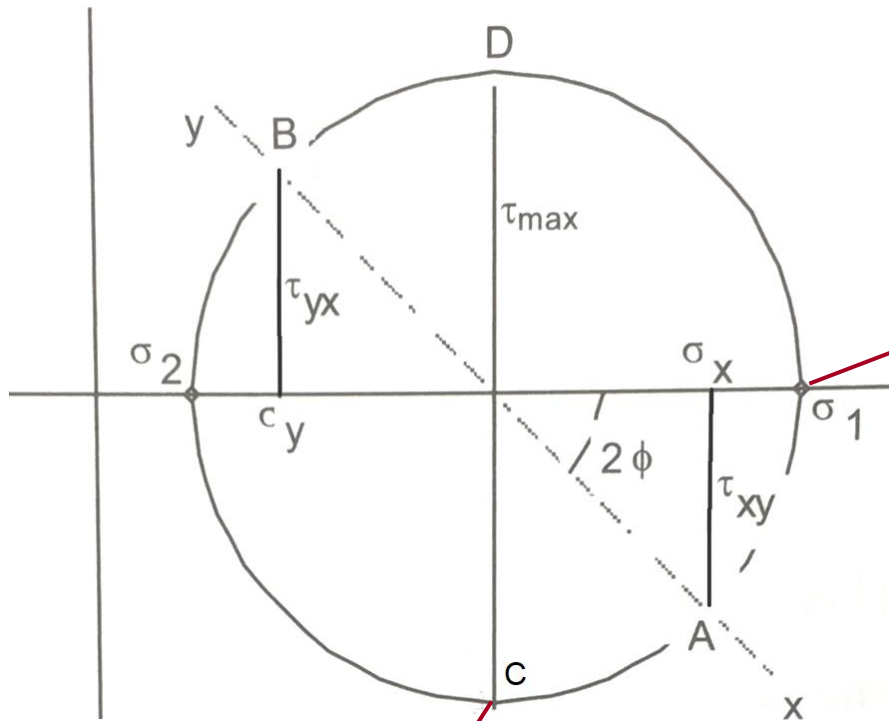
- A two dimensional Mohr's circle can be constructed if we know the normal stresses σ_x , σ_y and τ_{xy} .
- The sign conventions should be used.
 - Tensile stresses (positive) are to the right.
 - Compressive stresses (negative) are to the left.
 - Clockwise shear stresses are plotted upward.
 - Counterclockwise shear stresses are plotted downward.

Stress element



Stresses acting on faces A and B are represented as point A and B in the Mohr's circle, respectively.





Principal stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Principal direction

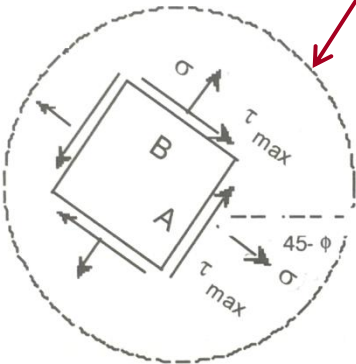
$$\tan 2\phi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Maximum shear stresses

$$\tau_{1,2} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Principal direction for max. shear stress

$$\tan 2\phi' = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

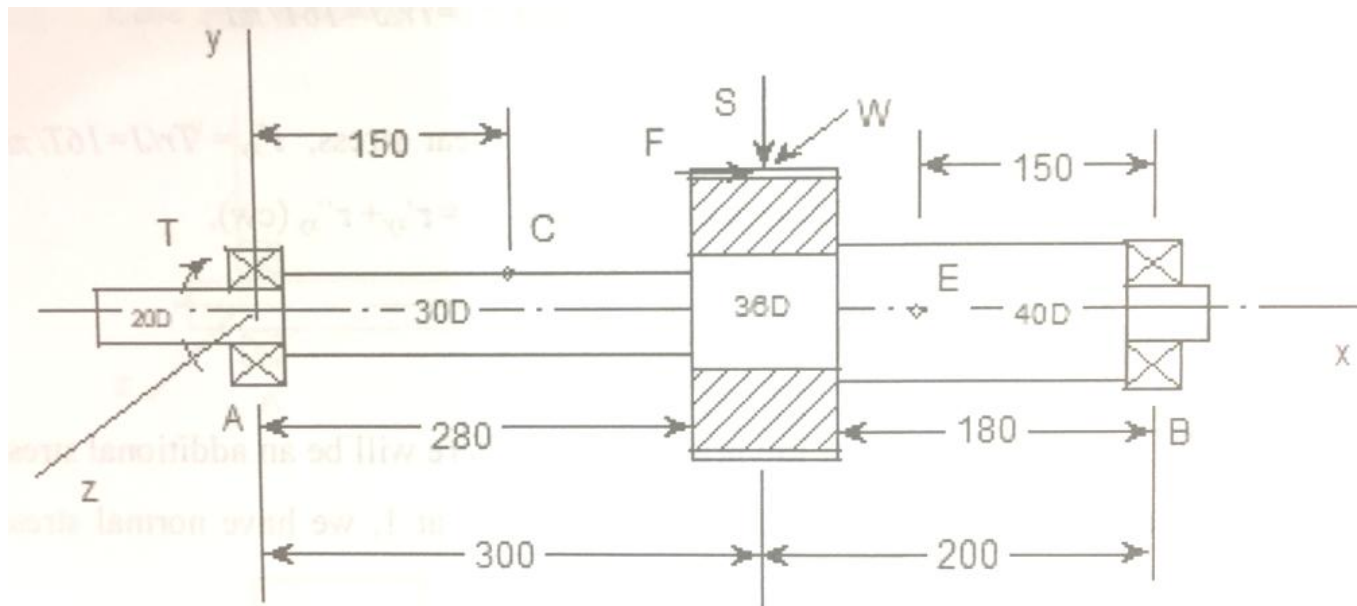


Example 2.2

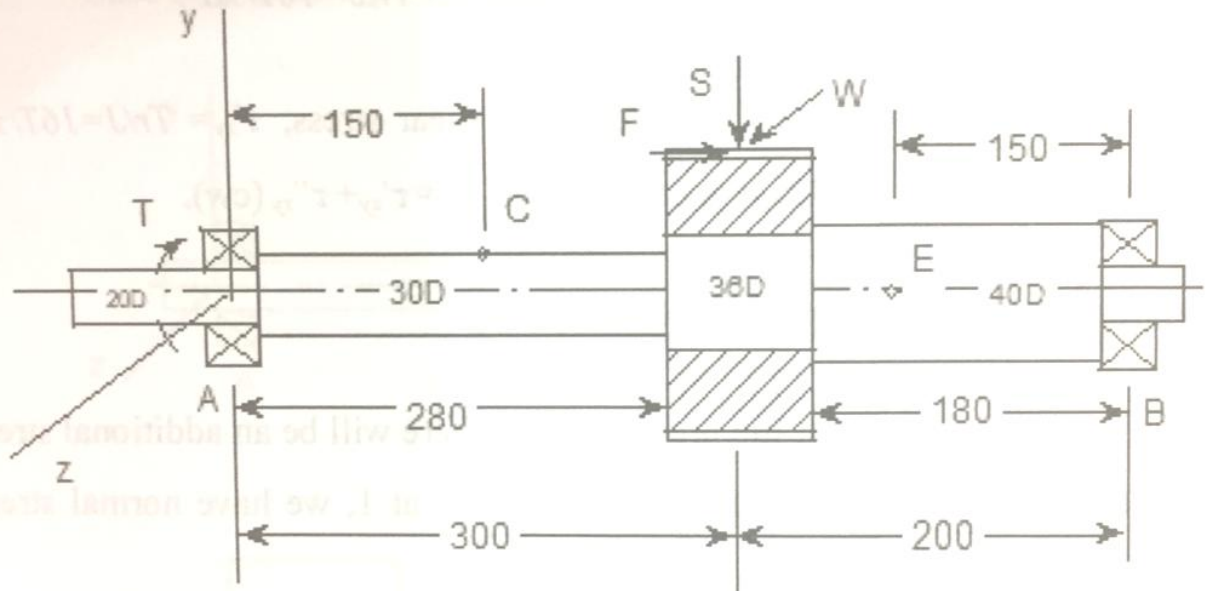


A shaft as shown in the Figure is subjected to a load which has three components; a vertical radial force $S=8.8$ kN, a rightward thrust force of $F=15$ kN and tangential force $W=10$ kN. Dimensions of the shaft are given in the figure. The left hand support (bearing at A) is considered to take thrust load. Two stress elements located on the surface and identified in the sections shown are: C on top; E is on the front side.

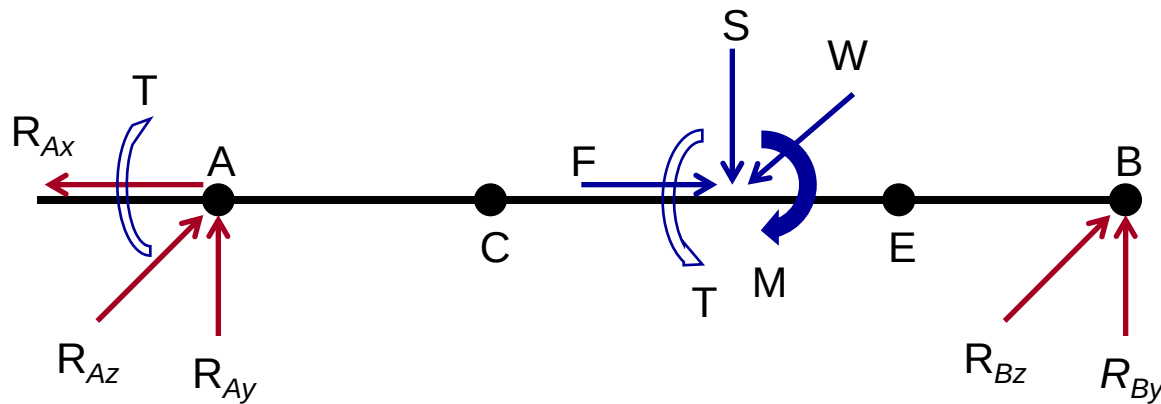
- Draw each of the stress elements properly oriented with respect to xyz, show the stresses which act upon them.
- Calculate the principle stresses at point E. Show the proper orientation (principal direction) of the element.



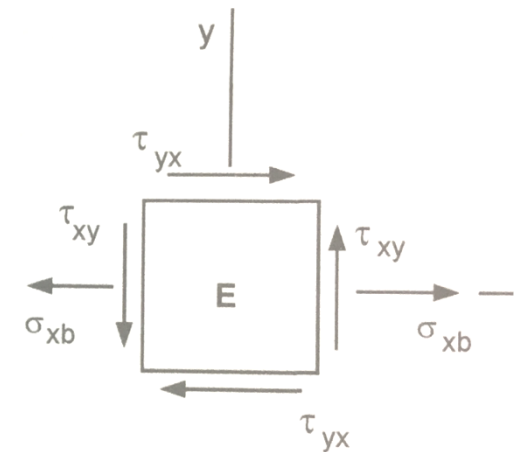
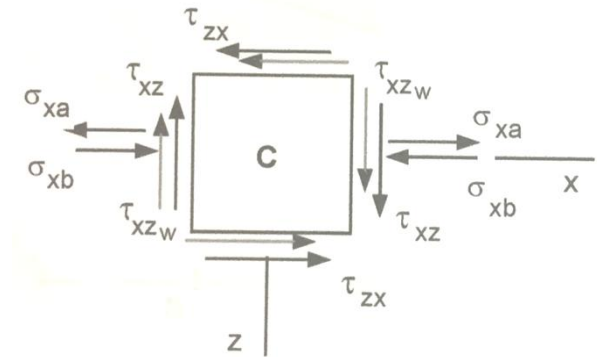
Solution of Example 2.2



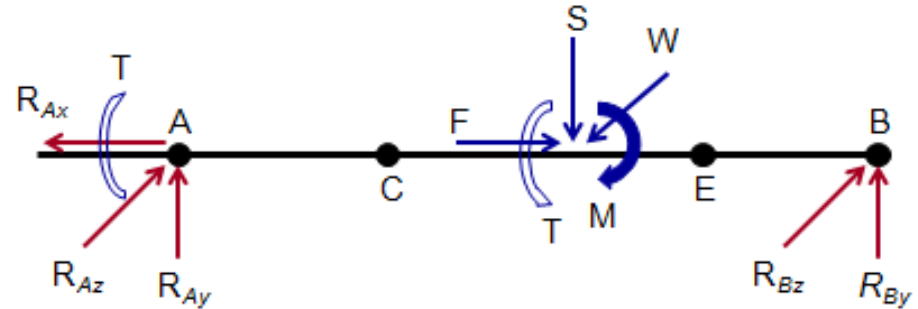
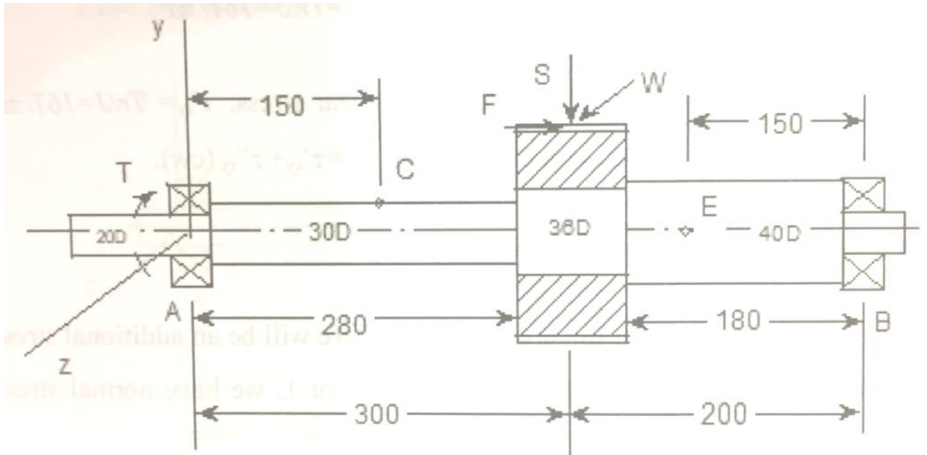
Free Body Diagram



Stress Elements



Solution of Example 2.2



Reaction in x-direction

$$R_{Ax} = F = 15kN$$

Reaction in y-direction

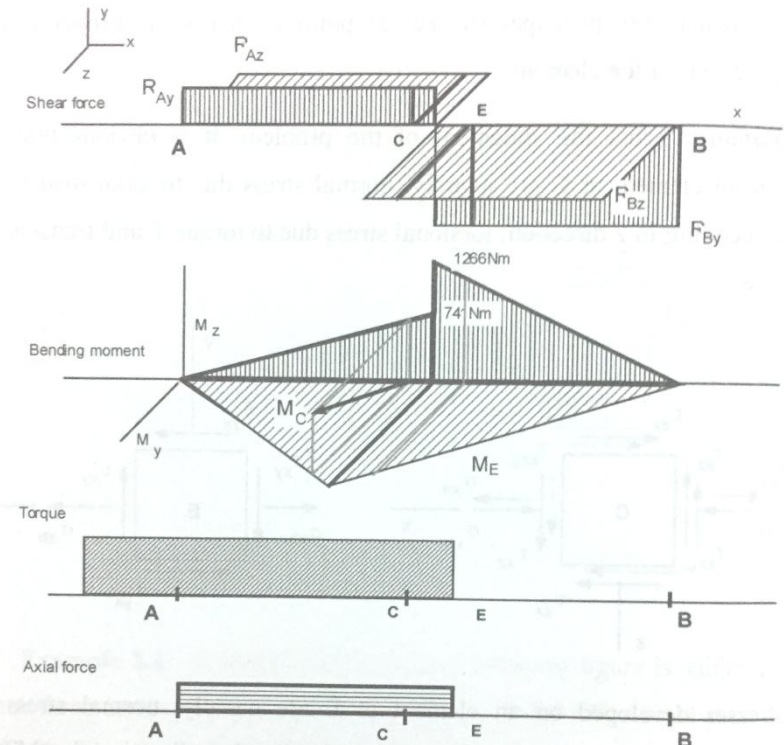
$$R_{Ay} = \frac{S(200) - F(35)}{500} = \frac{8.8(200) - 15(35)}{500} = 2.47kN$$

$$R_{By} = S - R_{Ay} = 8.8 - 2.47 = 6.33kN$$

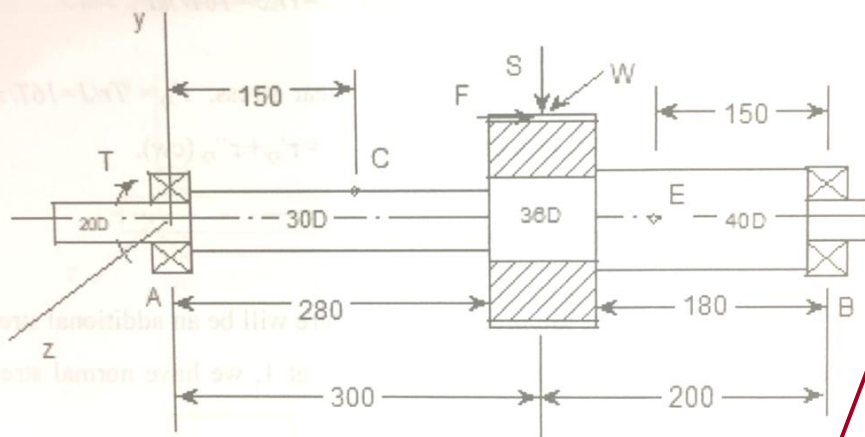
Reaction in z-direction

$$R_{Az} = \frac{W(200)}{500} = \frac{10(200)}{500} = 4kN$$

$$R_{Bz} = W - R_{Az} = 10 - 4 = 6kN$$



Solution of Example 2.2

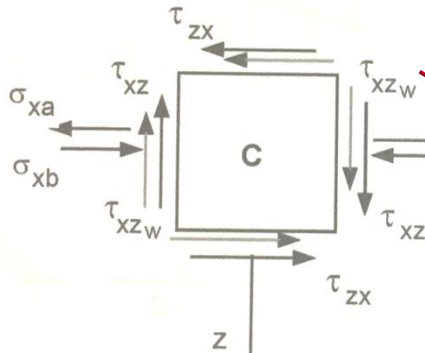


Bending moment at point C

$$M_C = R_{Ay} |AC| = 2.47 (150) = 370.5 \text{ Nm}$$

Torque at point C

$$T_C = W (35) = 10 (35) = 350 \text{ Nm}$$



At point C, we have normal stresses due to bending moment and axial load F

$$\sigma_{xb} = -\frac{32M_C}{\pi d^3} = \frac{32(370.5)(10)^3}{\pi (30)^3} = -139.77 \text{ MPa}$$

$$\sigma_{xa} = \frac{F}{A} = \frac{4F}{\pi d^2} = \frac{4(15000)}{\pi (30)^2} = 21.22 \text{ MPa}$$

The resultant normal stress

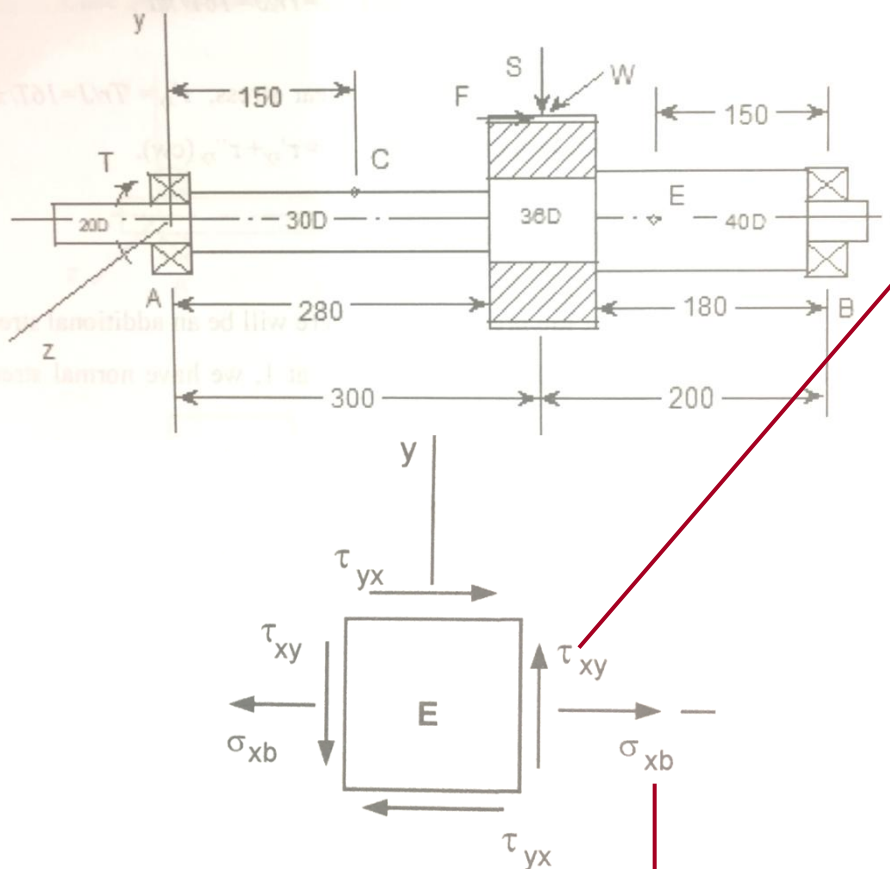
$$\sigma_x = \sigma_{xb} + \sigma_{xa} = -139.77 + 21.22 = -118.55 \text{ MPa}$$

There are two shear stress components at point C

$$\tau_{xz} = -\frac{16T_C}{\pi d^3} = \frac{16(350)(1000)}{\pi (30)^3} = 66.02 \text{ MPa}$$

$$\tau_{xzw} = \frac{4V}{3A} = \frac{4R_{Az}}{3A} = \frac{4(4)(4000)}{3\pi (30)^2} = 7.54 \text{ MPa}$$

Solution of Example 2.2



Bending moment at point E

$$M_E = R_{Bz} |BE| = 6(150) = 900 \text{ Nm}$$

Normal stress due to bending

$$\sigma_{xb} = -\frac{32M_E}{\pi d^3} = \frac{32(900)(10)^3}{\pi(40)^3} = 143.24 \text{ MPa}$$

Transverse shear stress due to force

$$\tau_{xy} = \frac{4V}{3A} = \frac{4(6330)}{3\pi(20)^2} = 6.72 \text{ MPa}$$

Principal stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{143.24}{2} \pm \sqrt{(71.62)^2 + (6.72)^2} = 71.62 \pm 71.93$$

$$\sigma_1 = 143.55 \text{ MPa} \quad \sigma_2 = -0.31 \text{ MPa}$$

$$\tan 2\phi = \frac{2\tau_{xy}}{\sigma_x} = \frac{2(6.72)}{143.24} = 0.094$$

$$\phi = 2.68^\circ \text{ (CCW)}$$