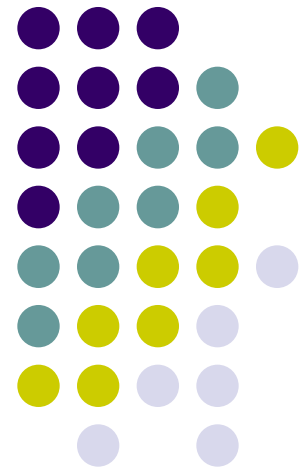


ME 307 – Machine Elements I

Chapter 2

Stress Analysis (Part II)



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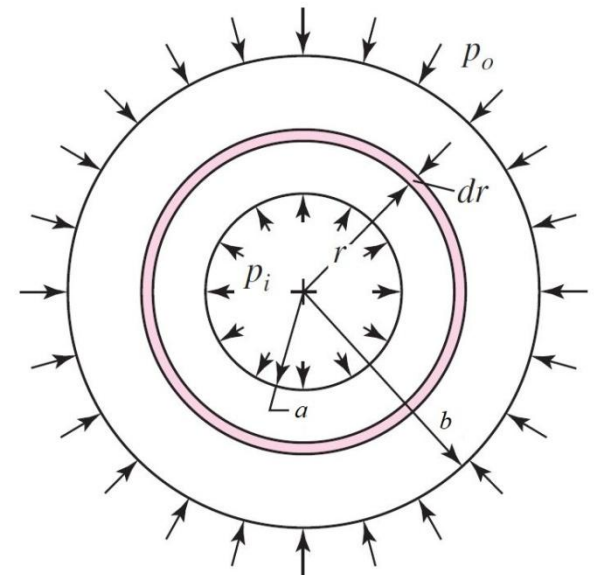
- **Pressurised cylinder** is a cylinder that **pressurised internally and externally**.
- Pressurised cylinders are used in many industrial application such as *pressure vessels, hydraulic cylinders, gun barrels and pipes carrying fluids at high pressure*.
- **By writing force equilibrium in vertical direction** and assuming that elongation in longitudinal direction remains constant (*plane stress case*), **tangential (hoop) stress** and **radial stress** can be calculated by **Lame's equations**.

$$\sigma_t = \frac{p_i a^2 - p_o b^2 - a^2 b^2 (p_o - p_i) / r^2}{b^2 - a^2}$$

$$\sigma_r = \frac{p_i a^2 - p_o b^2 + a^2 b^2 (p_o - p_i) / r^2}{b^2 - a^2}$$

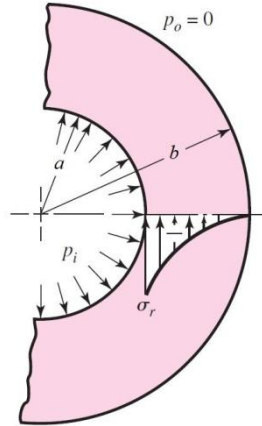
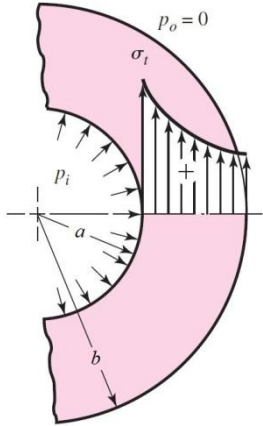
If ends are closed

$$\sigma_l = \frac{p_i a^2}{b^2 - a^2}$$





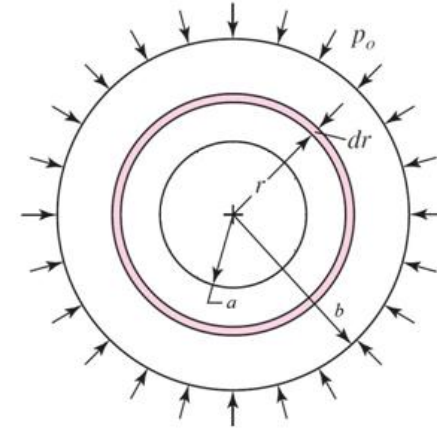
If the external pressure is zero ($p_o=0$)



$$\sigma_t = \frac{a^2 p_i}{b^2 - a^2} \left(1 + \frac{b^2}{r^2} \right)$$

$$\sigma_r = \frac{a^2 p_i}{b^2 - a^2} \left(1 - \frac{b^2}{r^2} \right)$$

If the internal pressure is zero ($p_i=0$)



$$\sigma_t = -p_o \frac{b^2}{b^2 - a^2} \left(1 + \frac{a^2}{r^2} \right)$$

$$\sigma_r = -p_o \frac{b^2}{b^2 - a^2} \left(1 - \frac{a^2}{r^2} \right)$$

At the inner surface

$$r = a$$

$$\sigma_r = -p_i$$

$$\sigma_t = p_i \frac{b^2 + a^2}{b^2 - a^2}$$

At the outer surface

$$r = b$$

$$\sigma_r = 0$$

$$\sigma_t = p_i \frac{2a^2}{b^2 - a^2}$$

At the inner surface

$$r = a$$

$$\sigma_r = 0$$

$$\sigma_t = -p_o \frac{2b^2}{b^2 - a^2}$$

At the outer surface

$$r = b$$

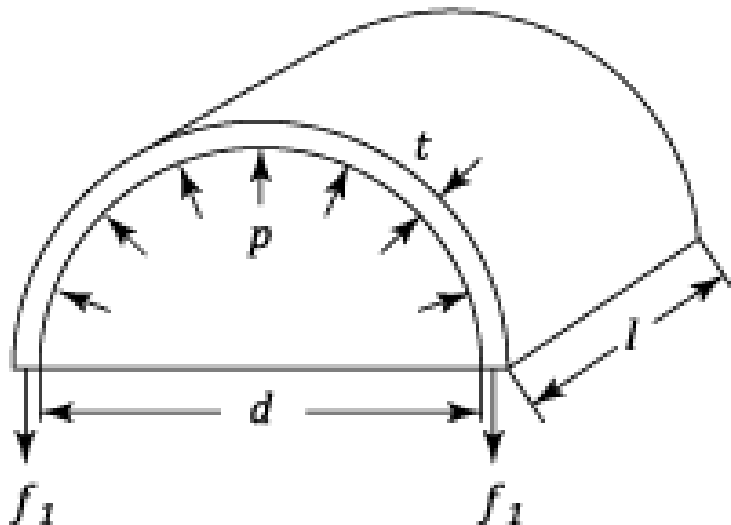
$$\sigma_r = -p_o$$

$$\sigma_t = -p_o \frac{b^2 + a^2}{b^2 - a^2}$$



- When the wall thickness of a cylindrical pressure vessel is about one-twentieth, or less, of its radius, the radial stress that results from pressurizing the vessel is quite small compared with the tangential stress.
- The internal force is resisted by tangential stress acting uniformly over the stressed area.

Internally pressurised thin walled cylinder
($t/r \leq 20$)



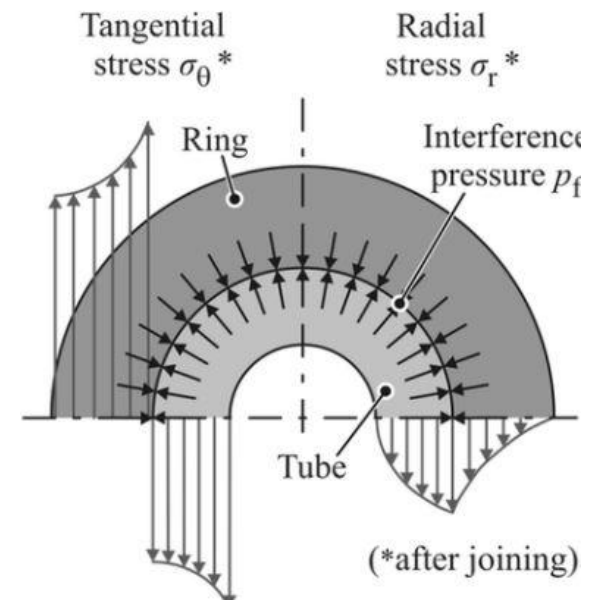
$$\sigma_t = \frac{pd_i}{2t}$$

$$\sigma_r = -p$$

$$\sigma_l = \frac{pd_i}{4t}$$



- **Interference fit** occurs when a shaft with a larger diameter is to be fitted to a hole with a smaller diameter.
- Interference fit can be obtained by heating and/or cooling the members or by applying a certain amount of force in axial direction.
- Interference fit is one of the common mechanical methods used to join two cylindrical components with a certain amount of interference;
 - to transmit torque (*shaft-gear connection*)
 - to create compressive type of residual stress in thick walled cylinders.
- After fitting, a certain amount of interface pressure develops between the members as illustrated in Figure.
- Interface pressure acts as an external pressure for the inner member and as an internal pressure for the outer member.



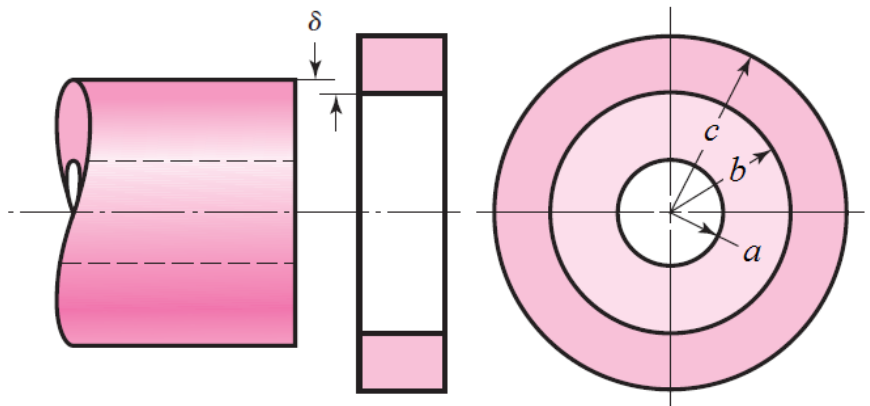


- The difference between shaft diameter and hole diameter is called as **diametral interference** and it is equal to sum of deformations experienced by inner and outer parts. ($\delta = \delta_o + \delta_i$)
- Increase in radius of the hole (δ_o) and decrease in radius of the shaft (δ_i) are obtained from **strain-stress relation**.

$$\left. \begin{aligned} \delta_o &= \frac{bp}{E_o} \left(\frac{c^2 + b^2}{c^2 - b^2} + \mu_o \right) \\ \delta_i &= -\frac{bp}{E_i} \left(\frac{b^2 + a^2}{b^2 - a^2} - \mu_i \right) \end{aligned} \right\} \delta = \delta_o + \delta_i = \frac{bp}{E_o} \left(\frac{c^2 + b^2}{c^2 - b^2} + \mu_o \right) + \frac{bp}{E_i} \left(\frac{b^2 + a^2}{b^2 - a^2} - \mu_i \right)$$

If the materials are the same ($E_o = E_i = E$)

$$p = \frac{E\delta}{b} \left[\frac{(c^2 - b^2)(b^2 - a^2)}{2b^2(c^2 - a^2)} \right]$$





► **Lame's equations** for each member of press fitted assembly;

For inner member

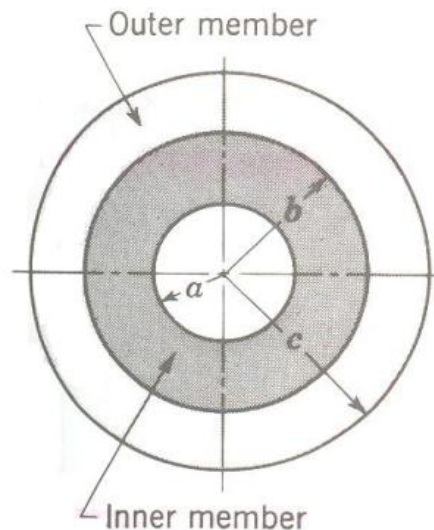
$$\sigma_{ir} = -p \frac{b^2}{b^2 - a^2} \left(1 - \frac{a^2}{r^2} \right)$$

$$\sigma_{it} = -p \frac{b^2}{b^2 - a^2} \left(1 + \frac{a^2}{r^2} \right)$$

For outer member

$$\sigma_{or} = p \frac{b^2}{c^2 - b^2} \left(1 - \frac{c^2}{r^2} \right)$$

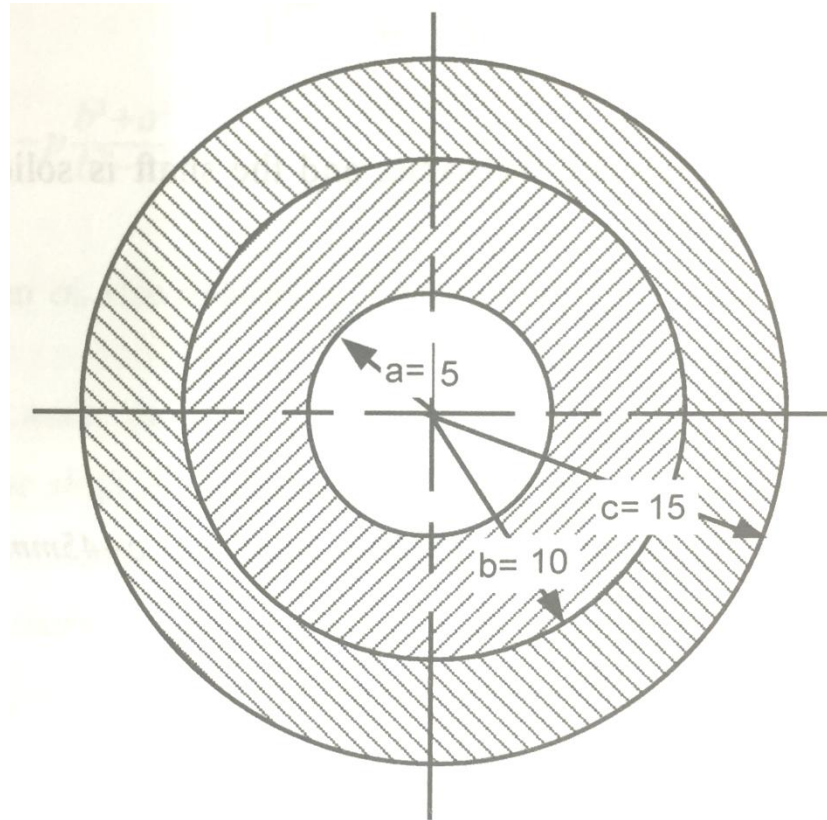
$$\sigma_{ot} = p \frac{b^2}{c^2 - b^2} \left(1 + \frac{c^2}{r^2} \right)$$



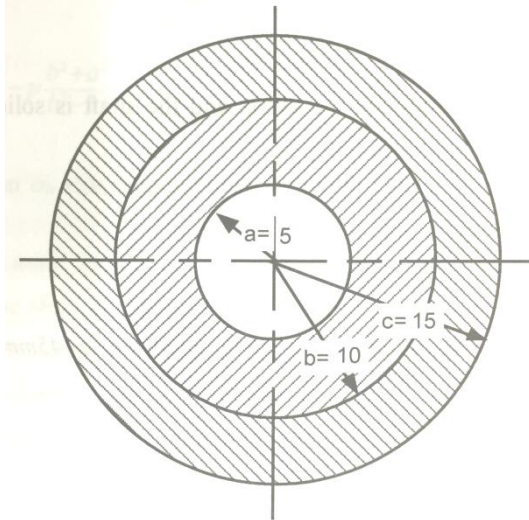
Example 2.10



A gun barrel is assembled by shrinking an outer barrel over an inner barrel so that the maximum principal stress equal to 70% of the yield strength of the material of the barrels. Both barrels are made from steel and have properties of $S_y = 530 \text{ MPa}$, $E = 207 \text{ GPa}$, $\mu = 0.292$ (*Poisson's ratio*). Inner and outer diameters of the inner barrel are 10 mm and 20 mm, respectively. Outer diameter of the outer barrel is 30 mm. What value of interference should be used in the assembly. Plot the stress distribution on the inner and outer barrel.



Solution of Example 2.10



When two element fitted with an interference fit, contact pressure is

$$\sigma_r = -p$$

Tangential stress on the outer surface of the inner member

$$\sigma_{it} = -p \frac{b^2 + a^2}{b^2 - a^2} = -p \frac{10^2 + 5^2}{10^2 - 5^2} = -1.67 p \text{ MPa}$$

Tangential stress on the inner surface of the outer member

$$\sigma_{ot} = p \frac{c^2 + b^2}{c^2 - b^2} = p \frac{15^2 + 10^2}{15^2 - 10^2} = 2.6 p \text{ MPa}$$

Hence, maximum principal stress is σ_{ot}

$$\sigma_{ot} = 0.7 S_y = 0.7 (530) = 371 \text{ MPa}$$

Interface pressure is

$$p = \frac{\sigma_{ot}}{2.6} = \frac{371}{2.6} = 142.7 \text{ MPa}$$

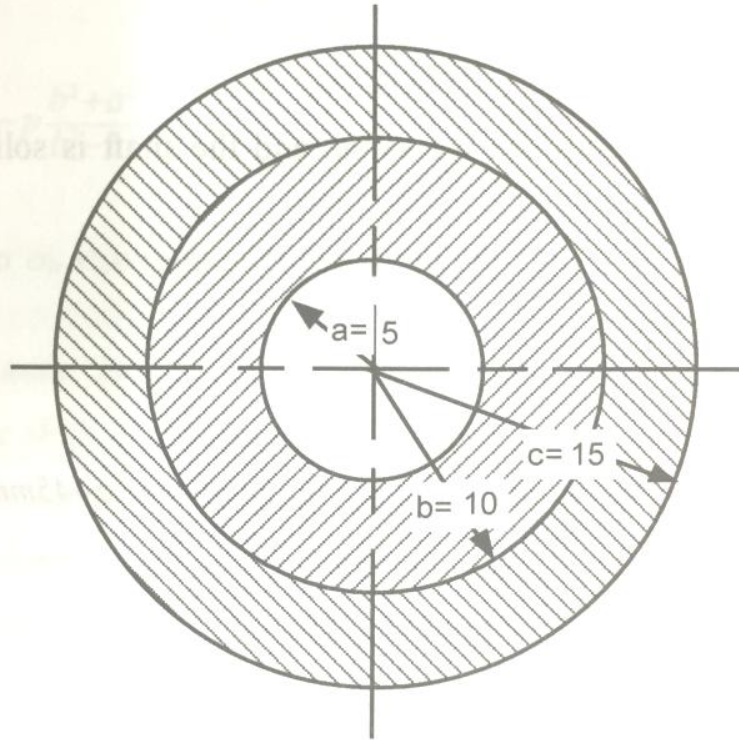
Since both barrels are made from same material

$$\delta = \frac{bp}{E} \left[\frac{2b^2 (c^2 - a^2)}{(c^2 - b^2)(b^2 - a^2)} \right]$$

$$\delta = \frac{10(142.7)}{207000} \left[\frac{2(10)^2 (15^2 - 5^2)}{(15^2 - 10^2)(10^2 - 5^2)} \right] = 0.029 \text{ mm in radiuswise}$$

$$\delta = 2(0.029) = 0.058 \text{ mm in diameterwise}$$

Solution of Example 2.10



Inner barrel is subjected to external pressure of $p=142.7$ Mpa. Tangential and radial stresses are;

$$\sigma_{it} = -p \frac{b^2}{b^2 - a^2} \left(1 + \frac{a^2}{r^2} \right) = -190.3 + \frac{4576.7}{r^2}$$

$$\sigma_{ir} = -p \frac{b^2}{b^2 - a^2} \left(1 - \frac{a^2}{r^2} \right) = -190.3 - \frac{4576.7}{r^2}$$

Outer barrel is subjected to internal pressure of $p=142.7$ Mpa. Tangential and radial stresses are;

$$\sigma_{ot} = p \frac{b^2}{c^2 - b^2} \left(1 + \frac{c^2}{r^2} \right) = 114.2 + \frac{25686}{r^2}$$

$$\sigma_{or} = p \frac{b^2}{c^2 - b^2} \left(1 - \frac{c^2}{r^2} \right) = 114.2 - \frac{25686}{r^2}$$

Solution of Example 2.10



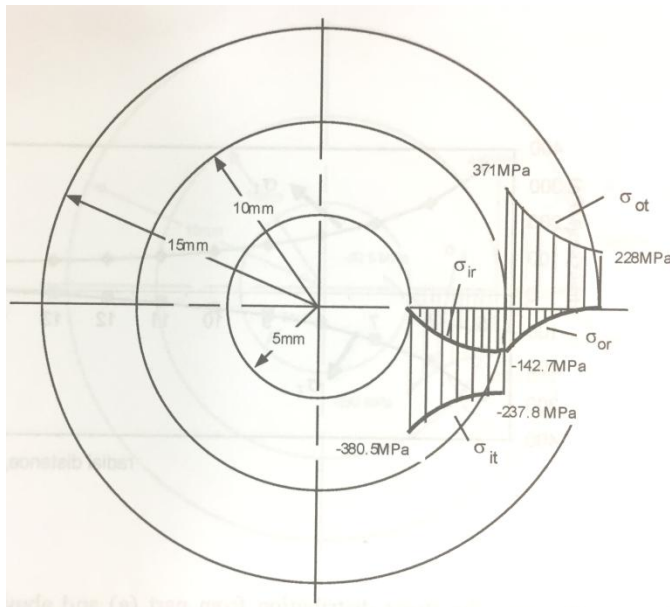
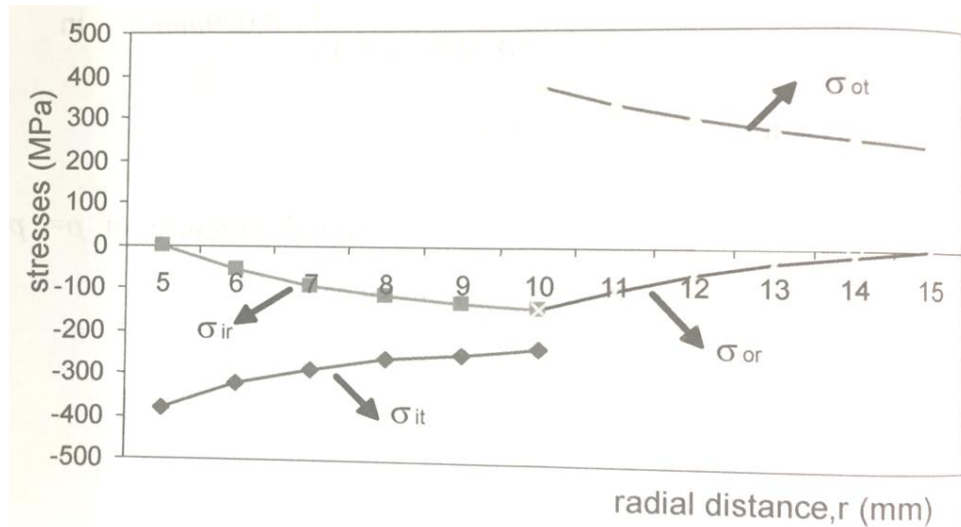
These stresses (σ_{ot} σ_{or} σ_{it} σ_{ir}) are functions of radius r .

$$\sigma_{it} = -190.3 + \frac{4576.7}{r^2}$$

$$\sigma_{ir} = -190.3 - \frac{4576.7}{r^2}$$

$$\sigma_{ot} = 114.2 + \frac{25686}{r^2}$$

$$\sigma_{or} = 114.2 - \frac{25686}{r^2}$$



On the inner part, both tangential and radial stresses are of compressive type. At the interface, maximum radial stress is equal to interface pressure. Tangential stress on the outer member is of tension type.