ME 308 MACHINE ELEMENTS II



WORM GEARS



•non-parallel_and

•non-intersecting <u>shafts</u>

•with relatively high speed reduction ratio









DIFFERENT TYPES OF GEARS AND SHAFT CONFIGURATIONS

Similar but not performing the same work







Parallel shafts are used to transmit motion via spur and helical gears Straight and spiral bevel gears are used in nonparallel and intersecting shafts configuration with 90° shaft angle usually

Non-intersecting non-parallel shafts are usually employed with crossed helical or worm gears







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Regarding the figure below,

there seems to be a transformation from spiral bevel to worm gearing if the pinion position is changed from center of the gear to top of the gear step-by-step.

The two cases between spiral bevel and worm are the hypoid and spiroid gearing.

The teeth of the gear shift from side to top of the ring too.







143

20

25

25

30

0.3683p_x

0.3683p

 $0.2865p_{x}$

0.2546

 $0.2228p_{\star}$

0.3683p.

0.3683p

 $0.3314p_{\star}$

0.2947p

0.2578p

0 - 15

15 - 30

30-35

35-40

40-45

FIGURE 14-13 Nomenclature of a single-enveloping worm gearset.

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A) Kinematics

gear

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- 1) Shafts do not intersect and non-parallel shaft angle is 90° (usually).
- 2) Worm may have 1,2,3,4,5,6 teeth (single tooth, double teeth, triple, quadraple, etc.) (T_w).
- 3) Worm gearsets are:
- Either single envoloping (gear envelopes the worm line contact occurs)
- Or double enveloping (both gear envelopes worm and worm envelopes gear - area contact occurs between the teeth)
 - •Right angle, non-crossing shafts
 - •<u>High level of speed reduction ratio</u> (1:20 1:50)
 - •<u>High friction and wear due to sliding</u> between worm and gear teeth
 - •Self locking may (and does) happen
 - Driver is always the worm and driven is the

As in crossed helical gears both worm and the gear have the same hand of helix







4) As in crossed helical gears both worm and the gear have the same hand but quite different helix angles ψ_W is very large but ψ_G is small.

The lead angle λ of the worm is generally specified which is the complement of ψ_w for 90° shaft angle



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 $C = \frac{d_G + d_W}{2}$

capacity of the gear set

 p_{t_G} = transverse pitch of gear

 $d_G = m_{t_G} T_G \quad d_G = \frac{p_{t_G}}{\pi} T_G$

 d_W is "independent" of T_W

and may have any value

in British Units

between
$$\frac{C^{0.875}}{3} \le d_W \le \frac{C^{0.875}}{1.7}$$

Pitch diameter,
$$d_w$$

Root diameter
Pitch cylinder
Helix
 ψ_w , helix angle
Lead, L
Worm

$$\psi_W + \lambda = 90^\circ \qquad \qquad \psi_W + \psi_G = 90^\circ$$

speed ratio
$$= \frac{N_W}{N_G} = \frac{T_G}{T_W}$$

 $\cos \psi_G = \left(\frac{\tan \phi_n}{\tan \phi_t}\right)_G$



5) For 90° shaft angle $\lambda_W = \psi_G$ and $p_{xW} = p_{tG}$ p_{x_w} = axial pitch of worm

6) The lead *L* of the worm for T_W number of teeth on the worm is:

$$L = T_W \times p_x$$

Also from the triangle





$$\left(\frac{d_W}{2} + a\right)^2 = \left(\frac{d_G}{2}\right)^2 + \left(\frac{F_G}{2}\right)^2 \quad F_G = ? \quad f(d_W, a)$$

a and b addendum and dedendum is given in Table 14.2 for different λ and ϕ_n values of worm gearing.

Effective facewidth of gear $(F_{eff, G})$ is taken as the minimum of the two

$$F_G$$
 and $\frac{2}{3}d_W$

(which ever is smaller)

 Table 14-2
 RECOMMENDED PRESSURE ANGLES AND TOOTH DEPTHS FOR WORM GEARING

Lead angle λ, degrees	Pressure angle ϕ_n , degrees	Addendum a	Dedendum b _G	
0-15	14 1	0.3683p _x	0.3683p _x	
15-30	20	0.3683p	0.3683p	
30-35	25	0.2865p	$0.3314p_{x}$	
35-40	25	0.2546p	0.2947p	
40-45	30	0.2228px	0.2578px	

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exerted upon it by the worm gear.



FIGURE 14-15 Drawing of the pitch cylinder of a worm, showing the forces exerted upon it by the worm gear.

$$W^{x} = W(\cos \phi_{n} \sin \lambda + \mu \cos \lambda)$$
$$W^{y} = W \sin \phi_{n}$$
$$W^{z} = W(\cos \phi_{n} \cos \lambda - \mu \sin \lambda)$$

- The relative motion between worm and wormgear teeth is <u>pure sliding</u>, and so we must expect that friction plays an important role in the performance of worm gearing.
- By introducing a coefficient of friction μ we can develop another set of relations.
 - In Fig. 14-15 we see that the force <u>W acting normal</u> to the worm-tooth profile produces a frictional force $W_f = \mu \ ^*W$

having a component

 $(\mu W \cos \lambda)$

in the negative *x* direction and

- another component
 (μ W sin λ)
 in the positive z direction.
- Equation therefore becomes 13





FIGURE 14-15 Drawing of the pitch cylinder of a worm, showing the forces exerted upon it by the worm gear. 13.3.2019

Efficiency η can be defined by using the equation

$$\eta = \frac{W_{Wt}(without \ friction); \ \mu = 0}{W_{Wt}(with \ friction)}$$

$$\eta = \frac{\cos \phi_n - \mu \tan \lambda}{\cos \phi_n + \mu \cot \lambda}$$

Table 14-3	EFFICIENCY OF WORM
z	GEARSETS FOR $\mu = 0.05$

Helix angle ψ , of gear degrees	Efficiency η, percent
1.0	25.2
2.5	46.8
5.0	62.6
7.5	71.2
10.0	76.8
15.0	82.7
20.0	86.0
25.0	88.0
13.3.2 30 90	89.2

$$W_{Wt} = W_{Gt} \frac{\cos \phi_n \sin \lambda + \mu \cos \lambda}{\mu \sin \lambda - \cos \phi_n \cos \lambda}$$
$$\cot \lambda = \frac{\cos \lambda}{\sin \lambda}$$
For 90° shaft angle $\lambda_W = \Psi_G$ and $p_{xW} = p_{tG}$





FIGURE 14-17 Representative values of the coefficient of friction for worm gearing. These values are based on good lubrication. Use curve B for highquality materials, such as a case-hardened worm mating with a phosphor-bronze gear. Use curve A when more friction is expected, as for example a cast-iron worm and worm gear.

gear

worm



- High gear tangential forces occur due to low speed of gear.
- Due to action reaction equality high axial forces occur on worm too.
- These high axial forces are usually carried by the strong tapered roller bearings of the worm shaft





F

Due to sliding between worm and gear teeth a friction force W_f occurs on the teeth as seen below in addition to the 3 components W_x , W_v and W_z of W_z .

Hence final force components on the worm teeth are:

$$W_{x}: W_{W_{t}} = W \times \cos \phi_{n} \sin \lambda + \mu \times W \times \cos \lambda$$
$$W_{y}: W_{W_{r}} = W \times \sin \phi_{n}$$
$$W_{z}: W_{W_{a}} = W \times \cos \phi_{n} \cos \lambda - \mu \times W \times \sin \lambda$$

Opposite but equal magnitude forces occur on the gear teeth:

$$W_{G_{t}} = -W_{W_{a}} \rightarrow W_{G_{t}} = \mu W \sin \lambda - W \cos \phi_{n} \cos \lambda = W(\mu \sin \lambda - \cos \phi_{n} \cos \lambda)$$
(1)

$$W_{G_{t}} = -W_{W_{t}} \rightarrow W_{G_{a}} = -W_{W_{t}}, \quad W_{W_{t}} = W(\cos \phi_{n} \sin \lambda + \mu \cos \lambda)$$
(2)

18

By using 1 & 2

$$W_{Wt} = W_{Gt} \frac{\cos \phi_n \sin \lambda + \mu \cos \lambda}{\mu \sin \lambda - \cos \phi_n \cos \lambda}$$
Also
$$W_{Wt} = \frac{\text{power input to worm}}{V_w (=\frac{\pi d_w N_w}{60})}$$

Power output = $\eta \times$ Power input

Since there is sliding velocity in worm-gear sets hence a loss due to friction between the teeth efficiency of worm gear sets is defined as:

$$\eta = \frac{W_{Wt}(without\ friction);\ \mu = 0}{W_{Wt}(with\ friction)}$$
$$\eta = W_{Gt}\frac{\cos\phi_n - \mu\tan\lambda}{\cos\phi_n + \mu\cot\lambda}\qquad \cot\lambda = \frac{\cos\lambda}{\sin\lambda}$$

For a typical μ value of 0.05 and standard \mathcal{Q}_n values in Table 14.2 of gear.

ψ_{G}	1°	5°	1 0 °	15°	20 °	25 °	30 °
η %	25	62	77	83	86	88	89

Coefficient of friction μ is dependent on sliding velocity V_s and given in



In Amer. System. (inches) $V_W = \frac{\pi d_W N_W}{12} \,(fpm)$ (rpm) $V_G = \frac{\pi d_G N_G}{12} (fpm)$



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Tooth Stress and Power Rating of Worm Gearing

Generally the teeth of the worm is stronger than the teeth of the gear in terms of bending strength. Thus the bending fatigue failure of worm gear set is based on the bending strength (stress) value of the gear tooth.

Buckingham adopts the Lewis equation of tooth stress as follows for the gear tooth stress.

$$\sigma = \frac{W_{Gt}}{p_n F_G y} \tag{14.28}$$

$$p_n = p_x \times \cos \lambda \qquad (14.29)$$

since
$$p_{xw} = p_{tG} \& \lambda_w = \psi_G$$

for 90° worm - gear set

Since the equation is only a rough approximation, stress concentration is not considered. Also, for this reason, the form factors are not referred to the number of teeth, but only to the normal pressure angle. The values of *y* are listed in Table 14.4.

where σ = bending stress, psi W_{Gt} = transmitted load, lb p_n = normal circular pitch of the gear, in p_x = axial circular pitch of the worm, in F_G = (*effective*) face width of gear, in y = Lewis form factor referred to the circular pitch λ = lead angle of worm = ψ of gear

Table 14-4 /VALUE	S OF P FOR
Normal pressure angle ϕ_n , degrees	Form factor y
141	0,100
20	0.125
25	0.150 -
30	0.175

In (SI) module.

$$\sigma = \frac{W_{Gt}}{F_G \times y \times p_{nG}} = \frac{W_{Gt}}{F_G \times y \times \pi . m_{nG}}$$
$$\sigma = \frac{W_{Gt}}{F_G \times y \times \pi . m_{tG} . \cos \psi_G}$$
$$m_{tG} = \frac{P_{tG}}{\pi} = \frac{P_{xw}}{\pi}$$

The AGMA equation for input-horse-power rating of worm gearing is:

$$H = \frac{W_{Gt} \times d_G \times N_W}{126000 \times m_G} + \frac{V_S \times W_f}{33000} \quad (14.30)$$

from gear shaft

Power loss due to friction between teeth of Worm and Gear.

 W_{Gt} = transmitted load, lb d_G = pitch diameter of the gear, in N_W = speed of worm, rpm m_G = gear ratio T_G / T_W V_S = sliding velocity at mean worm diameter, fpm W_f = frictional force, lb The permissible transmitted load W_{Gt} (transmitted load by the gear) in power equation is calculated by the equation

$$W_{Gt} = d_G^{0.8} \times F_e \times K_s \times K_m \times K_v \quad (14.31)$$

The notation of Eqs. (14.30) and (14.31) is as follows:

 W_{Gt} = transmitted load, lb

 d_G = pitch diameter of the gear, in

 N_W = speed of worm, rpm

 $m_G = \text{gear ratio } T_G / T_W$

 V_s = sliding velocity at mean worm diameter, fpm

 W_f = frictional force, lb

 F_e = effective face width of gear; the effective face width is the face width of the gear or two-thirds of the worm pitch diameter, whichever is less

 K_s = materials and size correction factor (Table 14.5)

 K_m = ratio correction factor (Table 14.6)

$$K_v$$
 = velocity factor (Table 14.7)

Module, m_n in SI



$$H = H_{out} + H_{loss}$$
$$H = W_{Gt} \times V_G + W_f \times V_s$$

$$H_{out} = W_{Gt} \left(\frac{d_G N_w}{m_G} \right) \times \frac{\pi}{60}$$

$$H_{loss} = V_s W_f = \frac{V_w}{\cos \lambda} \,\mu W$$

$$H_{loss} = W_{Gt} \frac{\mu}{\mu \sin \lambda - \cos \phi_n \sin \lambda} \times \frac{V_w}{\cos \lambda}$$

$$H_{out} = W_{Gt} \times V_{G}$$

$$V_{G} = \frac{\pi d_{G} N_{G}}{60}; \quad N_{G} = N_{w} \frac{T_{w}}{T_{G}}$$

$$V_{G} = \frac{\pi d_{G}}{60} \times N_{w} \times \left(\frac{T_{w}}{T_{G}}\right)$$

$$= \frac{1}{m_{G}}$$
Gear Ratio

Example 8.1 (14-19)



P = 10 teeth / inches

$$\phi_n = 14.5^\circ$$
$$\lambda = 9.083^\circ$$
$$d_W = 1.25^"$$

$$F_W = 2$$

Worm teeth are ground and polished

$$F_G = \frac{5}{8}$$
$$d_G = 4^"$$
$$N_W = 1720 rpm$$

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Find maximum safe horse power output and efficiency of the set.

40T sand cast bronze gear

$$H_{out} = ?$$

 $\eta = ?$

Force components on gear?

W = ? $W_{G_t} = ?$ $W_{G_a} = ?$ $W_{G_r} = ?$

1)

$$H_{out} = \frac{W_{Gt} \times d_G \times N_W}{126000 \times m_G} = ?$$

$$W_{Gt} = d_G^{0.8} \times F_e \times K_s \times K_m \times K_v$$

$$H_{loss} = \frac{V_s \times W_f}{33000}$$

$$4'' \quad \frac{2}{3} d_w = 0.833'' \qquad 0.820 \text{ (m}_G = 40/2 = 20)}{F_G = 0.625'' \therefore smaller}$$
Table 14.6

$$\eta = \frac{H_{out}}{H_{in}} = \frac{H_{out}}{H_{out} + H_{loss}} = ? \qquad W_{Gt} = (4)^{0.8} \times 0.625 \times 700 \times 0.820 \times 0.352$$
$$W_{Gt} = 383 \, lb$$

$$V_{s} = \frac{V_{W}}{\cos \lambda} = \frac{\frac{\pi d_{W} N_{W}}{12}}{\cos \lambda} = \frac{\frac{\pi \times 1.25 \times 1720}{12}}{\cos(9.083)} \qquad H_{out} = \frac{383 \times 4 \times 1720}{126000 \times 20} = \underbrace{1.045 \text{ hp}}{126000 \times 20}$$
$$V_{s} = 570 \text{ fpm} \rightarrow K_{v} = 0.352 \text{ (Table 14.7)}$$

$$V_{W} = \frac{\pi d_{W} N_{W}}{12} (fpm)$$
$$V_{G} = \frac{\pi d_{G} N_{G}}{12} (fpm)$$

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Thus $\mu = 0.032 \rightarrow$ from Fig.14.17 curve B for $V_s = 2.8956 m/s$

$$W_{f} = \mu W = \frac{\mu W_{Gt}}{\mu \sin \lambda - \cos \phi_{n} \cos \lambda} \qquad V_{s} = 570 \ fpm \times \frac{\min}{60 \sec} \times \frac{0.3048 \ m}{ft}$$
$$W_{f} = \frac{0.032 \times 383}{0.032 \times \sin(9.083) - \cos(14) \times \cos(9.083)} = -12.9 \ lb$$

$$H_{loss} = \frac{V_s \times W_f}{33000} \qquad H_{loss} = \frac{570 \times 12.9}{33000} = 0.222 \ hp$$

Maximum safe horse power output

Efficiency of the set.

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$$\eta = \frac{H_{out}}{H_{in}} = \frac{H_{out}}{H_{out} + H_{loss}} = \frac{1.045}{1.045 + 0.222}$$
$$\eta = 0.825 = 82.5\%$$

Coefficient of friction μ is dependent on sliding velocity V_s and given in Figure 14.17.



Force components on gear

$$W_{Gt} = W \cos \phi_n \cos \lambda$$
$$W = \frac{W_{Gt}}{\cos \phi_n \cos \lambda} = \frac{383}{\cos(14.5) \times \cos(9.083)} = 400.62 \, lb$$

$$W_{G_a} = W \times \cos \phi_n \times \sin \lambda = 400.62 \times \cos(14.5) \times \sin(9.083)$$
$$W_{G_a} = \underline{61.23 \, lb}$$
$$W_{G_r} = W \times \sin \phi_n = 400.62 \times \sin(14.5)$$
$$W_{G_r} = \underline{100.3 \, lb}$$

14-8 POWER RATING OF WORM GEARING.

When worm gearsets are used intermittently or at slow gear speeds, the bending strength of the gear tooth may become a principal design factor. Since the worm teeth are inherently stronger than the gear teeth, they are usually not considered, though the methods of Chap, 8 can be used to compute worm-tooth stresses. The teeth of worm gears are thick and short at the two edges of the face and thin in the central plane, and this makes it difficult to determine the bending stress. Buckingham* adapts the Lewis equation as follows:

W_{Gi} (14-28) $p_{e}F_{G}y$ (14-29) $p_{\pi} = p_x \cos \lambda$

where $\sigma =$ bending stress, psi

 $W_{Gt} = transmitted load, lb$

 $p_n = normal circular pitch, in$

 $p_x = axial circular pitch, in of$

 $F_{G} =$ face width of gear, in

y = Lewis form factor referred to the circular pitch $\lambda = \text{lead angle}$

Since the equation is only a rough approximation, stress concentration is not considered. Also, for this reason, the form factors are not referred to the number. of teeth, but only to the normal pressure angle. The values of y are listed in Table 14-4. ي مريد ج · · · · · · 1. 1.5

The AGMA equation for input-horsepower rating of worm gearing is

26 000m 33 000

The first term on the right is the output horsepower, and the second is the power loss.

14-30

hat s

TABLE 14.7 VELOCITY FACTOR K.

Velocity V ₅ , fpm	K.	Velocity V _s ,	K,	Velocity V ₂ , fpm	K.
	· 0 649	300	0.472	1400	0.216
15	0.647	350	0.446	1600	0.200
10	0.644	400	0.421	1800	0.187
20	0.634	450	0.398	2000	0.175
30	0.631	500	0.378	2200	0.165
40	0.625	550	0.358	2400	0.156
60	0.613	600	0,340	2600	0.148
80	0.600	700	0.310	2800	0:140
100	0.588	800	0.269	9000	0.134
150	0.558	900	0.269	4000	0.106
200	0.528	1000	0.258	5000	0.089
256	0.500	1200	0.235	6000	0.079

Source: Darie W. Dudley (ed.); Geor Handbook, McGraw-Hill, New York, 1962. ---- 13.3.2019

Tab	le 14-4	VALUES	OF y FOI JEARS	<u>R</u>		 		
Nor ang	mal pre- le ϕ_n , de	ssure grees	Form fact	огу			8 - 1 1 - 1	
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The	bermissible	e transmi	tted load 1	V _{gi} is c	omputed	from th	e equation	1
i	$W_{Gt} = K_{t}$	s d ^{0,8} F, F	K _m K _e		-			(1
The 1	otation	of Eqs. (1	4-30) and	(14-31) is as foll	ows:	ι. C	
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	m an	ar ratio	N /N					
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	$V_S = sm$	ting velo	city at me	an wor	m diame	ter, fpm		
	$W_f = \text{fri}$	ctional fo	rce <u>, lb</u>					
	$K_s = ma$	aterials a	nd size-cor	rection	factor			
· · ·	$F_e = eff$	ective fac	e width of	gear:	the effect	ive face	width is	the face -
	wi	dth of th	e gear or t	wowthin	vie of the	100000	nitob dian	antes
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5. <u>5.</u> 5. 5.	$\mathbf{X}_{\mathbf{r}} \approx \text{vel}$	locity fac	tor				-	•
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Table	14.5 14	ATEDIAL			ion au		·	
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·	VV.	UKM GI	AKING*			۳.		-
Face	width of F _G , in	gear	Sand-cast bronze	Stat	ic chill-c bronze	ust C	entrifuga broaz	l-cast
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1				·····				1 00
· · ·	Ratio <i>m</i> c	K _m	Ratio	K,	Ratio ^m G	K_		
	3.0	0.500	8.0	0.724	30.0	0.825		
	3.5	0.554	9.0	0.744	40.0	0.815	:	1.1
	4.0	0.593	10.0	0.760	50.0	0.785		
	4.5	0.620	12.0	0.783	60,0	0.745		
	5.0	0.645	14.0	0.799	70.0	0.687		
	6.0	0.679	16.0	0.809	80.0	0.622		1
	7.0	0.706	20.0	0.820	169.0	0 400		

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