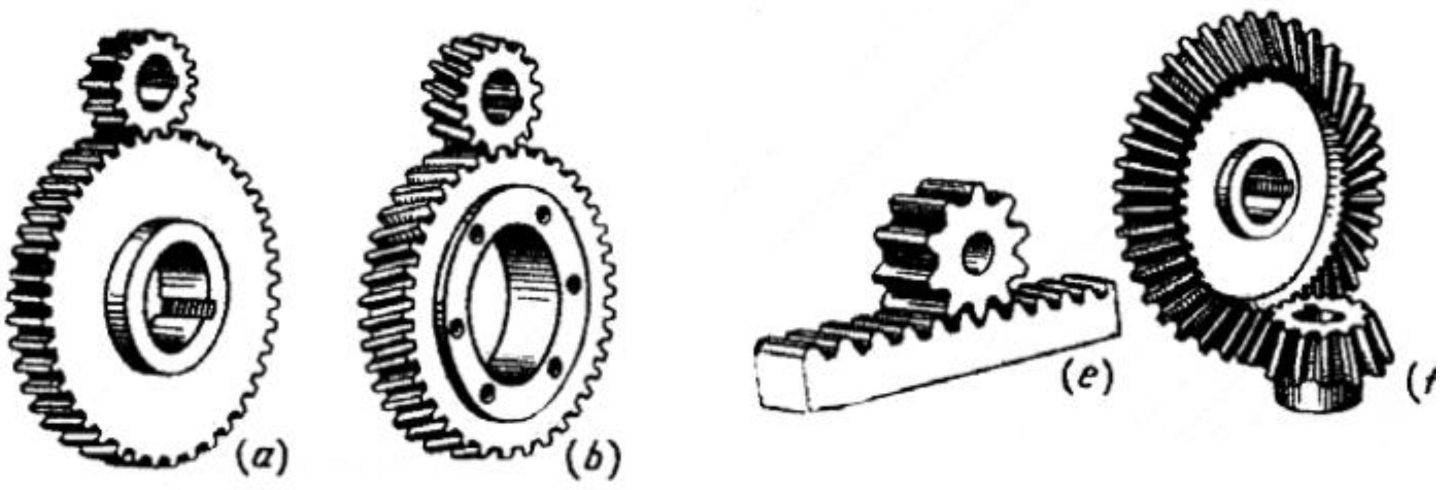
ME 308
MACHINE ELEMENTS II

CHAPTER 5

GEARS
PART_4



5.13 FATIGUE LOADING OF GEARS

While LEWIS equation is used for static bending stress calculation AGMA equation is used for fatigue condition and gives the bending stress in tooth root under a force of W_t acting tangent to the pitch circle and including effects of stress concentration (J).

For a fatigue-free safe operation the bending stress (σ) obtained from AGMA equation should be compared with the endurance strength (S_e) of the gear material with a global safety factor n_G, that is;

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \times S_e'$$

k_a: surface factor, k_b: size factor, k_c: relb. factor, k_d: temp. factor, k_e: str. con. factor, k_f: misc. factor

$$\sigma = \frac{W_t}{F \times m \times J \times K_v} \leq S_e / n_G$$

$$n_G = n \times K_o \times K_m$$

For Steels:

$$S_e' = 0.5 S_{ut} \quad \text{If } S_{ut} \leq 1400\ MPa$$

$$S_e' = 700\ MPa \quad \text{If } S_{ut} > 1400\ MPa$$

For Cast Irons:

$$S_e' = 0.45 S_{ut} \quad \text{If } S_{ut} \leq 600\ MPa$$

$$S_e' = 275\ MPa \quad \text{If } S_{ut} > 600\ MPa$$

Surface finish: for factor k_a, use machined surface always since the tooth root is always in machined or cast form even if the tooth flank is ground.

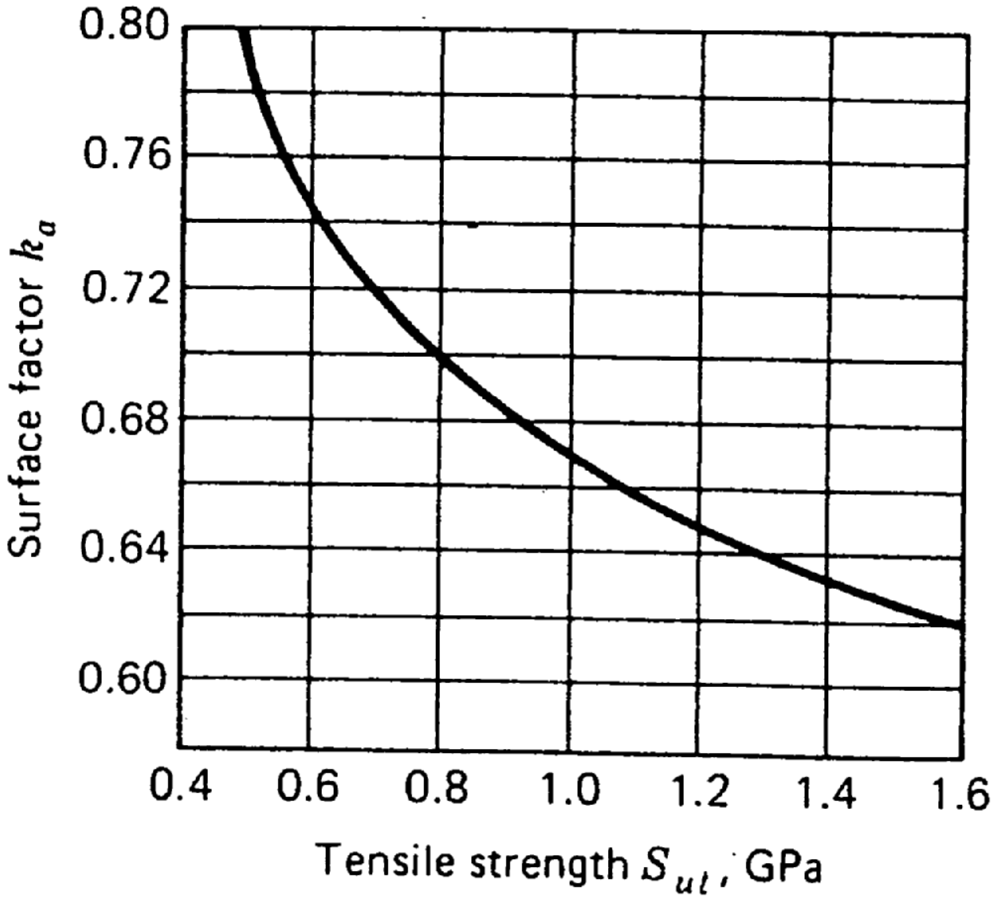


Figure 13–25 Surface finish factors k_a for cut, shaved and ground gear teeth.

<u>If the gear material is Cast Iron</u>

The S_e' values given in Table A-21 are fully corrected for surface factors (k_a), thus use k_a=1 but not corrected for other factors.

Size: The size factor, from eq. (7.16) is

$$k_b = \begin{cases} 1 & d \le 8\,mm \\ 1.189 d^{-0.097} & 8\,mm < d \le 250\,mm \end{cases} \qquad (7.16) \qquad \text{(a)}$$

For factor k_b, Eq. 7-16 is generally used but, in this equation the dimension d is the diameter of a round specimen. A spur gear tooth has a rectangular cross section and so the method of Sec. 7-7 must be used to get an equivalent value for d. For a rectangular cross section the formula for the equivalent diameter is

$$d = 0.808 (hb)^{1/2} \qquad \text{(b)}$$

where *h* is the height of the section and *b* is the width. For a gear tooth *h* is the tooth thickness which is about half the circilar pitch. And *b* is the face with *F*. Substituting $h = p/2$ and $F = 3p$ in Eq. (b) and solving gives

$$d_{eq} \cong p = \pi m \qquad \text{(c)}$$

Thus we can use these three equation to work out a set of size factors based on the module.

By using different module values *h* and *b* were determined and then k_b values were calculated. The results for k_b were then simply tabulated in Table 13-7 for different modules, and k_b values are taken from Table 13-7.

Table 13-7 SIZE FACTORS FOR SPUR-GEAR TEETH (Preferred modules in bold face

Module *m*	Factor k_b	Module *m*	Factor k_b
1 to 2	1.000	11	0.843
2.25	0.984	**12**	0.836
2.5	0.974	14	0.824
2.75	0.965	**16**	0.813
3	0.956	18	0.804
3.5	0.942	**20**	0.796
4	0.930	22	0.788
4.5	0.920	**25**	0.779
5	0.910	28	0.770
5.5	0.902	**32**	0.760
6	0.894	36	0.752
7	0.881	**40**	0.744
8	0.870	45	0.736
9	0.860	**50**	0.728
10	0.851		

Reliability factor, For factor k_c, Table13-8 is used.

Table 13-8 RELIABILITY FACTORS

Reliability R	0.50	0.90	0.95	0.99	0.999	0.9999
Factor k_c	1.000	0.897	0.868	0.814	0.753	0.702

Temperature: from eq. (7.26)

$$k_d = \begin{cases} 1 & T \le 350\ ^oC \\ 0.5 & 350\ ^oC < T \le 500\ ^oC \end{cases} \qquad (13.30)$$

Stress concentration: k_e = 1.0 for gears, since stress concentration due to fillet geometry is included in the geometry factor J.

Regarding factor k_f, miscellaneous effects, since most of the time gears rotate only in one direction the loading type is repeated, therefore tooth is subjected to one-way bending.

Since endurance limits S_e' and S_e are calculated for two-way bending specimens the actual endurance limits has to be increased for one-way bending teeth and k_f is taken from Fig 13-26.

For two-way bending (both direction rotating or idler) gears, however, k_f is taken as 1.0

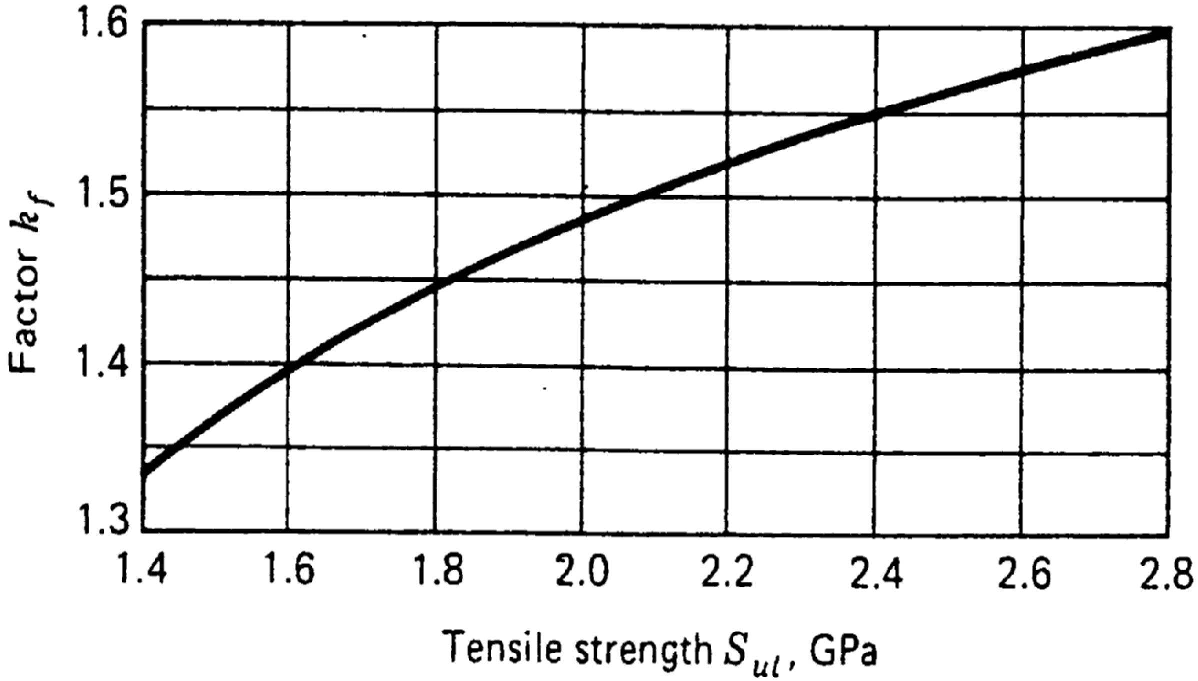


Figure 13–26
Miscellaneous effects factors k_f for one-way bending of gear teeth. Use k_f = 1.33 for values of S_{ut} less than 1.4 Gpa.

Once the S_e value is calculated from

$$S_e = k_a \times k_b \times k_c \times k_d \times k_e \times k_f \times S_e^{'}$$

and the actual stress value σ from;

$$\sigma = \frac{W_t}{F \times m \times J \times K_v}$$

Then global safety factor n_G can be calculated as:

$$n_G = \frac{S_e}{\sigma} \qquad n_G = n \times K_o \times K_m$$

Here

n is the usual safety factor (suggested to be $n>1.0$; even $n>2$)

K_o is the overload factor given in Table 13-9 for different power sources and driven machinery conditions ($K_o > 1$).

K_m is the load-distribution factor given in Table 13-10 for different face-width and mounting conditions ($K_m > 1$).

Table 13-9 OVERLOAD CORRECTION FACTOR K_o

Source of power	Driven machinery: Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Table 13-10 LOAD-DISTRIBUTION FACTOR K_m FOR SPUR GEARS

	Face width, mm			
Characteristics of support	0 to 50	150	225	400 up
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.3	1.4	1.5	1.8
Less rigid mountings, less accurate gears, contact across full face	1.6	1.7	1.8	2.2
Accuracy and mounting such that less than full-face contact exists	Over 2.2			

EXAMPLE: 5.5

A gear pair with 1.5 mm module, 25^o pressure angle, 15 teeth pinion driving a 64 teeth gear, 25 mm face width, hobbed teeth, a=1m, b=1.25m is made of BS080M30 HR steel and experiences light shock loads on both driving and driven machineries.

For n= 2.5, R= 50%, average mounting conditions, and a pitch line velocity 3.8 m/s. Find safe power capacity of the gear set? P=? kW

$$P = W_t \times V \quad W_t = ?$$

$$\sigma = \frac{W_t}{F \times m \times J \times K_v} = \frac{S_e}{n_G}, \quad n_G = K_o \times K_m \times n$$

$m = 1.5\ mm$

$T_P = 15$

$T_G = 64$

$F = 25\ mm$

$$W_t = F \times m \times J \times K_v \times \frac{S_e}{K_o \times K_m \times n},$$

$$J \cong 0.38952, \quad \text{from Table 13.5}$$

Obtained by interpolation

$$K_v = \frac{50}{50 + \sqrt{200V}} = 0.644 \quad \text{for hobbed teeth}$$

Ø= 25^o

S_{ut}= 490 MPa

From Table in Appendix A16 pp. 664

$$K_o = 1.5 \quad \text{Table 13.9}$$

$$K_m = 1.6 \quad \text{Table 13.10}$$

From Table 13.7 From Table 13.8

$$k_a = 0.8,\ k_b = 1.0,\ k_c = 1.0,\ k_d = 1.0,\ k_e = 1.0,\ k_f = 1.33$$

From Fig. 13.25

From Fig. 13.26

Table A-16 MECHANICAL PROPERTIES OF SOME STEELS
Although the values shown for the properties in this table can be easily obtained when carefully processed, they should probably be treated as *minimum values* for preliminary design. A minimum value is approximately several standard deviations below the arithmetic mean.

Material	British standard	Processing*	Maximum section size, mm	Yield strength S_y, MPa (N/mm^2)	Tensile strength S_{ut}, MPa (N/mm^2)	Elongation† $5.65\sqrt{A_0}$, %	Hardness number‡ HB
0.20C	070M20	HR	152	215	430	22	126–179
			254	200	400	20	116–170
		CD	13	385	530	12	154
			76	340	430	14	125
0.30C	080M30	HR	152	245	490	20	143–192
			254	230	460	19	134–183
		CD	13	470	600	10	174
			63	385	530	12	154
		H&T	63	385	550–700	13	152–207
0.40C	080M40	HR	150	280	550	16	152–207
		CD	63	430	570	10	165
		H&T	63	385	625–775	16	179–229
0.50C	080M50	HR	150	310	620	14	179–229
		CD	63	510	650	10	188
		H&T	150	430	625–775	11	179–229
1Cr	530M40	H&T	100	525	700–850	17	201–255
			29	680	850–1000	13	248–302

Table 13-4 AGMA GEOMETRY FACTOR J FOR TEETH HAVING $\phi = 20°$, $a = 1m$, $b = 1.25m$, AND $r_f = 0.300m$

Number of teeth	Number of teeth in mating gear: 1	17	25	35	50	85	300	1000
18	0.244 86	0.324 04	0.332 14	0.338 40	0.344 04	0.350 50	0.355 94	0.361 12
19	0.247 94	0.330 29	0.338 78	0.345 37	0.351 34	0.358 22	0.364 05	0.369 63
20	0.250 72	0.336 00	0.344 85	0.351 76	0.358 04	0.365 32	0.371 51	0.377 49
21	0.253 23	0.341 24	0.350 44	0.357 64	0.364 22	0.371 86	0.378 41	0.384 75
22	0.255 52	0.346 07	0.355 59	0.363 06	0.369 92	0.377 92	0.384 79	0.391 48
24	0.259 51	0.354 68	0.364 77	0.372 75	0.380 12	0.388 77	0.396 26	0.403 60
26	0.262 89	0.362 11	0.372 72	0.381 15	0.388 97	0.398 21	0.406 25	0.414 18
28	0.265 80	0.368 60	0.379 67	0.388 51	0.396 73	0.406 50	0.415 04	0.423 51
30	0.268 31	0.374 62	0.385 80	0.395 00	0.403 59	0.413 83	0.422 83	0.431 79
34	0.272 47	0.383 94	0.396 71	0.405 94	0.415 17	0.426 24	0.436 04	0.445 86
38	0.275 75	0.391 70	0.404 46	0.414 80	0.424 56	0.436 33	0.446 80	0.457 35
45	0.280 13	0.402 23	0.415 79	0.426 85	0.437 35	0.450 10	0.461 52	0.473 10
50	0.282 52	0.408 08	0.422 08	0.435 55	0.444 48	0.457 78	0.469 75	0.481 93
60	0.286 13	0.417 02	0.431 73	0.443 83	0.455 42	0.469 60	0.482 43	0.495 57
75	0.289 79	0.426 20	0.441 63	0.454 40	0.466 68	0.481 79	0.495 54	0.509 70
100	0.293 53	0.435 61	0.451 80	0.465 27	0.478 27	0.494 37	0.509 09	0.524 35
150	0.297 38	0.445 30	0.462 26	0.476 45	0.490 23	0.507 36	0.523 12	0.539 54
300	0.301 41	0.455 26	0.473 04	0.487 98	0.502 56	0.520 78	0.537 65	0.555 33
Rack	0.305 71	0.465 54	[illegible]	[illegible]	[illegible] 29	0.534 67	0.552 72	0.571 73

Table 13-5 AGMA GEOMETRY FACTOR J FOR TEETH HAVING $\phi = 25°$, $a = 1m$, $b = 1.25m$, AND $r_f = 0.300m$

Number of teeth	Number of teeth in mating gear							
	1	17	25	35	50	85	300	1000
13	0.286 65	0.346 84	0.352 92	0.357 44	0.361 38	0.365 72	0.369 25	0.372 51
14	0.293 64	0.359 24	0.365 87	0.370 81	0.375 14	0.379 94	0.383 86	0.387 49
15	0.300 09	0.370 27	0.377 40	0.382 75	0.387 44	0.392 67	0.396 94	0.400 92
16	0.305 58	0.380 16	0.387 75	0.393 46	0.398 49	0.404 11	0.408 73	0.413 03
17	0.310 43	0.389 07	0.397 09	0.403 14	0.408 49	0.414 48	0.419 41	0.424 02
18	0.314 75	0.397 14	0.405 56	0.411 93	0.417 56	0.423 90	0.429 13	0.434 03
19	0.318 62	0.404 49	0.413 28	0.419 94	0.425 85	0.432 50	0.438 01	0.443 18
20	0.322 11	0.411 21	0.420 34	0.427 27	0.433 44	0.440 39	0.446 16	0.451 59
21	0.325 28	0.417 38	0.426 82	0.434 01	0.440 42	0.447 65	0.453 67	0.459 33
22	0.328 16	0.423 06	0.432 80	0.440 23	0.446 86	0.454 36	0.460 60	0.466 50
24	0.333 22	0.433 18	0.443 46	0.451 32	0.458 36	0.466 35	0.473 01	0.479 32
26	0.337 52	0.441 93	0.452 68	0.460 93	0.468 33	0.476 74	0.483 78	0.490 46
28	0.341 22	0.449 57	0.460 75	0.469 33	0.477 05	0.485 85	0.493 23	0.500 23
30	0.344 43	0.456 31	0.467 85	0.476 75	0.484 75	0.493 89	0.501 57	0.508 68
34	0.349 76	0.467 63	0.479 81	0.489 23	0.497 72	0.507 46	0.515 66	0.523 49
38	0.354 00	0.476 78	0.489 48	0.499 33	0.508 24	0.518 47	0.527 10	0.535 36
45	0.359 67	0.489 19	0.502 61	0.513 05	0.522 52	0.533 44	0.542 68	0.551 54
50	0.362 78	0.496 08	0.509 91	0.520 68	0.530 47	0.541 77	0.551 36	0.560 56
60	0.367 50	0.506 83	0.521 09	0.532 38	0.542 67	0.554 57	0.564 69	0.574 44
75	0.372 32	0.517 47	0.532 57	0.544 40	0.555 20	0.567 73	0.578 42	0.588 73
100	0.377 26	0.528 60	0.544 36	0.556 76	0.568 10	0.581 29	0.592 57	0.603 48
150	0.382 37	0.540 05	0.556 51	0.569 51	0.581 38	0.595 26	0.607 16	0.618 69
300	0.387 72	[illegible]	[illegible]	[illegible]	0.595 07	0.609 67	0.622 22	[illegible]
Rack	0.393 42	0.564 05	0.581 94	0.596 13	0.609 21	0.624 56	0.637 78	0.650 68

Linear interpolation between two known points

In mathematics, **linear interpolation** is a method of curve fitting using linear polynomials.

If the two known points are given by the coordinates (x_0,y_0) and (x_1,y_1), the **linear interpolant** is the straight line between these points. For a value x in the interval (x_0,x_1), the value y along the straight line is given from the equation

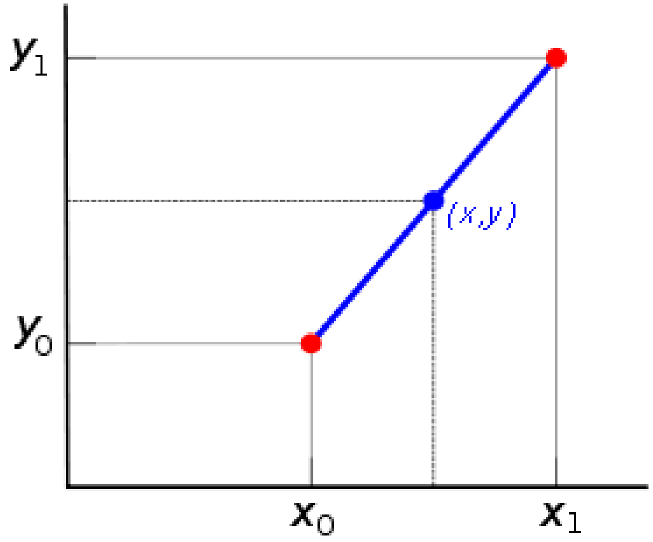
$$\frac{y-y_0}{x-x_0}=\frac{y_1-y_0}{x_1-x_0}$$

which can be derived geometrically from the figure on the right. It is a special case of polynomial interpolation with $n = 1$.

Solving this equation for y, which is the unknown value at x, gives

$$y=y_0+(y_1-y_0)\frac{x-x_0}{x_1-x_0}$$

which is the formula for linear interpolation in the interval (x_0,x_1).



Given the two red points, the blue line is the linear interpolant between the points, and the value *y* at *x* may be found by linear interpolation.

$$y=0.38744+(0.39267-0.38744)\frac{64-50}{85-50}$$

$$y=0.38952$$

$$S_e^{'} = 0.5 S_{ut}$$

$$S_e^{'} = 0.5 \times 490 = 245\ MPa$$

$$S_e = k_a \times k_b \times k_c \times k_d \times k_e \times k_f \times S_e^{'}$$

$$S_e = 0.8 \times 1.0 \times 1.0 \times 1.0 \times 1.0 \times 1.33 \times 245$$

$$S_e = 260\ MPa$$

$$W_t = \frac{P}{V} \qquad P = W_t \times V$$

$$V = \frac{\pi \times d \times N}{60}$$

$$W_t = F \times m \times J \times K_v \times \frac{S_e}{K_o K_m n}$$

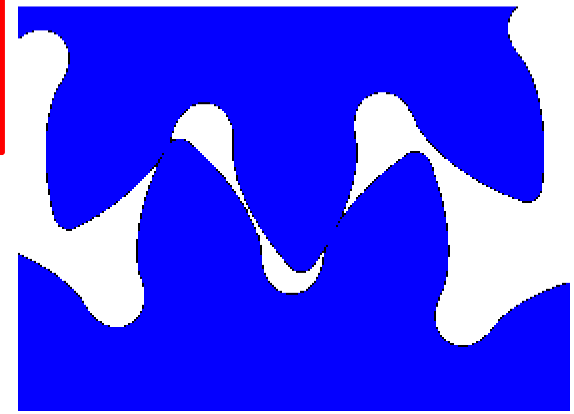
$$W_t = 25 \times 1.5 \times 0.38952 \times 0.644 \times \frac{260}{1.5 \times 1.6 \times 2.5}$$

$$W_t = 407.6\ N$$

$$P = 407.6 \times 3.8 = 1549\ \ Watt \text{ or less}$$

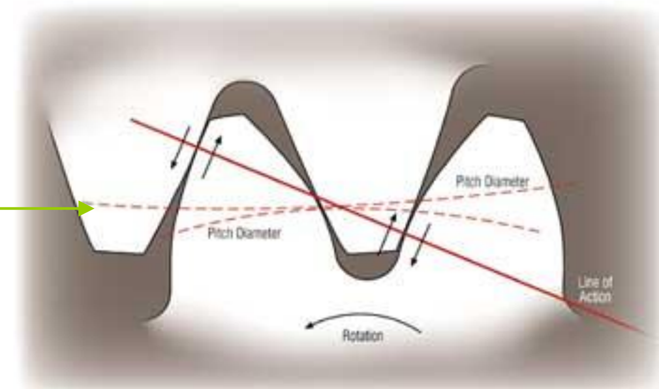
$$P \cong 1.55\ kW \text{ or less}$$

5.14 SURFACE DURABILITY OF GEAR TEETH



Other than the fatigue failure of tooth due to bending the teeth surfaces may also fail due to high contact stresses on the teeth.

The tooth surface will wear or pit due to high contact stresses along with the sliding action near tip and root of the tooth.

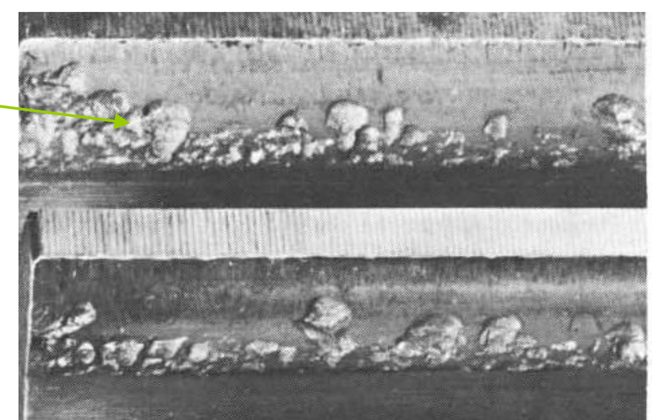


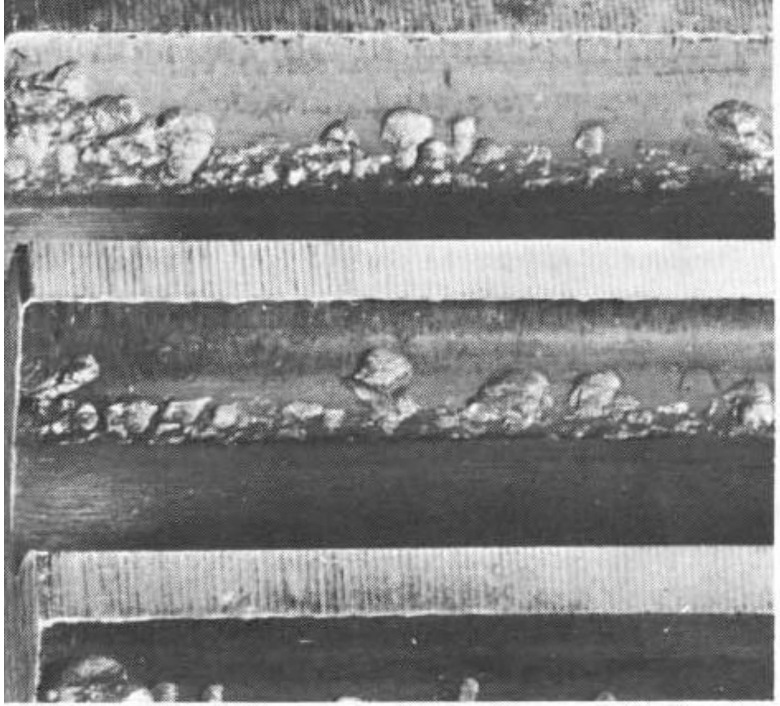
The common failure types are:

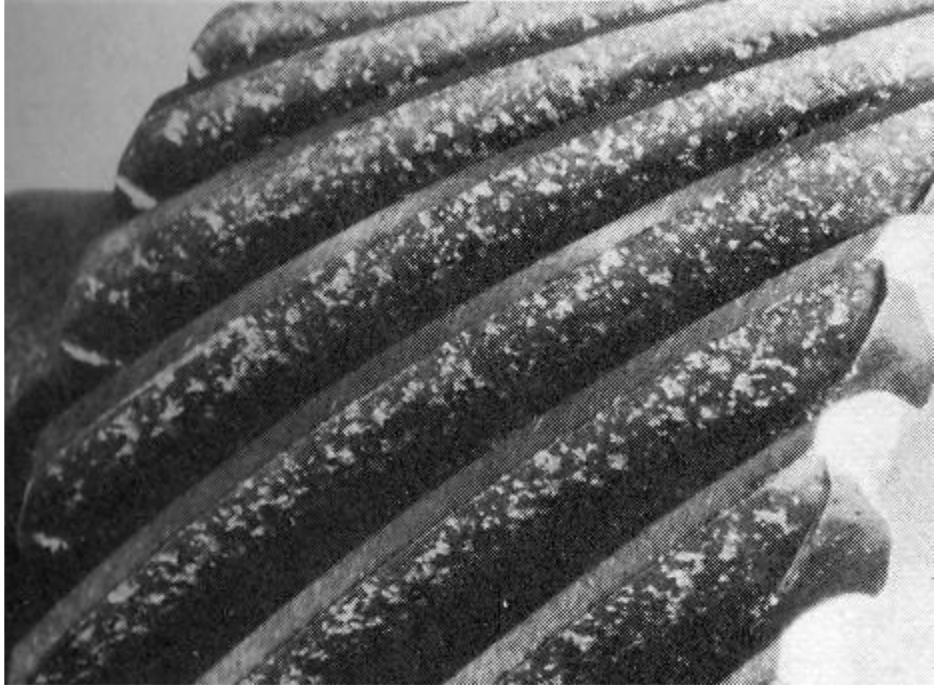
-pitting (due to many repitations of high contact stresses) (karıncalanma, yüzeyden malzeme kaybı)

-scoring (due to lubricant failure)(çizik yapma)

-abrasion (due to presence of foreign material) (aşındırma, aşınma, yıpranma, yenme, aşınmış kısım, aşınma sonucu kopan parçalar, sıyrık, sıyırma)







Surface failures due to contact stresses

Contours of a particle to be spalled out.

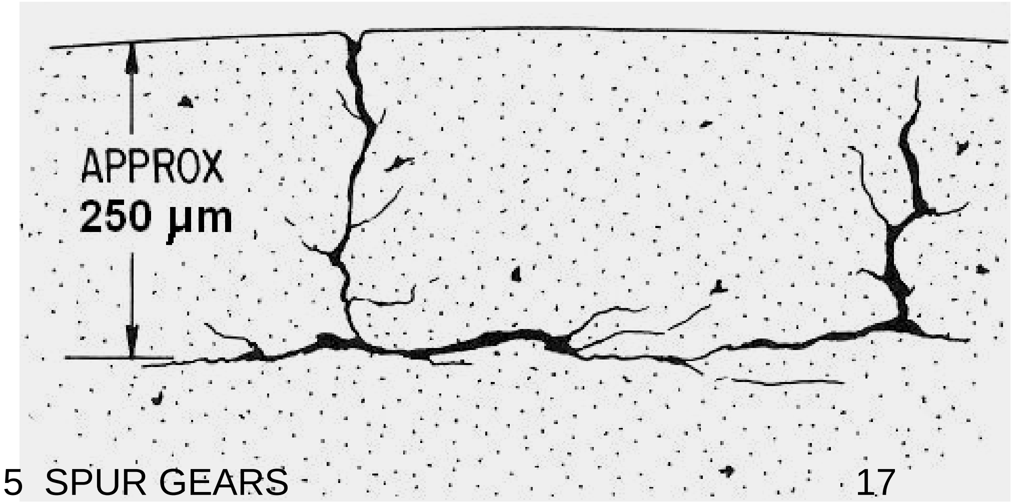
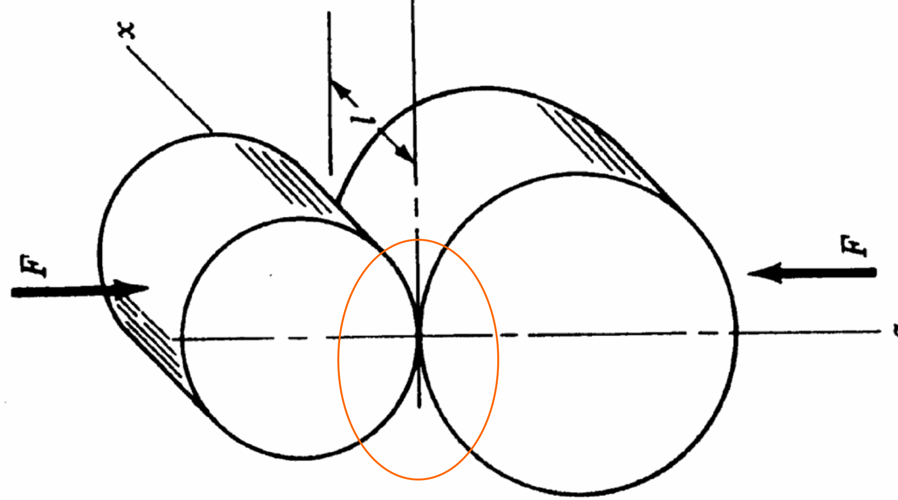
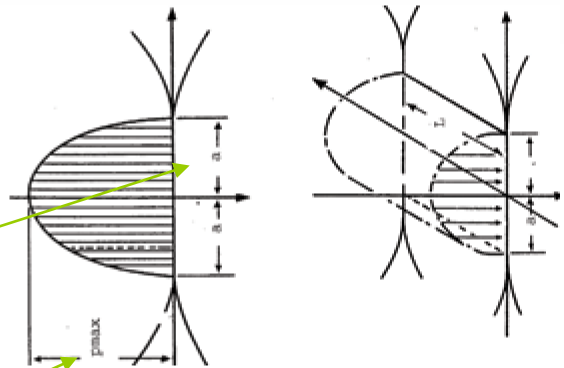


Figure illustrates a case in which the contacting elements are two cylinders of length l and diameters d_1 and d_2 .



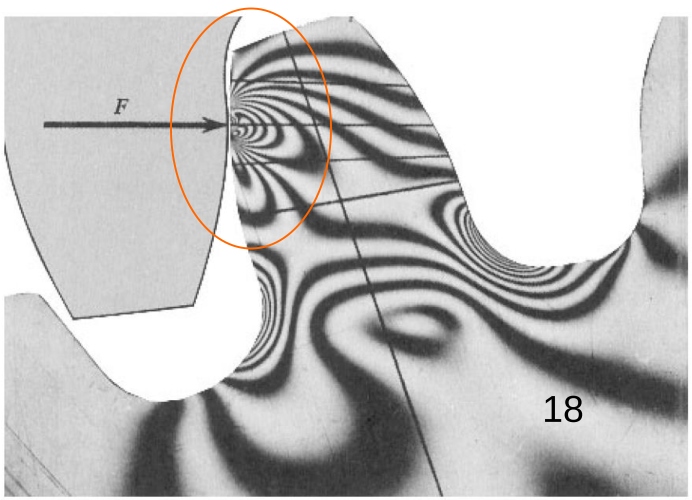
The area of contact is a narrow rectangle of width *2b* and length *l*, and the pressure distribution is elliptical. The half-width *b* is given by the equation



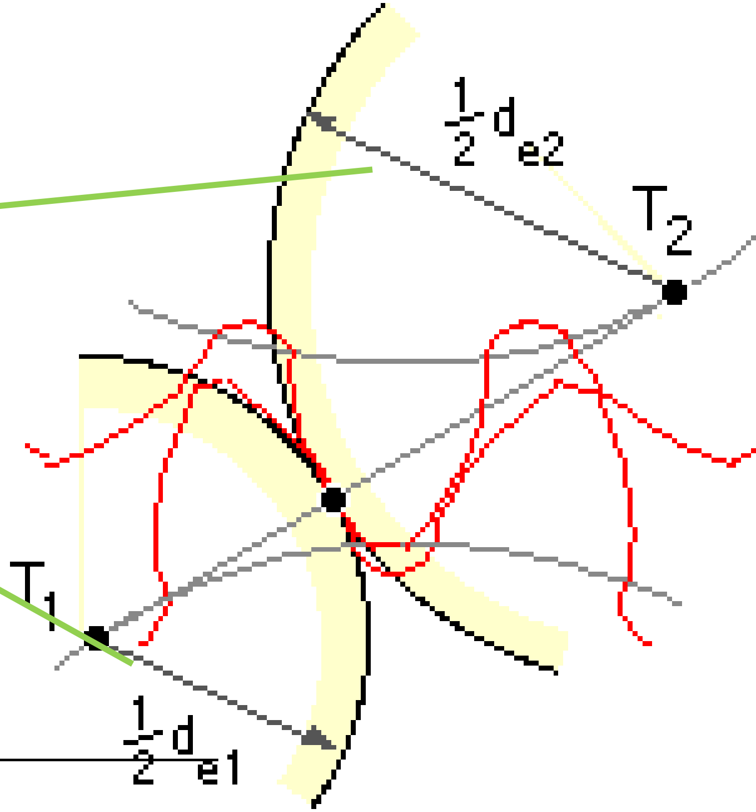
$$b = \sqrt{\frac{2F}{\pi l} \cdot \frac{\left[\left(1-v_1^2\right)/E_1\right]+\left[\left(1-v_2^2\right)/E_2\right]}{\left(1/d_1\right)+\left(1/d_2\right)}}$$

The maximum pressure is

$$p_{\max} = \frac{2F}{\pi b l}$$



$$b=\sqrt{\frac{2F}{\pi l}\cdot\frac{\left[\left(1-v_1^2\right)/E_1\right]+\left[\left(1-v_2^2\right)/E_2\right]}{\left(1/d_1\right)+\left(1/d_2\right)}}$$



For contacting gears at pitch point the contact stress is given as:

$$\sigma_H=-\sqrt{\frac{W_t}{Fd_p}\cdot\frac{1}{\pi\left(\frac{1-v_P^2}{E_P}+\frac{1-v_G^2}{E_G}\right)}\cdot\frac{1}{\frac{\cos\phi\cdot\sin\phi}{2}\cdot\frac{m_G}{m_G+1}}}$$

Let:

$$C_P=\sqrt{\frac{1}{\pi\left(\frac{1-v_P^2}{E_P}+\frac{1-v_G^2}{E_G}\right)}}$$

and

$$I=\frac{\cos\phi\cdot\sin\phi}{2}\cdot\frac{m_G}{m_G+1}$$

With this simplification, and the addition of a velocity factor C_v, then equation can be written as

$$\sigma_H = -C_P \sqrt{\frac{W_t}{C_v F d_p I}}$$

This is the Hertzian contact stress on teeth surfaces created by the tangential load W_t.

$$C_P = \sqrt{\frac{1}{\pi\left(\frac{1-\nu_P^2}{E_P} + \frac{1-\nu_G^2}{E_G}\right)}}$$

where the sign is negative because σ_H is a compressive stress.

This contact stress (σ_H) should be smaller than the surface strength (S_H) of the gear material for a safe design ($\sigma_H < S_H$)

$$S_H = \frac{C_L C_H}{C_T C_R} S_C \qquad S_C = 2.76HB - 70 \; MPa$$

where

- S_H corrected surface fatigue strength, or Hertzian strength
- C_L life factor
- C_H hardness-ratio factor; use 1.0 for spur gears
- C_T temperature factor; use 1.0 for temperatures less than 120°C
- C_R reliability factor

$$W_{t,p} = n_G \times W_t$$

$$n_G = C_o \times C_m \times n$$

$$\sigma_H = -C_P \sqrt{\frac{W_t}{C_v \times F \times d_p \times I}}$$

In contact stress analysis, safety factor is not used as the ratio of strength to stress but rather used as a load safety factor to increase the applied load W_t

Therefore contact stress equation is re-written as:

$$W_{tp} = n_G \times W_t \quad and \quad \sigma_H = S_H$$

$$S_H = C_P \sqrt{\frac{W_{t\,p}}{C_v \times F \times d_p \times I}}$$

Table 13-11 VALUES OF THE ELASTIC COEFFICIENT C_p FOR SPUR AND HELICAL GEARS WITH NONLOCALIZED CONTACT AND FOR $\nu = 0.30$

The units of C_p are $(\text{MPa})^{1/2}$.

Pinion	Modulus of elasticity E, GPa	Gear: Steel	Gear: Malleable iron	Gear: Nodular iron	Gear: Cast iron	Gear: Aluminum bronze	Gear: Tin bronze
Steel	200	191	181	179	174	162	158
Mall. iron	170	181	174	172	168	158	154
Nod. iron	170	179	172	170	166	156	152
Cast iron	150	174	168	166	163	154	149
Al bronze	120	162	158	156	154	145	141
Tin bronze	110	158	154	152	149	141	137

In case of contact stresses,

$C_v = K_v$ (as in the case of bending stress)

$C_o = K_o$ and $C_m = K_m$ (Tables 13-9 &10)

I is called the geometry factor and given as:

$$C_v = K_v = \frac{a}{a+V}$$

$$C_v = K_v = \frac{b}{b+V}$$

$$C_v = K_v = \sqrt{\frac{c}{c+\sqrt{200V}}}$$

$$I = \frac{\cos\phi \times \sin\phi}{2} \times \frac{m_G}{m_G+1}$$

where m_G is the gear ratio (T_G/T_P)

Factors C_L and C_R are given in Table 13-12

Table 13-12 LIFE AND RELIABILITY MODIFICATION FACTORS

Cycles of life	Life factor C_L	Reliability R	Reliability factor C_R
10^4	1.5	Up to 0.99	0.80
10^5	1.3	0.99 to 0.999	1.00
10^6	1.1	0.999 up	1.25 up
10^8 up	1.0		

EXAMPLE: 5.6

A Spur gear set with 14T precision made pinion is to drive a 21T gear.

A module of 3 mm is selected with a 58 mm face width based upon a 25^{o} pressure angle and dedendum of 1.25 m.

20 kW is to be trasmitted at pinion speed of 1150 rpm, under steady load conditions.

The material selected is forged BS080M40 steel heat treated (HT) to a hardness of 235 BHN. (21 HRC)

Determine the <u>factor of safety</u>

i) guarding against a bending fatigue failure; for 99 % reliability with better-than -average mountings and cutting accuracy.

ii) guarding against contact fatigue failure

Given:

$$T_P = 14$$
$$T_G = 21$$
$$m = 3\ mm$$
$$F = 58\ mm$$
$$\phi = 25^o$$
$$b = 1.25 \times \text{m}$$
$$N_P = 1150\ rpm$$
$$P = 20\ kW$$
$$R = 99\ \%$$

- Material- (appendix)
- Steady load condition and
- better than average mounting
- BHN= 235

$$S_{ut} = 775\ MPa$$
$$\rightarrow K_o = 1.0 \quad \text{Table 13.9}$$
$$\rightarrow K_m = 1.4 \quad \text{Table 13.10}$$

$$S_e = k_a \times k_b \times \cdots \times k_f \times S_e^{'}$$
$$k_a = 0.70 \quad \text{Fig. 13.25}$$
$$k_b = 0.956 \quad \text{Table 13.7}$$
$$k_c = 0.814 \quad \text{Table 13.8}$$
$$k_d = 1.0$$
$$k_e = 1.0$$
$$k_f = 1.33 \quad \text{Fig. 13.26}$$
$$S_e^{'} = 0.5 S_{ut} = 0.5 \times 775 = 387.5\ MPa$$
$$S_e = 0.7 \times 0.956 \times 0.814 \times 1.0 \times 1.0 \times 1.33 \times 387.5$$
$$S_e = 280\ MPa$$

$$n = \frac{n_G}{K_o \times K_m} \quad \rightarrow \quad n_G = \frac{S_e}{\sigma}$$

Required

n=? Based on bending

$$\sigma = \frac{W_t}{F \times m \times J \times K_v}$$

$$W_t = \frac{P}{V} = \frac{60 \times P}{\pi \times d \times N}$$

$$n_G = \frac{S_e}{\sigma} = ? \quad n = \frac{n_G}{K_o \times K_m} = ?$$

$$W_t = \frac{60 \times 20000}{\pi (3 \times 14 \times 10^{-3}) \times 1150}$$

$$W_t = 7908\ N$$

$$J \cong 0.36256 \quad \text{(Table13.5 interpolation)}$$

$$For \rightarrow T_P = 14; \quad and \quad T_G = 21$$

	1	17	21 (X)	25	35
13					
14 →			Y		
15					
16					

$$y = 0.35924 + (0.36587 - 0.35924)\frac{21-17}{25-17}$$

$$y = 0.36256$$

$$V = \frac{\pi \times d \times N}{60} = \frac{\pi \times 0.042 \times 1150}{60} = 2.529\ m/s$$

$$K_v = \frac{78}{78 + \sqrt{200V}} \quad \text{precision made}$$

$$K_v = 0.881$$

$$\sigma = \frac{7908}{58 \times 3 \times 0.36256 \times 0.881}$$

$$\sigma = 142.28 MPa$$

$$n_G = \frac{S_e}{\sigma} = \frac{280}{142.28} = 1.968$$

$$n_b = \frac{n_G}{K_o \times K_m} = \frac{1.968}{1.0 \times 1.4} = 1.406 \geq 1.0$$

It is safe, based on bending stress

ii) n=? Based on contact stress

$$W_{t\,p} = W_t \times n_G = W_t \times (n \times C_o \times C_m) \rightarrow n = \frac{W_{t\,p}}{W_t \times C_o \times C_m} = ?$$

$C_0 = K_0 = 1.0$

$C_m = K_m = 1.4$

$$W_t = ? \rightarrow = \frac{60 \times P}{\pi \times d \times N} = \frac{60 \times 20}{\pi (3 \times 14 \times 10^{-3}) \times 1150} = 7.908\ kN$$

as already calculated in bending stress condition

$$S_H = C_P \sqrt{\frac{W_{t\,p}}{C_v \times F \times d_p \times I}} = ? \quad S_H = \frac{C_L C_H}{C_T C_R} S_C$$

S_H → corrected surface fatigue strength

$C_L = 1.0$	Life factor (Tbl. 13.12)
$C_R = 1.0$	Rel. factor(Tbl. 13.12)
$C_H = 1.0$	Hard.-ratio factor
$C_T = 1.0$	Temp. factor <120°C

$$W_{t\,p} = \left(\frac{S_H}{C_p} \right)^2 \times C_v \times F \times d_p \times I = ?$$

$$S_C = 2.76 HB - 70\ MPa$$

$$S_C = (2.76 \times 235) - 70 = 578.6\ MPa$$

$$S_H = \frac{1.0 \times 1.0}{1.0 \times 1.0} \times 578.6 = 578.6\ MPa$$

ii)n=? Based on contact stress

$$C_p = ? \quad Table\, 13.11 \quad steel\ on\ steel = 191 \;\; MPa$$

$$C_v = K_v = 0.881 \quad already\ calculated$$

$$F = 58\ mm$$

$$d_p = 3\times 14 = 42\ mm$$

$$I = \frac{\cos\phi\times\sin\phi}{2}\times\frac{m_G}{m_G+1} = \frac{\cos 25\times\sin 25}{2}\cdot\frac{1.5}{1.5+1}$$

$$I = 0.115$$

$$\phi = 25^o \quad m_G = \frac{T_G}{T_P} = \frac{21}{14} = 1.5$$

Table 13-11 VALUES OF THE ELASTIC COEFFICIENT C_p FOR SPUR AND HELICAL GEARS WITH NONLOCALIZED CONTACT AND FOR $\nu = 0.30$

The units of C_p are $(\text{MPa})^{1/2}$.

Pinion	Modulus of elasticity E, GPa	Gear: Steel	Malleable iron	Nodular iron	Cast iron	Aluminum bronze	Tin bronze
Steel	200	191	181	179	174	162	158
Mall. iron	170	181	174	172	168	158	154
Nod. iron	170	179	172	170	166	156	152
Cast iron	150	174	168	166	163	154	149
Al. bronze	120	162	158	156	154	145	141
Tin bronze	110	158	154	152	149	141	137

Table 13-12 LIFE AND RELIABILITY MODIFICATION FACTORS

Cycles of life	Life factor C_L	Reliability R	Reliability factor C_R
10^4	1.5	Up to 0.99	0.80
10^5	1.3	0.99 to 0.999	1.00
10^6	1.1	0.999 up	1.25 up
10^8 up	1.0		

$$W_{t\,p} = \left(\frac{S_H}{C_p}\right)^2\times C_v\times F\times d_p\times I = \left(\frac{578.6}{191}\right)^2\times 0.881\times 58\times 42\times 0.115$$

$$W_{t\,p} = 2265\ N = 2.265\ kN$$

$$W_{t\,p} = W_t\times n_G = W_t\times(n\times C_o\times C_m)\rightarrow n = \frac{W_{t\,p}}{W_t\times C_o\times C_m} = ?$$

$$n_c = \frac{W_{t\,p}}{W_t\times C_o\times C_m} = \frac{2.265}{7.908\times 1.0\times 1.4} = 0.2 < 1.0$$

$$n_G = \frac{S_e}{\sigma} = \frac{280}{142.28} = 1.968$$

$$n_b = \frac{n_G}{K_o K_m} = \frac{1.968}{1.0\times 1.4} = 1.406 \geq 1.0$$

It is not safe,
based on contact stresses,

It is safe, based on bending stress

Note if we increase the hardness from 235 BHN to 500 BHN. What hapens?

$$S_C = 2.76HB - 70\ MPa$$

$$S_C = (2.76 \times 500) - 70 = 1310\ MPa$$

$$S_H = \frac{1.0 \times 1.0}{1.0 \times 1.0} \times 1310 = 1310\ MPa$$

$$W_{t\,p} = \left(\frac{S_H}{C_p}\right)^2 \times C_v \times F \times d_p \times I = \left(\frac{1310}{191}\right)^2 \times 0.881 \times 58 \times 42 \times 0.115$$

$$W_{t\,p} = 11609.9\ N = 11.61\,kN$$

$$n_c = \frac{W_{t\,p}}{W_t \times C_o \times C_m} = \frac{11.61}{7.908 \times 1.0 \times 1.4} = 1.05 > 1.0$$

Now, It is safe also based on <u>contact stresses</u>

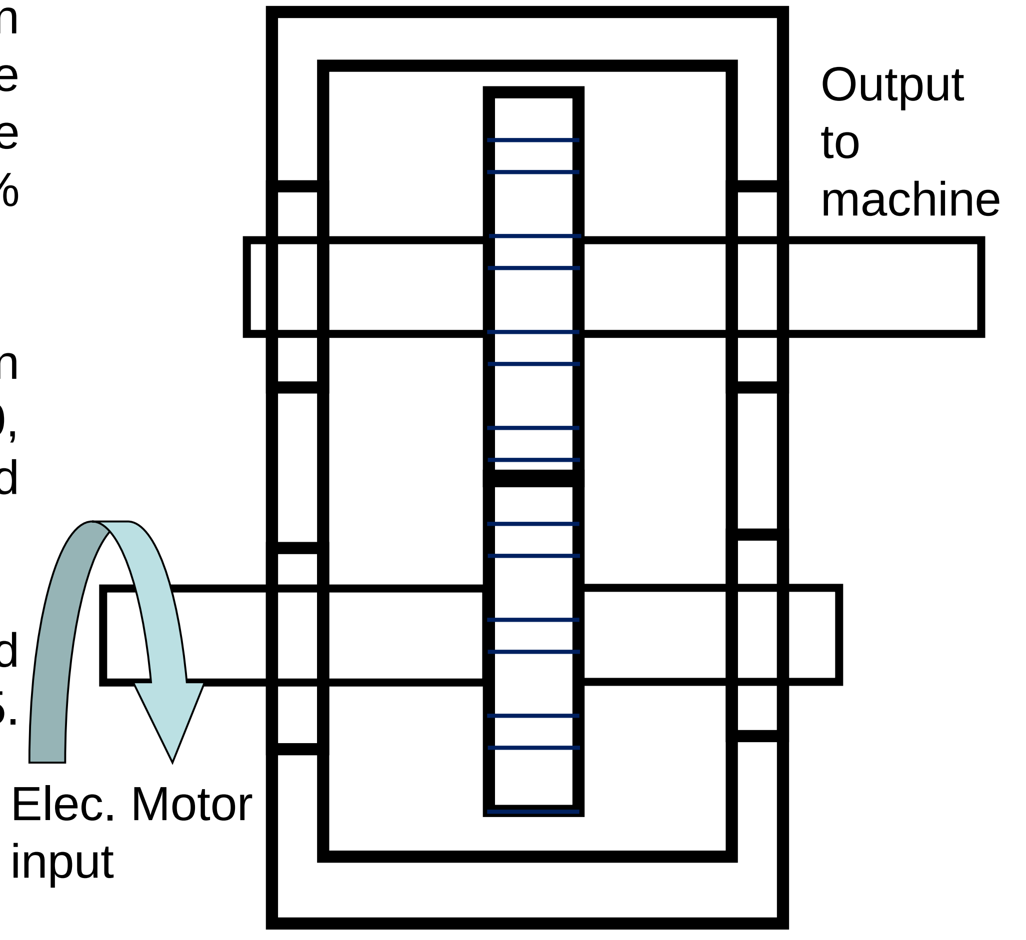
EXAMPLE: 5.7

In the reduction unit shown, the pinion shaft is driven by a 7.5 kW motor by means of a coupling. The motor speed is 1200 rpm and the output shaft is to rotate at 400 rpm.

a) Assuming that the gear and the pinion are made of UN5G10500 cold drawn steel (with S_{ut}=*689 MPa*) design the gear and the pinion based on the bending fatigue failure with 90 % relaibility.

b) Assuming that the gear and the pinion are made of UN5G10500 at HB=310, design both gear and the pinion based on the surface durability.

For both cases the load is steady and continious and the factor of safety is 1.5. Also state any assumptions you use.



SOLUTION:

Assume that: 1) Ø= 20°

2) full depth gear a=1.0 m, b=1.25 m,

3) T_{min}= *18 teeth*

Designing a gear means :

Determine: $i)\ T_P = ? \quad T_G = ? \rightarrow \ for \quad T_P = T_{\min} = 18 \rightarrow \quad T_G = \frac{1200}{400} \times T_P = 54$

$ii)\ m = ? \rightarrow d_P = ?, \quad d_G = ?$

$iii)\ F = ?$

a) Based on bending fatigue failure:

We have 2 criteria : 1) $n_G = \frac{S_e}{\sigma}$

2) $3p_c \le F \le 5p_c$

$n_G = ? \quad n \times K_o \times K_m$

$S_e = ? \quad k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \times S_e'$

$\sigma = ?$

$$\sigma = \frac{W_t}{K_v \times F \times J \times m}$$

Since most parameters are dependent as more than 1 variable (m, F, T) we have to use trial and error method based on 2 criteria

$$n_G = \frac{S_e}{\sigma}$$

$$3p_c \le F \le 5p_c$$

$$W_t = \frac{60 \times P}{\pi \times d \times N}$$

Since both pinion and gear are made of the same material with same S_{ut} design will be based on pinion since it rotates more.

$$d = T \times m, \quad V = \frac{\pi \times d \times N}{60}$$

$$\sigma = \frac{S_e}{n_G}, \quad F = \frac{W_t}{\sigma \times J \times m \times K_v}$$

$$3p_c = 3 \times \pi \times m$$

$$5p_c = 5 \times \pi \times m$$

$$K_v = \frac{50}{50 + \sqrt{200V}}$$

For hobbed teeth

$$\sigma = \frac{W_t}{F \times m \times J \times K_v}$$

$$y = 0.34404 + (0.35050 - 0.34404)\frac{54-50}{85-50}$$

$$y = 0.34478$$

$$W_t = \frac{P}{V}$$

$$n_G = n \times K_o \times K_m = 1.5 \times 1.0 \times 1.3 = 1.95$$

$$n = 1.5$$

$K_o = 1.0$ uniform loading

$K_m = 1.3$ assuming $F \leq 50$ mm and accurate mounting

$J \cong 0.34478$ for 18T $\rightarrow$ 54T Table 13.4

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \times S_e'$$

$$S_e = 0.72 \times k_b \times 0.897 \times 1.0 \times 1.0 \times 1.33 \times (0.5 \times 689)$$

$$S_e = k_b \times 296.0\ MPa \quad (k_b\ dependent)$$

$$\sigma = \frac{S_e}{n_G} = \frac{296.0 \times k_b}{1.95} = 151.79 \times k_b\ MPa$$

m (mm)	d_p (mm)	k_b	V (m/sec)	K_v	W_t (N)	σ *(MPa)*	$3p_c$	F	$5p_c$	Notes
3	54	0.956	3.392	0.657	2211.0	145.28	28.27	22.38	47.12	Not Suitable
4	72	0.930	4.523	0.624	1658.2	141.33	37.7	26.2	62.83	Get worse to the left
2	36	1.000	2.262	0.701	3315.0	151.79	18.85	45.1	31.4	To the right and not suitable
2.5	45	0.974	2.827	0.677	2653.0	148.01	23.56	30.7	39.26	SUITABLE

assuming $F \leq 50$ mm *is correct*

Thus, based on bending stress

$$m = 2.5\ mm$$

$$T_P = 18 \quad d_P = 45$$

$$T_G = 54 \quad d_G = 135$$

$$F = 31 - 35\ mm$$

b) Based on surface durability we have 2 criteria:

$$W_{tp} = W_t \times n_G$$

1)

$$S_H = -C_P \sqrt{\frac{W_{tp}}{C_v \times F \times d_p \times I}} \quad or \quad F = \frac{W_{tp}}{\left(\frac{S_H}{C_p}\right)^2 C_v \times d_p \times I}$$

2) $3p_c < F < 5p_c$

$$3p_c = 3 \times \pi \times m$$

$$5p_c = 5 \times \pi \times m$$

T_P	T_G	m (mm)	d_p (mm)	V (m/sec	C_v	W_t (N)	W_{tp} (N)	$3p_c$	F	$5p_c$	Notes
18	54	2	36	2.262	0.701	3315	6464.25	18.85	100.5	31.40	NO
		4	72	4.523	0.624	1658	3233.00	37.69	25.44	62.83	NO
		3	54	3.392	0.657	2211	4311.50	28.27	42.90	47.12	OK
		3.5	63	3.958	0.640	1895	3695.00	33.00	32.30	55	OK

assuming $F \leq 50$ mm *is correct*

Assume *F<50 mm*

$$C_m = 1.3$$

$$C_o = 1.0$$

$$n = 1.5$$

$$n_G = 1.95$$

$$S_H = \frac{C_L \times C_H}{C_T \times C_R} \times S_C = \frac{1.0 \times 1.0}{1.0 \times 0.8}(2.76 \times 310 - 70) = 982\ MPa$$

$$C_p = 191 \text{ steel on steel}$$

$$F = \frac{W_{tp}}{\left(\frac{S_H}{C_p}\right)^2 C_v \times d_p \times I} = \frac{4310}{\left(\frac{982}{191}\right)^2 0.657 \times 54 \times 0.107} = 42.9\ mm$$

$$I = \frac{\cos\phi \times \sin\phi}{2} \times \frac{m_G}{m_G + 1} = \frac{\cos 20 \times \sin 20}{2} \times \frac{3.0}{3.0 + 1}$$

$$I = 0.107$$

$$W_{t,p} = \frac{60 \times P}{\pi\ d\ N} \times n_G = \frac{60 \times 7500}{\pi \times 0.054 \times 1200} \times 1.95 = 4310\ N$$

$$W_{t\,p} = W_t \times n_G$$

Results

$$m = 3.0$$

$$T_P = 18 \quad \rightarrow \quad d_P = 54\ mm$$

$$T_G = 54 \quad \rightarrow \quad d_G = 162\ mm$$

$$F = 43 - 47\ mm$$

EXAMPLE: 5.8

A set of spur gears (pinion cut from hot rolled *BS 08040M* steel with *HB 180* and gear cut from *BS 220* cast-iron) is to be designed to transmit *1.25 kW* at a pinion speed of *400 rpm* and a speed reduction of *1.5:1*.

Use a safety factor of 4 and determine suitable values for:

- module:
- Pitch diameters:
- Tooth numbers:
- Face width of the pinion and gear.

Use 20^o full-depth teeth with $b=1.25\ m$ and make necessary assumptions if required.

SOLUTION: Two criteria:

1) Bending stress fatigue (for pinion and gear)
2) Contact stress fatigue (for pinion and gear)

4 total analysis
Or designs

Bending stress:

$$S_{ut_{Pin}} = 550\ MPa$$

$$S_{ut_{Gear}} = 220\ MPa$$

$$n_G = \frac{S_e}{\sigma} = \frac{k_a \times k_b \times \cdots S_e^{'}}{\dfrac{W_t}{F \times K_v \times J \times m}} = k_a \times k_b \times \cdots S_{ut} / 2$$

Gear with S_{utGear}*= 220 MPa* (<S_{utPin}*= 550 MPa*) is more critical than the pinion. Thus base the fatigue bending design on gear.

Contact stress:

$$n_G = \frac{W_{t\,p}}{W_t} \quad or \quad \frac{1}{n_G} = \frac{\sigma_H}{S_H} = \frac{C_P \sqrt{\dfrac{W_{tp}}{C_v F d_p I}}}{\dfrac{C_L C_H}{C_T C_R} S_C}$$

$$HB_{pin} = 180$$

$$HB_{gear} = 196$$

$$n_G = \frac{\dfrac{C_L C_H}{C_T C_R}(2.76HB - 70)}{C_P \sqrt{\dfrac{W_{tp}}{C_v F d_p I}}}$$

$$S_C = (2.76HB - 70)\ MPa$$

Since HB pinion is less than HB gear <u>the contact stress design has to be based on pinion.</u>
Also pinion rotates more.

Bending stress fatigue design of gear:

$$\sigma \le \frac{S_e}{n_G} \quad and \quad 3p_c < F < 5p_c$$

$$\sigma = \frac{W_t}{F \times J \times m \times K_v}$$

$$S_e = k_a \times k_b \times \cdots S_e^{'}$$

a) $$W_t = \frac{P(power)}{V(lin.vel.)} = \frac{1250\,watt}{\dfrac{\pi \times d_G \times N_G}{60}}$$

$$W_t = \frac{60 \times 1250}{\pi (T_G \times m) \times (400/1.5)} = \frac{60 \times 1250}{\pi (400) \times (T_P \times m)}$$

$$W_t = \frac{60 \times 1250}{\pi \times 400 \times T_P \times m}$$

Let $T_P = 18 \quad and \quad T_G = 27$

$$W_t = \frac{3.3157}{m} \quad N$$ m in meters

b) $$K_v = \frac{3}{3+\sqrt{V}} \quad \text{for a Cast Iron gear}$$

$$\mathrm{V} = \frac{\pi \times d \times N}{60} = \frac{\pi \times T_G \times m \times 266.67}{60} = 376.99\, m \quad m/\sec$$

m in meters

c) $J = 0.33339 \quad \text{from Table 13.4}$

$\phi = 20^o \quad b = 1.25m$

$$y = 0.33214 + (0.33840 - 0.33214)\frac{27-25}{35-25}$$

$$y = 0.33339$$

d) m is already unknown

e) F is also unknown but limited to $3p_c<F<5p_c$.

$$F = \frac{W_t}{\sigma \times K_v \times J \times m} \qquad \sigma = \frac{S_e}{n_G} = \frac{k_a \cdot k_b \cdots S_e^{'}}{n \times K_o \times K_m}$$

$$S_e^{'} = 0.5 S_{ut} \quad \text{if} \quad S_{ut} < 1400\ MPa$$

$$S_e^{'} = 0.5 \times 220 = 110\ MPa$$

$$k_a = 0.80 \quad \text{from the Fig.13.25}$$

$$k_b = f(m)$$

$$k_c = 0.897 \quad \text{for } 90\,\%\ \text{rel. (assumed)}$$

$$k_d = 1.0 \quad (\text{for} \quad \mathrm{T} < 350^{o}\ \mathrm{C})$$

$$k_e = 1.0$$

$$k_f = 1.33 \quad \text{from the Fig.13.26}$$

$$n_G = K_o \times K_m \times n;$$

Table 13.9 $\quad K_o = 1.5$ light mode shock

Table 13.10 $\quad K_m = 1.7$ F assumed as $50 < F < 150$

$$S_e = 0.80 \times k_b \times 0.897 \times 1.0 \times 1.0 \times 1.33 \times 110$$

$$S_e = 104.985 \times k_b\ MPa$$

$$\sigma \le \frac{S_e}{n_G}$$

$$n_G = 1.5 \times 1.7 \times 4 = 10.2$$

$$\sigma = \frac{104.985 \times k_b}{10.2}\ MPa$$

$$S_e = 104.985 \times k_b = 104.985 \times 1.0 = 104.985 MPa$$

$$\sigma \leq \frac{S_e}{n_G}$$

$$n_G = 1.5 \times 1.7 \times 4 = 10.2$$

$$V = 376.99\, m = 376.99 \times 0.002 = 0.754\, m/\sec$$

$$\sigma = \frac{104.985 \times k_b}{10.2}\, MPa$$

$$K_v = \frac{3}{3+\sqrt{V}} = \frac{3}{3+\sqrt{0.754}}$$

$$K_v = 0.776$$

$$\sigma = \frac{104.985 \times k_b}{10.2} = \frac{104.985 \times 1}{10.2} = 10.292\, MPa$$

$$W_t = \frac{3.3157}{m} = \frac{3.3157}{0.002} = 1657.85\, N$$

$$F = \frac{W_t}{\sigma \times K_v \times J \times m} = \frac{1657.85}{10.292 \times 0.776 \times 0.33334 \times 2} = 311.36\, mm$$

Since many parameters are dependent on module (m) we use tabulation method of

$$p_c = \pi \times m$$

m (mm)	k_b	V (m/sec)	K_v	W_t (N)	S_e (MPa)	σ (MPa)	$3p_c$	F (mm)	$5p_c$	**Notes**
2	1	0.754	0.776	1657.85	104.985	10.292	18.85	311.4	31.41	NOT GOOD
4	0.93	1.508	0.742	829	97.636	9.572	37.70	79.16	62.83	NOT GOOD
5	0.91	1.885	0.720	663.14	95.536	9.366	47.12	53.35	78.53	OKEY
4.5	0.92	1.696	0.73	737	96.586	9.4692	42.40	64.27	70.7	OKEY

Thus $T_P = 18$ $\quad d_P = T_P \times m = 90\ mm$

$T_G = 27$ $\quad d_G = T_G \times m = 135\ mm$

$n = 4.0$

$m = 5.0$

$F \cong 54\ mm \rightarrow K_m = 1.7$ is correct

assumption of $50 < F < 150$ is correct

Design will be satisfactory from the bending fatigue strength point of view considering the gear material.

Then pinion must be checked from the bending fatigue strength point of view considering the pinion material.

$$\sigma \leq \frac{S_e}{n_G}$$

Checking the pinion $(S_{ut} = 550\ MPa)$

$$n_G = K_o \times K_m \times n$$

$$S_e = 0.76 \times 0.91 \times 0.897 \times 1.0 \times 1.0 \times 1.33 \times \frac{550}{2} = 226.9\ MPa$$

$$\sigma = \frac{W_t}{K_v \times F \times J \times m} = \frac{663.15}{0.72 \times 54 \times (0.33334) \times 5.0} = 10.23\ MPa$$

$$V = 376.99\, m = 376.99 \times 0.005 = 1.885\ m/\sec$$

$$K_v = \frac{50}{50 + \sqrt{200V}} = \frac{50}{50 + \sqrt{200 \times 1.885}}$$

$$K_v = 0.72$$

$$n_G = \frac{226.9}{10.23} = 22.18$$

$$n = \frac{n_G}{K_o \times K_m} = \frac{22.18}{1.5 \times 1.7} = 8.70$$

$$n = 8.70 > 4$$

So pinion is safe also!

Contact stress design of pinion

Rather than re-designing the pinion, check the present design based on bending stress for the contact stress

$$n_G = \frac{W_{t\,p}}{W_t} = n \times C_0 \times C_m$$

$$S_H = \sigma_H = C_P \sqrt{\frac{W_{tp}}{C_v F d_p I}}$$

$$C_p = 174 \text{ Steel} - \text{Cast Iron}$$

$$C_v = K_v = 0.72$$

$$S_H = \frac{C_L \times C_H}{C_T \times C_R} \times S_C$$

C_L = 1.0

C_R = 0.8

C_H = 1.0

C_T = 1.0

$$S_C = 2.76HB - 70 \; MPa$$

$$S_C = (2.76 \times 180) - 70 = 426.8 \; MPa$$

$$S_H = \frac{1.0 \times 1.0}{1.0 \times 0.8} \times 426.8 = 533.5 \; MPa$$

$$F = 54 \; mm$$

$$d_p = T_P \times m = 18 \times 5 = 90 \; mm$$

$$I = \frac{\cos\phi \times \sin\phi}{2} \times \frac{m_G}{m_G + 1} = \frac{\cos 20 \times \sin 20}{2} \times \frac{1.5}{1.5 + 1}$$

$$I = 0.0964$$

$$W_{tp} = \left(\frac{S_H}{C_p}\right)^2 \times C_v \times F \times d_P \times I = \left(\frac{533.5}{174}\right)^2 \times 0.72 \times 54 \times 90 \times 0.0964$$

$$W_{tp} = 3171\,N$$

$$n_G = \frac{W_{t\,p}}{W_t} = n \times C_0 \times C_m$$

$$n_G = \frac{3171}{663.15} = 4.7804 = n \times C_0 \times C_m \qquad n = \frac{4.7804}{1.5 \times 1.7} = 1.875 < 4$$

So pinion is not safe in terms of contact stress for the data of

$$m = 5, \quad F = 54\,mm, \quad T_P = 18, \quad T_G = 27$$

Now re-design the pinion based on contact stress

$$S_H = C_P \sqrt{\frac{W_{tp}}{C_v \times F \times d_p \times I}} \quad and \quad 3p \le F \le 5p$$

$$W_{t\,p} = n \times C_0 \times C_m$$

$$C_0 = 1.5$$

$$C_m = 1.7 \quad 50 < F < 150$$

$$S_H = 533.5\,MPa$$

$$C_p = 174\,\text{Steel} - \text{Cast Iron}$$

$$I = 0.0964$$

$$d_p = m \times T_P = 18m$$

$$\mathrm{W}_t = \frac{60 \times P}{\pi \times d \times N} = \frac{60 \times P}{\pi \times (T_P \times m) \times N}$$

$$\mathrm{W}_t = \frac{3.3157}{m}\,N \quad \mathrm{W}_{tp} = \frac{33.82014}{m}$$

$$V = 0.377\ m$$

$$V = \frac{\pi \times d \times N}{60} = \frac{\pi \times (T \times m) \times N}{60}$$

$$C_v = \frac{50}{50 + \sqrt{200V}} \rightarrow C_v = \frac{50}{50 + \sqrt{200 \times 0.377m}}$$

$$F = \frac{W_{tp}}{\left(\frac{S_H}{C_p}\right)^2 C_v \times d_p \times I}$$

Use tabulation method

m (mm)	V (m/sec)	C_v	$W_{t,p}$ (N)	d_p (mm)	$3p_c$ (mm)	F (mm)	$5p_c$ (mm)	Notes
5	1.885	0.72	6764.0	90	47.0	115.00	78.00	Not Good
8	3.016	0.67	4227.5	144	75.4	48.35	125.60	Not Good
6	2.262	0.7015	5636.7	108	56.5	82.00	94.25	OKEY
7	2.359	0.697	4831.5	126	66.0	60.70	110.00	Not Good
3	1.131	0.768	11273.4	54	28.2	300.00	47.12	Not Good

$$3p_c = 3 \times \pi \times m$$

$$5p_c = 5 \times \pi \times m$$

$$W_t = \frac{60 \times P}{\pi \times d \times N} = \frac{60 \times P}{\pi \times (T_P \times m) \times N}$$

$$W_t = \frac{3.3157}{m}\ N \qquad W_{tp} = \frac{33.82014}{m}$$

$$m = 6\ mm$$

$$T_P = 18 \qquad d_P = 108\ mm$$

$$T_G = 27 \qquad d_G = 162\ mm$$

$$F = 82\ mm$$

Now check <u>gear</u> for bending strength fatigue

$$n_G = \frac{S_e}{\sigma} = n \times K_0 \times K_m$$

$$S_e = 0.80 \times 0.894 \times 0.897 \times 1.33 \times \frac{220}{2} = 93.85\ MPa$$

$$\sigma = \frac{W_t}{F \times K_V \times J \times m} = \frac{552.6}{82 \times 0.7015 \times 0.33334 \times 6}$$

$$\sigma = 4.8\ MPa$$

$$n_G = \frac{93.85}{4.8} = 19.55 = n \times 1.5 \times 1.7$$

$$n = \frac{19.55}{1.5 \times 1.7} = 7.67 > 4 \quad OKEY!$$

Satisfies bending stress fatigue conditions.

$$m = 6\ mm$$

$$T_P = 18 \qquad d_P = 108\ mm$$

$$T_G = 27 \qquad d_G = 162\ mm$$

$$F = 82\ mm$$

Also check gear surface durability

$$W_{tp} = \left(\frac{S_H}{C_p}\right)^2 C_v \times F \times d_P \times I$$

$$S_{H_G} = \frac{1.0\times1.0}{1.0\times0.8}(2.76\times196 - 70) = 588.7\ MPa$$

$$W_{tp} = \left(\frac{588.7}{174}\right)^2 \times 0.7015\times82\times108\times0.0964$$

$$W_{tp} = 6855.40\ N$$

$$n_G = \frac{93.85}{4.8} = 19.55 = n\times1.5\times1.7$$

$$n = \frac{19.55}{1.5\times1.7} = 7.67 > 4 \quad OKEY!$$

$$n_G = \frac{W_{tp}}{W_t} = \frac{6855.4}{552.6} = 12.41 = n\times1.5\times1.7$$

$$n = \frac{n_G}{C_o\times C_m} = \frac{12.41}{1.5\times1.7} = 4.87 > 4 \quad OKEY!$$

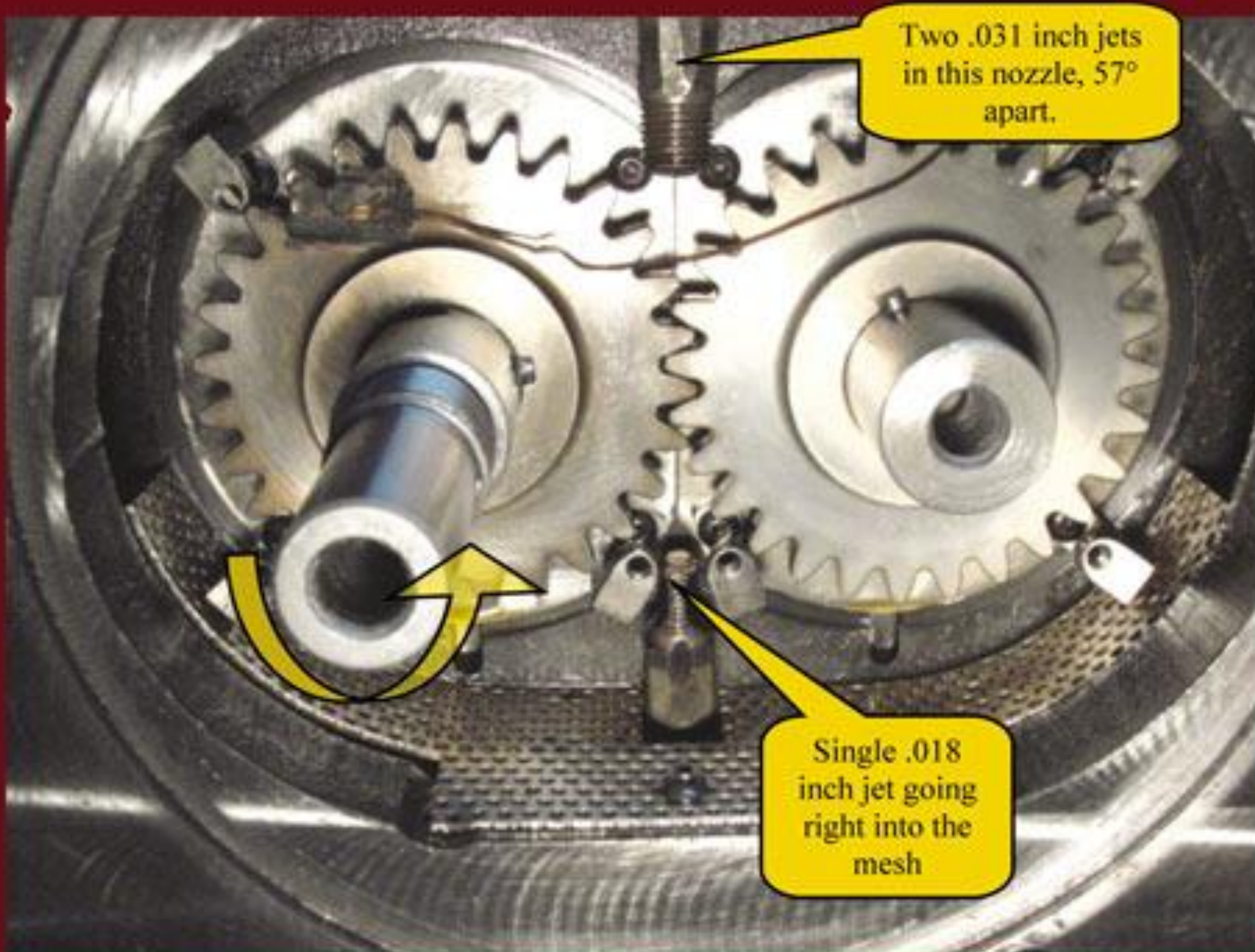
Satisfies both bending stress fatigue and contact stress conditions.

Table 13-9 OVERLOAD CORRECTION FACTOR K_o

	Driven machinery		
Source of power	**Uniform**	**Moderate shock**	**Heavy shock**
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Table 13-10 LOAD-DISTRIBUTION FACTOR K_m FOR SPUR GEARS

	Face width, mm			
Characteristics of support	**0 to 50**	**150**	**225**	**400 up**
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.3	1.4	1.5	1.8
Less rigid mountings, less accurate gears, contact across full face	1.6	1.7	1.8	2.2
Accuracy and mounting such that less than full-face contact exists	Over 2.2			


Two .031 inch jets in this nozzle, 57° apart.
Single .018 inch jet going right into the mesh

EXAMPLE: 5.8(New)

A set of spur gears (pinion cut from hot rolled *BS 08040M* steel with *HB 180* and gear cut from *BS 220* cast-iron) is to be designed to transmit *1.25 kW* at a pinion speed of *400 rpm* and a speed reduction of *1.5:1*.

Use a safety factor of 4 and determine suitable values for:

- module:
- Pitch diameters:
- Tooth numbers:
- Face width of the pinion and gear.

Use 20^o full-depth teeth with $b=1.25\ m$ and make necessary assumptions if required.

SOLUTION: Two criteria:

1) Bending stress fatigue (for pinion and gear)
2) Contact stress fatigue (for pinion and gear)

4 total analysis
Or designs

Bending stress:

$$n_G = \frac{S_e}{\sigma} = \frac{k_a \times k_b \times \cdots S_e^{'}}{\dfrac{W_t}{F \times K_v \times J \times m}} = k_a \times k_b \times \cdots S_{ut}/2$$

$$S_{ut_{Pin}} = 550\ MPa$$

$$S_{ut_{Gear}} = 220\ MPa$$

Gear with $S_{utGear} = 220\ MPa$ ($< S_{utPin} = 550\ MPa$) is more critical than the pinion. Thus base the fatigue bending design on gear.

Contact stress:

$$n_G = \frac{W_{tp}}{W_t} \quad or \quad \frac{1}{n_G} = \frac{\sigma_H}{S_H} = \frac{C_P\sqrt{\dfrac{W_{tp}}{C_v F d_p I}}}{\dfrac{C_L C_H}{C_T C_R} S_C}$$

$$HB_{pin} = 180$$

$$HB_{gear} = 196$$

$$n_G = \frac{\dfrac{C_L C_H}{C_T C_R}(2.76HB - 70)}{C_P\sqrt{\dfrac{W_{tp}}{C_v F d_p I}}}$$

$$S_C = (2.76HB - 70)\ MPa$$

Since HB pinion is less than HB gear <u>the contact stress design has to be based on pinion.</u>
Also pinion rotates more.

Bending stress fatigue design of gear:

$$\sigma \leq \frac{S_e}{n_G} \quad and \quad 3p_c < F < 5p_c$$

$$\sigma = \frac{W_t}{F \times J \times m \times K_v}$$

$$S_e = k_a \times k_b \times \cdots S_e^{'}$$

a) $W_t = \dfrac{P(power)}{V(lin.vel.)} = \dfrac{1250\ watt}{\dfrac{\pi \times d_G \times N_G}{60}}$

$$W_t = \frac{60 \times 1250}{\pi(T_G \times m) \times (400/1.5)} = \frac{60 \times 1250}{\pi(T_P \times m) \times (400)}$$

$$W_t = \frac{60 \times 1250}{\pi \times 400 \times T_P \times m}$$

Let $T_P = 20 \quad and \quad T_G = 30$

$$W_t = \frac{2.984}{m} \quad N \quad \text{(m in meters)}$$

b) $$K_v = \frac{3}{3+\sqrt{V}} \quad \text{for a Cast Iron gear}$$

$$\text{V} = \frac{\pi \times d \times N}{60} = \frac{\pi \times T_G \times m \times 266.67}{60} = 418.884\, m \quad m/\sec$$

m in meters

c) $$J = 0.33339 \quad \text{from Table 13.4}$$

$$\phi = 20^o \quad b = 1.25m$$

$$y = 0.34485 + (0.35176 - 0.34485)\frac{30-25}{35-25}$$

$$y = 0.3483$$

d) m is already unknown

e) F is also unknown but limited to $3p_c < F < 5p_c$.

$$F = \frac{W_t}{\sigma \times K_v \times J \times m} \qquad \sigma = \frac{S_e}{n_G} = \frac{k_a \cdot k_b \cdots S_e^{'}}{n \times K_o \times K_m}$$

$$S_e^{'} = 0.5 S_{ut} \quad \text{if} \quad S_{ut} < 1400\ MPa$$

$$S_e^{'} = 0.5 \times 220 = 110\ MPa$$

$$S_e = 0.80 \times k_b \times 0.897 \times 1.0 \times 1.0 \times 1.33 \times 110$$

$$S_e = 104.985 \times k_b\ MPa$$

$$k_a = 0.80 \quad \text{from the Fig. 13.25}$$

$$k_b = f(m)$$

$$k_c = 0.897 \quad \text{for } 90\,\% \text{ rel. (assumed)}$$

$$\sigma \le \frac{S_e}{n_G}$$

$$k_d = 1.0 \quad (\text{for} \quad \mathrm{T} < 350^{\mathrm{o}}\ \mathrm{C})$$

$$k_e = 1.0$$

$$k_f = 1.33 \quad \text{from the Fig. 13.26}$$

$$n_G = 1.5 \times 1.7 \times 4 = 10.2$$

$$\sigma = \frac{104.985 \times k_b}{10.2}\ MPa$$

$$n_G = K_o \times K_m \times n;$$

$$\text{Table 13.9} \qquad K_o = 1.5 \quad \text{light mode shock}$$

$$\text{Table 13.10} \qquad K_m = 1.7 \quad F \text{ assumed as } 50 < F < 150$$

$$S_e = 104.985 \times k_b = 104.985 \times 1.0 = 104.985 MPa$$

$$\sigma \le \frac{S_e}{n_G}$$

$$n_G = 1.5 \times 1.7 \times 4 = 10.2$$

$$V = 418.884\, m = 418.884 \times 0.002 = 0.838\ (m/\sec)$$

$$\sigma = \frac{104.985 \times k_b}{10.2}\ MPa$$

$$K_v = \frac{3}{3+\sqrt{V}} = \frac{3}{3+\sqrt{0.838}}$$

$$K_v = 0.7662$$

$$\sigma = \frac{104.985 \times k_b}{10.2} = \frac{104.985 \times 1}{10.2} = 10.292\ MPa$$

$$W_t = \frac{2.984}{m} = \frac{2.984}{0.002} = 1492.0\ N$$

$$F = \frac{W_t}{\sigma \times K_v \times J \times m} = \frac{1492.0}{10.292 \times 0.7662 \times 0.3483 \times 2} = 271.61\, mm$$

Since many parameters are dependent on module (m) we use tabulation method of

$$p_c = \pi \times m$$

m (mm)	k_b	V (m/sec)	K_v	W_t (N)	S_e (MPa)	σ (MPa)	$3p_c$	F (mm)	$5p_c$	**Notes**
2	1	0.838	0.766	1492.0	104.985	10.292	18.85	271.6	31.41	NOT GOOD
4	0.93	1.675	0.698	746.0	97.636	9.572	37.70	80.14	62.83	NOT GOOD
5	0.91	2.094	0.674	596.8	95.536	9.366	47.12	54.24	78.53	OKEY
4.5	0.92	1.696	0.73	737	96.586	9.4692	42.40	64.27	70.7	OKEY

Thus

$T_P = 20$ $\quad d_P = T_P \times m = 100\ mm$

$T_G = 30$ $\quad d_G = T_G \times m = 150\ mm$

$n = 4.0$

$m = 5.0$

$F \cong 55\ mm \rightarrow K_m = 1.7$ is correct

assumption of $50 < F < 150$ is correct

Design will be satisfactory from the bending fatigue strength point of view considering the gear material.

Then pinion must be checked from the bending fatigue strength point of view considering the pinion material.

$$\sigma \leq \frac{S_e}{n_G}$$

Checking the pinion $(S_{ut} = 550\ MPa)$

$$n_G = K_o \times K_m \times n$$

$$S_e = 0.76 \times 0.91 \times 0.897 \times 1.0 \times 1.0 \times 1.33 \times \frac{550}{2} = 226.9\ MPa$$

$$\sigma = \frac{W_t}{K_v \times F \times J \times m} = \frac{663.15}{0.72 \times 54 \times (0.33334) \times 5.0} = 10.23\ MPa$$

$$V = 376.99\ m = 376.99 \times 0.005 = 1.885\ m/\sec$$

$$K_v = \frac{50}{50 + \sqrt{200V}} = \frac{50}{50 + \sqrt{200 \times 1.885}}$$

$$K_v = 0.72$$

$$n_G = \frac{226.9}{10.23} = 22.18$$

$$n = \frac{n_G}{K_o \times K_m} = \frac{22.18}{1.5 \times 1.7} = 8.70$$

$$n = 8.70 > 4$$

So pinion is safe also!

Contact stress design of pinion

Rather than re-designing the pinion, check the present design based on bending stress for the contact stress

$$n_G = \frac{W_{t\,p}}{W_t} = n \times C_0 \times C_m$$

$$S_H = \sigma_H = C_P \sqrt{\frac{W_{tp}}{C_v F d_p I}}$$

$$C_p = 174 \text{ Steel} - \text{Cast Iron}$$

$$C_v = K_v = 0.72$$

$$S_H = \frac{C_L \times C_H}{C_T \times C_R} \times S_C$$

$C_L = 1.0$

$C_R = 0.8$

$C_H = 1.0$

$C_T = 1.0$

$$S_C = 2.76HB - 70\ MPa$$

$$S_C = (2.76 \times 180) - 70 = 426.8\ MPa$$

$$S_H = \frac{1.0 \times 1.0}{1.0 \times 0.8} \times 426.8 = 533.5\ MPa$$

$$F = 55\ mm$$

$$d_p = T_P \times m = 20 \times 5 = 100\ mm$$

$$I = \frac{\cos\phi \times \sin\phi}{2} \times \frac{m_G}{m_G + 1} = \frac{\cos 20 \times \sin 20}{2} \times \frac{1.5}{1.5 + 1}$$

$$I = 0.0964$$

$$W_{tp} = \left(\frac{S_H}{C_p}\right)^2 \times C_v \times F \times d_P \times I = \left(\frac{533.5}{174}\right)^2 \times 0.72 \times 54 \times 90 \times 0.0964$$

$$W_{tp} = 3171\, N$$

$$n_G = \frac{W_{t\,p}}{W_t} = n \times C_0 \times C_m$$

$$n_G = \frac{3171}{663.15} = 4.7804 = n \times C_0 \times C_m \qquad n = \frac{4.7804}{1.5 \times 1.7} = 1.875 < 4$$

So pinion is not safe in terms of contact stress for the data of

$$m = 5, \quad F = 54\, mm, \quad T_P = 18, \quad T_G = 27$$

<u>Now re-design the pinion based on contact stress</u>

$$S_H = C_P \sqrt{\frac{W_{tp}}{C_v \times F \times d_p \times I}} \quad and \quad 3p \le F \le 5p$$

$$W_{t\,p} = n \times C_0 \times C_m$$

$$C_0 = 1.5$$

$$C_m = 1.7 \quad 50 < F < 150$$

$$S_H = 533.5\, MPa$$

$$C_p = 174 \,\mathrm{Steel - Cast\ Iron}$$

$$I = 0.0964$$

$$d_p = m \times T_P = 20m$$

$$\mathrm{W}_t = \frac{60 \times P}{\pi \times d \times N} = \frac{60 \times P}{\pi \times (T_P \times m) \times N}$$

$$\mathrm{W}_t = \frac{3.3157}{m}\, N \qquad \mathrm{W}_{tp} = \frac{33.82014}{m}$$

$$V = 0.377\ m$$

$$V = \frac{\pi \times d \times N}{60} = \frac{\pi \times (T \times m) \times N}{60}$$

$$C_v = \frac{50}{50 + \sqrt{200V}} \rightarrow C_v = \frac{50}{50 + \sqrt{200 \times 0.377m}}$$

$$F = \frac{W_{tp}}{\left(\frac{S_H}{C_p}\right)^2 C_v \times d_p \times I}$$

<u>Use tabulation method</u>

m (mm)	V (m/sec)	C_v	$W_{t,p}$ (N)	d_p (mm)	$3p_c$ (mm)	F (mm)	$5p_c$ (mm)	Notes
5	1.885	0.72	6764.0	90	47.0	115.00	78.00	Not Good
8	3.016	0.67	4227.5	144	75.4	48.35	125.60	Not Good
6	2.262	0.7015	5636.7	108	56.5	82.00	94.25	OKEY
7	2.359	0.697	4831.5	126	66.0	60.70	110.00	Not Good
3	1.131	0.768	11273.4	54	28.2	300.00	47.12	Not Good

$$3p_c = 3 \times \pi \times m$$

$$5p_c = 5 \times \pi \times m$$

$$\mathrm{W}_t = \frac{60 \times P}{\pi \times d \times N} = \frac{60 \times P}{\pi \times (T_P \times m) \times N}$$

$$\mathrm{W}_t = \frac{3.3157}{m}\ N \qquad \mathrm{W}_{tp} = \frac{33.82014}{m}$$

$$m = 6\ mm$$

$$T_P = 18 \qquad d_P = 108\ mm$$

$$T_G = 27 \qquad d_G = 162\ mm$$

$$F = 82\ mm$$

Now check <u>gear</u> for bending strength fatigue

$$n_G = \frac{S_e}{\sigma} = n \times K_0 \times K_m$$

$$S_e = 0.80 \times 0.894 \times 0.897 \times 1.33 \times \frac{220}{2} = 93.85\ MPa$$

$$\sigma = \frac{W_t}{F \times K_V \times J \times m} = \frac{552.6}{82 \times 0.7015 \times 0.33334 \times 6}$$

$$\sigma = 4.8\ MPa$$

$$n_G = \frac{93.85}{4.8} = 19.55 = n \times 1.5 \times 1.7$$

$$n = \frac{19.55}{1.5 \times 1.7} = 7.67 > 4 \quad OKEY!$$

Satisfies bending stress fatigue conditions.

$m = 6\ mm$

$T_P = 18 \qquad d_P = 108\ mm$

$T_G = 27 \qquad d_G = 162\ mm$

$F = 82\ mm$

Also check gear surface durability

$$W_{tp} = \left(\frac{S_H}{C_p}\right)^2 C_v \times F \times d_P \times I$$

$$S_{H_G} = \frac{1.0 \times 1.0}{1.0 \times 0.8}(2.76 \times 196 - 70) = 588.7\ MPa$$

$$W_{tp} = \left(\frac{588.7}{174}\right)^2 \times 0.7015 \times 82 \times 108 \times 0.0964$$

$$W_{tp} = 6855.40\ N$$

$$n_G = \frac{93.85}{4.8} = 19.55 = n \times 1.5 \times 1.7$$

$$n = \frac{19.55}{1.5 \times 1.7} = 7.67 > 4 \quad OKEY!$$

$$n_G = \frac{W_{tp}}{W_t} = \frac{6855.4}{552.6} = 12.41 = n \times 1.5 \times 1.7$$

$$n = \frac{n_G}{C_o \times C_m} = \frac{12.41}{1.5 \times 1.7} = 4.87 > 4 \quad OKEY!$$

Satisfies both bending stress fatigue and contact stress conditions.