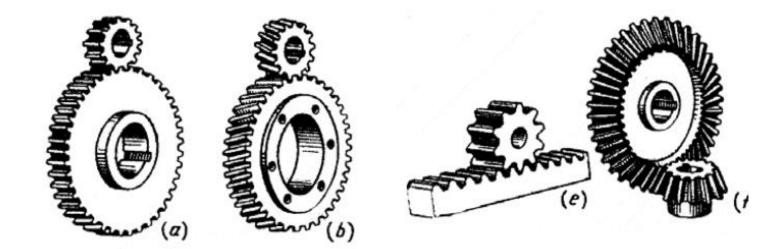
# ME 308 MACHINE ELEMENTS II







13.3.2019

## **5.8 GEAR TRAINS**

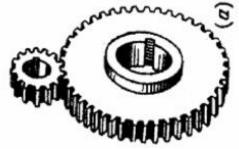
Gear trains are usually used to:

- increase speed and decrease torque
- decrease speed and increase torque
- change the direction of rotation.

A small gear driving a large gear is speed reducing gear train whereas a large gear driving a small gear is a speed increasing gear train

Gear trains are classified based on two criteria:

- number of stages of gear train
- positioning of the gears on shafts



There are two main types of gear trains based on number of stages

 Single stage gear train is the one where there is only one driving and one driven gear

2) <u>Multiple stage gear trains</u> have more gear pairs in mesh at the same time.



This is han a starge spear traines

# There are two main types of gear trains based on positioning of the gears on shafts

- 1) <u>Simple gear trains</u> is the one where there is only one gear on each shaft (the same gear becomes both driving and driven gear on intermediary shafts)
- 2) <u>compound gear trains</u> have multiples of gears on intermediary shafts.

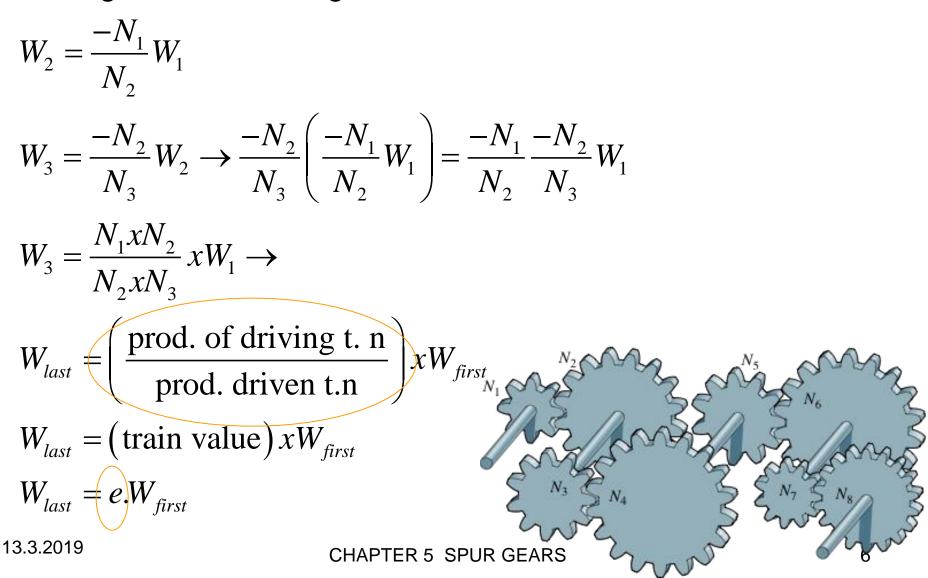
1) In single stage gear trains speed ratio is inversly proportional to the radii of the gears:

$$\frac{W_1}{W_2} = \frac{r_2}{r_1} = \frac{mN_2/2}{mN_1/2} = \frac{N_2}{N_1} = \frac{\text{driven gear t. no}}{\text{driving gear t .no}}$$
$$W_2 = \frac{N_1}{N_2} W_1 = \frac{\text{driving tooth no}}{\text{driven t. no}} x \text{driving speed}$$
$$W_{\text{last}} = \frac{-\text{driving t.no}}{\text{driven t.no}} x W_{\text{first}}$$

Now we also see that speed ratio is inversly proportional to the number of tooth of gears.



2) In multiple-stage trains speed ratio of each stage is inversly proportional to the (radii or) tooth numbers of the gears in that stage, hence



<u>A) For :</u>simple gear trains (gears of each one on one shaft)

$$W_{4} = \left(\frac{N_{1}}{N_{2}}\right) \left(\frac{N_{2}}{N_{3}}\right) \left(\frac{N_{3}}{N_{4}}\right) x W_{1} = \frac{-N_{1}}{N_{4}} W_{1}$$

$$W_{last} = \frac{\text{tooth no on first gear}}{\text{tooth no on last gear}} x W_{first} = \frac{N_{f}}{N_{L}} x W_{f}$$

$$W_{last} = e.W_{first} e=\text{train values}$$

B) For gears with more than one on each shaft (compound gear trains)

$$W_{6} = \frac{N_{1}xN_{3}xN_{5}}{N_{2}xN_{4}xN_{6}} xW_{1} = \frac{-N_{1}}{N_{2}} x \frac{-N_{3}}{N_{4}} x \frac{-N_{5}}{N_{6}} xW_{1} = -e.W_{f}$$

$$W_{last} = \frac{\text{prod. of driving teeth no}}{\text{prod. of driven teeth no}} xW_{first}$$

$$W_{13,3,2019} \qquad \text{CHAPTER 5 SPUR GEARS}$$

#### EXAMPLE: 5.2

Design a two stage compound spur gear train for an overall ratio of approximately 47:1

Specify tooth numbers for each gear in the train.

$$\frac{W_{first}}{W_{last}} = 47:1$$

$$\frac{n_i}{n_o} = 47 = e_1 x e_2 \text{ let } e_1 = e_2$$
$$e_1 = \sqrt{47} \cong 6.855(<10)$$

Train value must be smallar than 10 as given in Hand Books

a) for 20 degrees pressure angle gears T<sub>min</sub> is theoretically 18 tooth and practically 14

$$T_{1} = 18 \rightarrow T_{2} = e_{1}xT_{1} = 123.4?$$

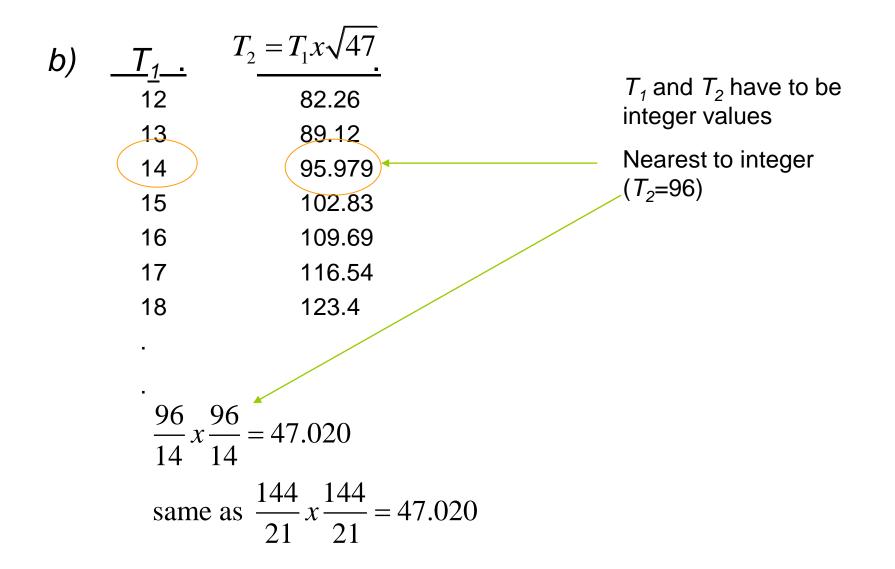
$$T_{1} = 19 \rightarrow T_{2} = 130.25?$$

$$T_{1} = 20 \rightarrow T_{2} = 137.1?$$

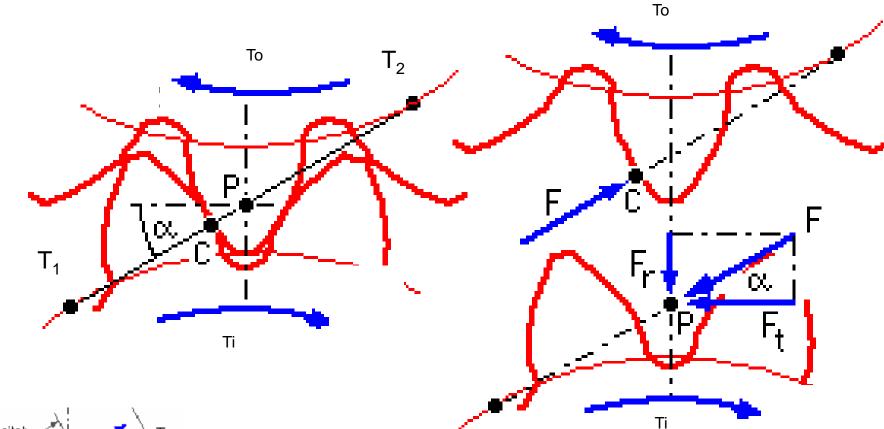
$$T_{1} = 21 \rightarrow T_{2} = 143.955 \cong 144$$
If  $T_{2} = 144$ 

$$T_{1} = 21 \rightarrow e_{1} = \frac{144}{21} = 6.857$$

$$e_{1}xe_{2} = 47.020$$
 acceptable



#### **5.9 FORCE ANALYSIS IN SPUR GEAR SETS**



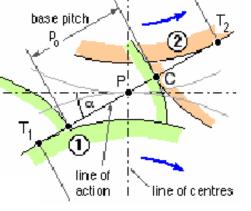
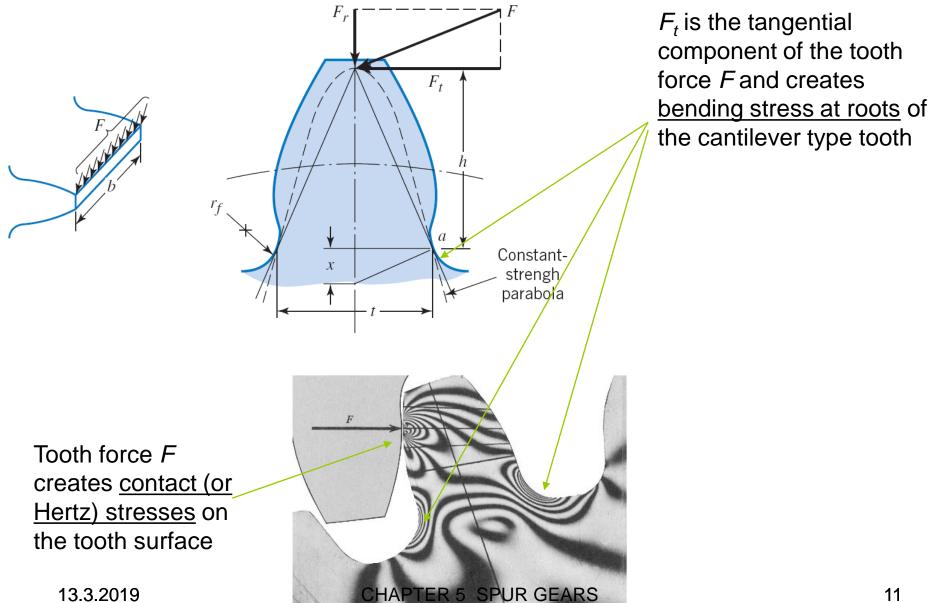


Fig. 3 Tooth forces

## **5.9 FORCE ANALYSIS IN SPUR GEAR SETS**



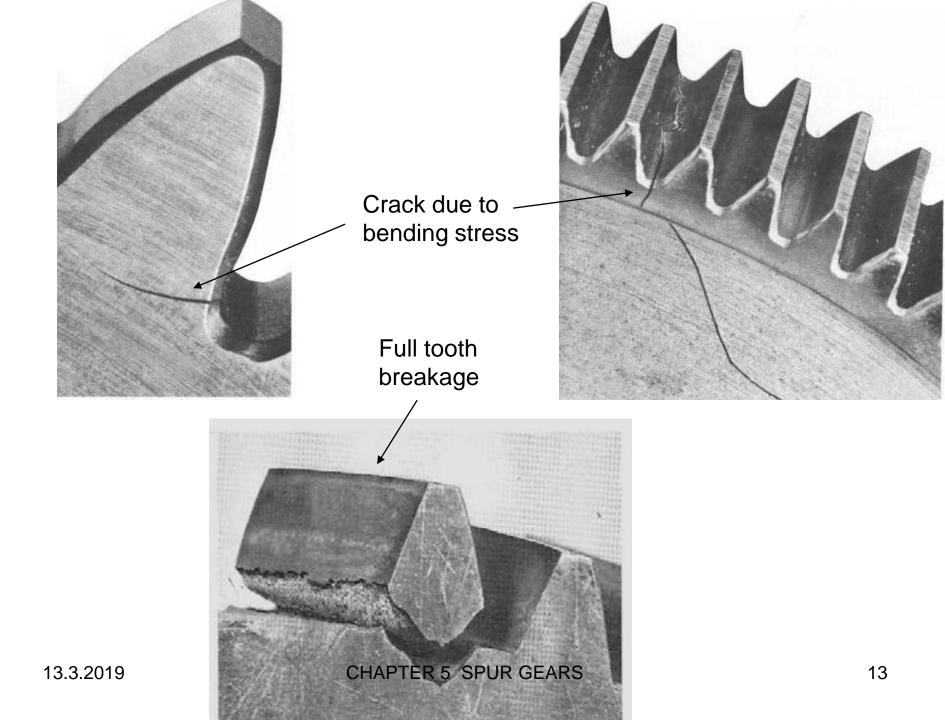
#### **5.10 TYPES OF FAILURES OF SPUR GEARS**

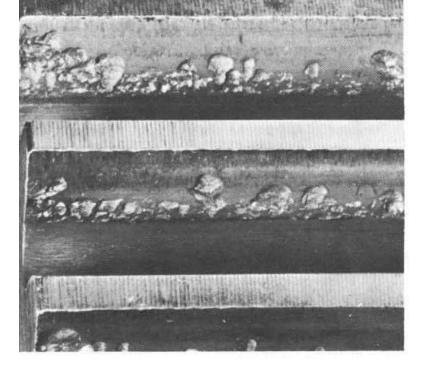
In terms of the design criteria of gears there are basically two important limiting design factors :

- Failure of tooth <u>due to bending stress</u> (static and fatigue)
- Failure of tooth surfaces due to contact stresses

Thus, in terms of strength of gear teeth, design is based on

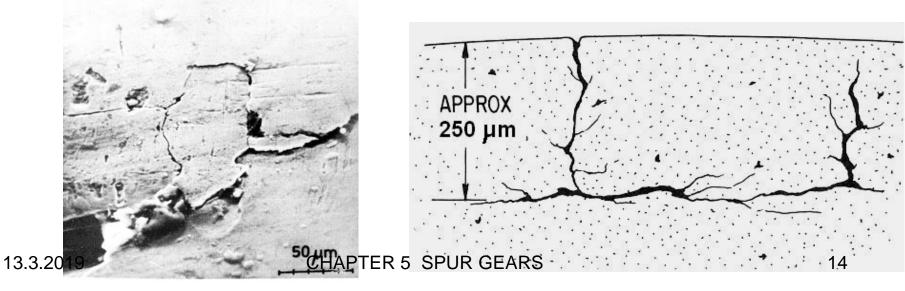
- Static (strength) failure due to bending stress
- Fatigue failure due to bending stress and
- Surface fatigue failure due to contact or Hertzian stresses







#### Surface failures due to contact stresses

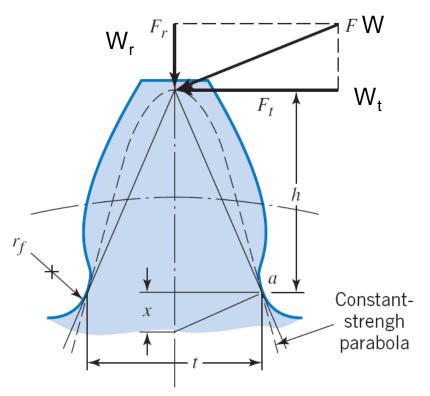




Failure due to bending stress

Failure due to contact stress-





## **5.11 LEWIS EQUATION**

Bending stress in tooth root (especially tensile stress) is created by the force component  $F_t(W_t)$  and calculated from general eqn:

*σ=M\*c/I* 

Here

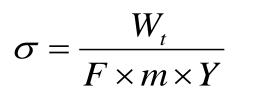
 $M = W_t^* h, c = t/2$  and  $I = F^* t^3/12$ 

*F* is the facewidth of tooth

Force  $F_r(W_r)$  does actually help reducing the tensile bending stress, but this is not taken into consideration and the likely worst condition is analysed.

Bending stress formula for gear tooth is first derived by LEWIS and his formula is given as:

$$\sigma = \frac{W_t}{F \times m \times Y}$$



In LEWIS equation,

- $W_t$  is the tangential load,
- *F* is the tooth face width,
- *m* is the module and
- Y is the modified form factor :

Lewis equation is used only for static design and is not accurate enough for dynamic conditions

- The factor Y is given in tables(Table 13.3) and assumes that the load is not shared between the teeth in contact and the maximum load happens at the tip of the teeth.
- However in actual case maximum load does not reach the tip of the tooth but stays somewhere below where the moment arm is not maximum.

Here again the likely worst condition is analysed.

Fable 13-3VALUES	5 OF	THE	LEWIS	FORM	FACTOR	Y
------------------	------	-----	-------	------	--------	---

Number of Teeth	Y	Number of Teeth	Ŷ	Number o teeth
12	0.245	28	0.353	12
13	0.261	30	0.359	13
14	0.277	34	0.371	14
15	0.290	38	0.384	15
16	0.296	43	0.397	16 17
17	0.303	50	0.409	18
18	0.309	60	0.422	19
19	0.314	75	0.435	20
20	0.322	100	0.447	21
21	0.328	150	0.460	22 24
22	0.331	300	0.472	24
24	0.337	400	0.480	28
26	0.346	Rack	0.485	30 34

#### Table 14–2

Values of the Lewis Form Factor Y (These Values Are for a Normal Pressure Angle of 20°, Full-Depth Teeth, and a Diametral Pitch of Unity in the Plane of Rotation) 13.3.2019

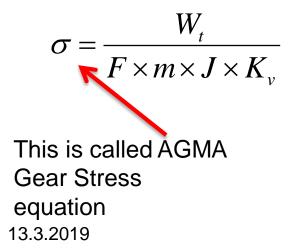
	$\phi = 20^{\circ}$	$\phi = 20^{\circ}$	$\phi = 25^{\circ}$	$\phi = 25^{\circ}$
mber of	$a=0.8m^*$	a = m	a = m	a = m
teeth	b = m	b=1.25m	b = 1.25m	$b = 1.35m^{\dagger}$
12	0.335 12	0.229 60	0.276 77	0.254 73
13	0.348 27	0.229 00	0.276 77	0.254 75
13				
	0.359 85	0.255 30	0.307 17	0.287 11
15	0.370 13	0.266 22	0.320 09	0.301 00
16	0.379 31	0.276 10	0.331 78	0.313 63
17	0.387 57	0.285 08	0.342 40	0.325 17
18	0.395 02	0.293 27	0.352 10	0.335 74
19	0.401 79	0.300 78	0.360 99	0.345 46
20	0.407 97	0.307 69	0.369 16	0.354 44
21	0.413 63	0.314 06	0.376 71	0.362 76
22	0.418 83	0.319 97	0.383 70	0.370 48
24	0.428 06	0.330 56	0.396 24	0.384 39
26	0.436 01	0.339 79	0.407 17	0.396 57
28	0.442 94	0.347 90	0.416 78	0.407 33
30	0.449 02	0.355 10	0.425 30	0.416 91
34	0.459 20	0.367 31	0.439 76	0.433 23
38	0.467 40	0.377 27	0.451 56	0.446 63
45	0.478 46	0.390 93	0.467 74	0.465 11
50	0.484 58	0.398 60	0.476 81	0.475 55
60	0.493 91	0.410 47	0.490 86	0.491 77
75	0.503 45	0.422 83	0.505 46	0.508 77
100	0.513 21	0.435 74	0.520 71	0.526 65
150	0.523 21	0.449 30	0.536 68	0.545 56
300	0.533 48	0.463 64	0.553 51	0.565 70
Rack	0.544 06	0.478 97	0.571 39	0.587 39

## 5.11.1 DYNAMIC EFFECTS

When the gears are run at high speeds small disturbances create high dynamic loads and stresses. Also at high speeds resonant speeds could be reached thus creating high vibrations hence high dynamic loads and stresses again.

To compensate for these cases a velocity factor,  $K_v$  (or called dynamic factor) is used in stress equation.

LEWIS's bending stress equation was further modified to include likely speed factor  $K_v$  for dynamic conditions and the bending stress concentration factor in the tooth root for fillet geometry and re-written as

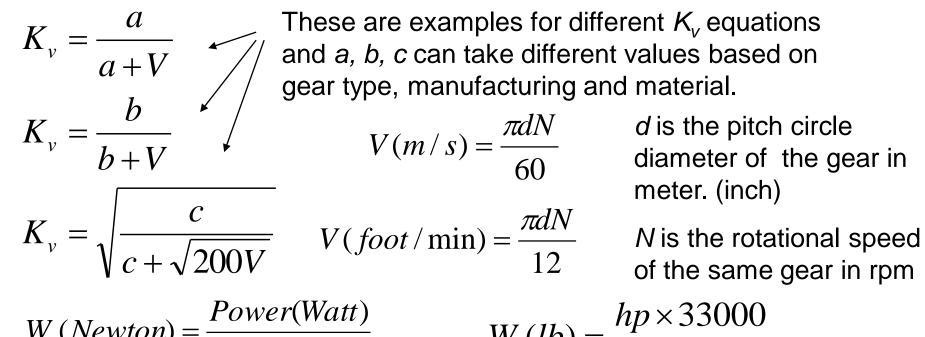


Here,

- $W_t$  is the tangential load,
  - *F* is the tooth face width,
  - *m* is the module,
  - J is the geometry factor and
  - $K_v$  is the speed factor:

V is the pitch line velocity of meshing gears used.

There are different  $K_{\nu}$  equations for different gear types and gear materials (like spur, helical, bevel etc. and cut or milled teeth, not carefully generated, finished by hobbing or shaping, high precision shaved or ground teeth, etc.)



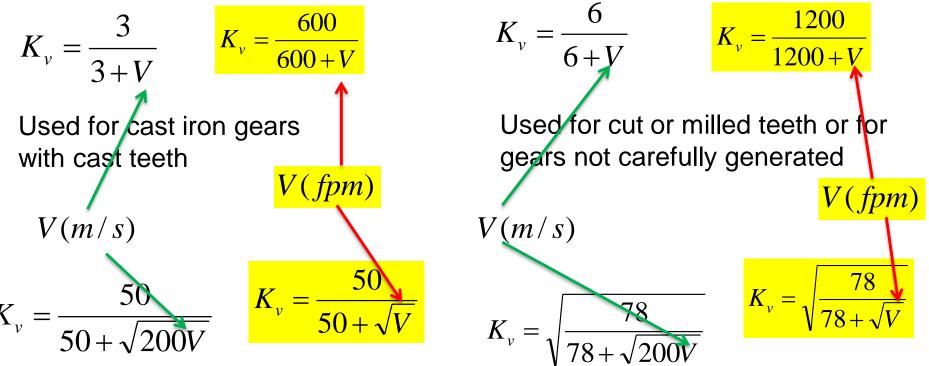
meter. (inch)

of the same gear in rpm

$$W_t(Newton) = \frac{Power(Watt)}{V(m/s)} \qquad \qquad W_t(lb) = \frac{hp \times 33000}{V(fpm)}$$

13.3.2019

 $K_v$  is first expressed by Carl G. Barth and called velocity factor, dynamic factor or Barth equation.



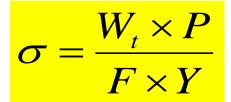
Teeth finished by hobbing or shaping

If the gears have high precision shaved or ground teeth

**Finally** 

$$\sigma = \frac{W_t}{F \times m \times Y}$$

**TT**7



LEWIS equation is used for quick estimate of gear size when there is no risk of fatigue failure of teeth (Y - from Table 13-3).

$$\sigma = \frac{W_t}{F \times m \times J \times K_v}$$

$$\sigma = \frac{W_t \times P}{F \times J \times K_v}$$

This is called AGMA equation and is (must be) used for analysis of gears when there is the risk of bending fatigue failure of teeth (J – from Table 13-4,5).

### **5.12 ESTIMATING GEAR SIZE**

For a pair of gear transmitting power or motion the given information generally are:

- -the power (watt) to be transmitted
- -the speed N (rpm) of the gear to be sized and
- -the speed ratio of the pair

The designer is then required to determine other parameters to satisfy a safe gear pair. For such a case, designer has to determine:

- -the number of teeth (T) on the gears to be sized
- -the Lewis form factor Y (Table 13-3) for the gears to be sized
- -the bending stress in the gear material (root bending stress)
- -the gear material and its specifications ( $S_{y}$ ,  $S_{ut}$ , etc.) and
- -the size of the gear specified by the parameters (diameter and Face width).

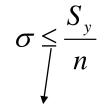
For a safe (failure-free) operation the bending stress in  $\sigma = \frac{W_t}{F \times m \times J \times K} \leq S_y$ the tooth root should be smaller than the strength of gear material that is:  $W_t$  can be found from power relation  $W_t(Newton) = \frac{Power(Watt)}{V(m/s)}$ V can be found linear pitch line velocity relation However, V depends on diameter which then  $\checkmark V(m/s) = \frac{\pi dN}{60}$ depends on tooth number and module both of which are already unknowns.  $> d = N_T \times m$ F is also not known yet and has to be determined by designer J can be found from Table if the tooth number is known (which is not determined yet)  $\rightarrow K_v = \frac{a}{a+V}$ <u> $K_v$  requires velocity V which was dependent on both</u> tooth number and module (as above again)  $S_{v}$  is material dependent and designer can determine it by selecting a proper material from catalogues CHAPTER 5 SPUR GEARS 13.3.2019 24

So many parameters depend on few unknowns like  $N_{T1}$ ,  $N_{T2}$ , m and F

 $N_{T1}$  and  $N_{T2}$  can be determined by speed ratio (by keeping in mind the undercut and minimum tooth number restriction (18 theoretically for 20 degree pressure angle).

Determining *m* and *F*, however, are not so easy, and an analytical solution is difficult to get. Rather an iteration technique of assuming different values *m* and *F* and then checking whether correct or not is more suitable (as in the case of springs and RCB's).

The checking criteria are usually the strength safety and the geometric suitability: (Use a safety factor more than 2 or sometimes 3 for gear size determination).



To prevent tooth root crack or breakage due to bending 13.3.2019 To prevent requirement of larger diameter gears

$$3p_c \leq F \leq 5p_c$$

To prevent mal-distribution of tooth load over face width

After determining the proper tooth numbers,  $\sigma = \frac{W_t}{F \times m \times J \times K_y} \leq S_y$ 

iteration technique starts as follows:

- 1) Assume a module *m*,
- Calculate pitch diameters ( $d_1$  and  $d_2$ ) 2)
- Calculate pitch line velocity V 3)
- $\longrightarrow W_t(Newton) = \frac{Power(Watt)}{V(m/s)}$ Calculate  $W_t$  from power relation 4)
- Calculate  $K_{\nu}$  from relation 5)
- Find Y or J-factors: from Tables 13-3 or 4,5 6) Calculate face width *F*, \_\_\_\_\_
- Check if *F* is in limits? 7)
- If YES, design (gear sizing) is finished 8)
- If NO, re-try another iteration until satisfied 9)

 $p_{c}$ 

 $3p_c \leq F \leq 5p_c$ 

 $\frac{W_t}{(\frac{S_y}{(-)}) \times m \times J \times K_v}$ 

 $d = N_T \times m$ 

 $V(m/s) = \frac{\pi dN}{60}$ 

 $\longrightarrow K_v = \frac{a}{a+V}$ 

**Table 13-4** AGMA GEOMETRY FACTOR J FOR TEETH HAVING  $\phi = 20^{\circ}$ , a = 1m, b = 1.25m, AND  $r_f = 0.300m$ 

Number of	Number of teeth in mating gear								
teeth	1	17	25	35	50	85	<b>300</b> ·	1000	
18	0.244 86	0.324 04	0.332 14	0.338 40	0.344 04	0.350 50	0.355 94	0.361 12	
19	0.247 94	0.330 29	0.338 78	0.345 37	0.351 34	0.358 22	0.353 94	0.369 63	
20	0.250 72	0.336 00	0.344 85	0.351 76	0.358 04	0.365 32	0.371 51	0.309 03	
21	0.253 23	0.341 24	0.350 44	0.357 64	0.364 22	0.371 86	0.378 41	0.384 75	
22	0.255 52	0.346 07	0.355 59	0.363 06	0.369 92	0.377 92	0.384 79	0.391 48	
24 -	0.259 51	0.354 68	0.364 77	0.372 75	0.380 12	0.388 77	0.396 26	0.403 60	
26	0.262 89	0.362 11	0.372 72	0.381 15	0.388 97	0.398 21	0.406 25	0.414 18	
-28	0.265 80	0.368 60	0.379 67	0.388 51	0.396 73	0.406 50	0.415 04	0.423 51	
30	0.268 31	0.374 62	0.385 80	0.395 00	0.403 59	0.413 83	0.422 83	0.431 79	
34	0.272 47	0.383 94	0.396 71	0.405 94	0.415 17	0.426 24	0.436 04	0.445 86	
38	0.275 75	0.391 70	0.404 46	0.414 80	0.424 56	0.436 33	0.446 80	0.457 35	
45	0.280 13	0.402 23	0.415 79	0.426 85	0.437 35	0.450 10	0.461 52	0.473 10	
50	0.282 52	0.408/08	0.422 08	0.435 55	0.444 48	0.457 78	0.469 75	0.481 93	
60	0.286 13	0.417 02	0.431 73	0.443 83	0.455 42	0.469 60	0.482 43	0.495 57	
75	0.289 79	0.426 20	0.441 63	0.454 40	0.466 68	0.481 79	0.495 54	0.509.70	
100	0.293 53	0.435 61	0.451 80	0.465 27	0.478 27	0.494 37	0.509 09	0.524 35	
150	0.297 38	0.445 30	0.462 26	0.476 45	0.490 23	0.507 36	0.523 12	0.539 54	
300	0.301 41	0.455 26	0.473 04	0.487 98	0.502 56	0.520 78	0.537 65	0.555 33	
18.352019	0.305 71	0.465 54	0484P15R		0.535 29	0.534 67	0.552 72	0.571273	

# Table 13-5AGMAGEOMETRYFACTORJFORTEETHHAVING $\phi = 25^{\circ}$ , $a = 1m, b = 1.25m, AND r_f = 0.300m$

	Number of teeth in mating gear								
Number of teeth	1	17	25	35	5(0	85	300	1000	
13	0.286 65	0.346 84	0.352 92	0.357 44	0.361 38	0.365 72	0.369 25	0.372 51	
14	0.293 64	0.359 24	0.365 87	0.370 81	0.375 14	0.379 94	0.383 86	0.387 49	
15	0.300 09	0.370 27	0.377 40	0.382 75	0.387 44	0.392 67	0.396 94	0.400 92	
16	0.305 58	0.380 16	0.387 75	0.393 46	0.398 49	0.404 11	0.408 73	0.413 03	
17	0.310 43	0.389 07	0.397 09	0.403 14	0.408 49	0.414 48	0.419 41	0.424 02	
18	0.314 75	0.397 14	0.405 56	0.411 93	0.417 56	0.423 90	0.429 13	0.434 03	
19	0.318 62	0.404 49	0.413 28	0.419 94	0.425 85	0.432 50	0.438 01	0.443 18	
20	0.322 11	0.411 21	0.420 34	0.427 27	0.433 44	0.440 39	0.446 16	0.451 59	
21	0.325 28	0.417 38	0.426 82	0.434 01	0.440 42	0.447 65	0.453 67	0.459 33	
22	0.328 16	0.423 06	0.432 80	0.440 23	0.446 86	0.454 36	0.460 60	0.466 50	
24	0.333 22	0.433 18	0.443 46	0.451 32	0.458 36	0.466 35	0.473 01	0.479 32	
26	0.337 52	0.441 93	0.452 68	0.460 93	0.468 33	0.476 74	0.483 78	0.490 46	
28	0.341 22	0.449 57	0.460 75	0.469 33	0.477 05	0.485 85	0.493 23	0.500 23	
30	0.344 43	0.456 31	0.467 85	0.476 75	0.484 75	0.493 89	0.501 57	0.508 68	
34	0.349 76	0.467 63	0.479 81	0.489 23	0.497 72	0.507 46	0.515 66	0.523 49	
38	0.354 00	0.476 78	0.489 48	0.499 33	0.508 24	0.518 47	0.527 10	0.535 36	
45	0.359 67	0.489 19	0:502 61	0.513 05	0.522 52	0.533 44	0.542 68	0.551 54	
50	0.362 78	0.496 08	0.509 91	0.520 68	0.530 47	0.541 77	0.551 36	0.560 56	
60	0.367 50	0.506 83	0.521 09	0.532 38	0.542 67	0.554 57	0.564 69	0.574 4	
75	0.372 32	0.517 47	0.532 57	0.544 40	0.555 20	0.567 73	0.578 42	0.588 7	
100	0.377 26	0.528 60	0.544 36	0.556 76	0.568 10	0.581 29	0.592 57	0.603 4	
150	0.382 37	0.540 05	0.556 51	0.569 51	0.581 38	0.595 26	0.607 16	0.618 6	
300	0.387 72	0.551485	ER569 92	J 8.582 AR	S 0.595 07	0.609 67	0.622 22	0.634 42	
Rack	0.393 42	0.564 05	0.581 94	0.596 13	0.609 21	0.624 56	0.637 78	0.650 68	

13.3.20

Regarding material strength, when both pinion and gear are made of the same material the pinion is always the weaker one of the two because it has more undercut shape due to less number of teeth (it rotates more hence loaded up more frequently when fatigue is considered).

Thus design can be done for pinion only and concluded with same F (face width) for gear.

If <u>materials are different</u> for pinion and gear <u>the weaker</u> <u>material</u> strength and specifications are used in design and analysis processes.

#### EXAMPLE: 5.3

A steel pinion with m=4 mm,  $Ø= 20^{\circ}$  pressure angle and 22 T runs at 900 rpm and transmits 12.5 hp to a 60 T gear.

Calculate the bending stress on the pinion tooth using the <u>Lewis stress</u> equation based on a face width of 38 mm.

Solution:

Lewis stress equation

$$\sigma = \frac{W_t}{F \times m \times Y}; \ W_t = \frac{60P}{\pi dN} = \frac{60 \times (12.5 \times 746)}{\pi \times (4 \times 22 \times 10^{-3}) \times 900} = 2249 \ N$$

Y = 0.31997 (from Table 13.3 for 22 T)

Now 
$$\sigma = \frac{2249}{38 \times 4 \times 0.31997} = 46.242 MPa$$

#### EXAMPLE: 5.4

A set of BS Grade 180 Gray Cast Iron (with  $S_{ut}$ = 180 *MPa*) gears is to be designed to transmit 1.2 *kW* at a pinion speed of 400 *rpm* and a speed reduction of 1.5:1. Use a safety factor of 4 and determine suitable values of m,  $T_P$ ,  $T_G$ ,  $d_P$ ,  $d_G$ , and F based on the Lewis stress equation. Use 20° full depth teeth with *b*= 1.25 *m*.

$$\sigma = \frac{W_t}{FmY} = \frac{S_{ut}}{n} \quad W_t = \frac{P}{V} = \frac{60P}{\pi dN} \quad d = ?$$
$$V = \frac{\pi dN}{60} \quad d = ?$$

Solution:

 $1 - \sigma \leq \frac{O_P}{C}$ 

This is a gear design problem with criteria of

n

Since both *d and F* are unknown, iteration is suitable for

$$\begin{array}{ll} 2-3p_{c} \leq F \leq 5p_{c} & \phi = 20^{o} \rightarrow T_{\min} = 18 \rightarrow T_{2} = 1.5T_{1} = 27\\ 3-\frac{T_{G}}{T_{P}} = 1.5 = \frac{d_{G}}{d_{P}} & T_{P} = 18 \ and \ T_{G} = 27 \quad and\\ Y = 0.29327 \quad for \ 18 \ T \ pinion \end{array}$$

 $3p_c \le F \le 5p_c$ 

т	$d_1 = T_1 m$	V	Y	$W_t$	$F = \frac{W_t \times n}{2}$	Зр <sub>с</sub>	5p <sub>c</sub>	
(mm)	(mm)	(m/s)		(N)	$S_{ut}  imes m  imes Y$ (mm)	(mm)	(mm)	Notes
2	36	0.754	0.29327	1591.5	60.30	18.84	31.4	Not Suitable
3	54	1.131	0.29327	1061.0	26.80	28.30	47.1	Not Suitable
2.5	45	0.942	0.29327	1273.24	38.59	23.56	39.27	SUITABLE

Solution: $p_c = \pi m$ m = 2.5 mm $W_t = \frac{P}{V} = \frac{60P}{\pi dN} = \frac{60 \times 1.2 \times 10^3}{\pi \times 0.036 \times 400} = 1591.5 N$  $d_p = 45 \text{ mm}$  $V = \frac{\pi dN}{60} = \frac{\pi \times 0.036 \times 400}{60} = 0.754 \text{ m/sec}$  $d_G = 67.5 \text{ mm}$  $F = \frac{W_t \times n}{S_{ut} \times m \times Y} = \frac{1591.5 \times 4}{180 \times 2 \times 0.29327} = 60.3 \text{ mm}$