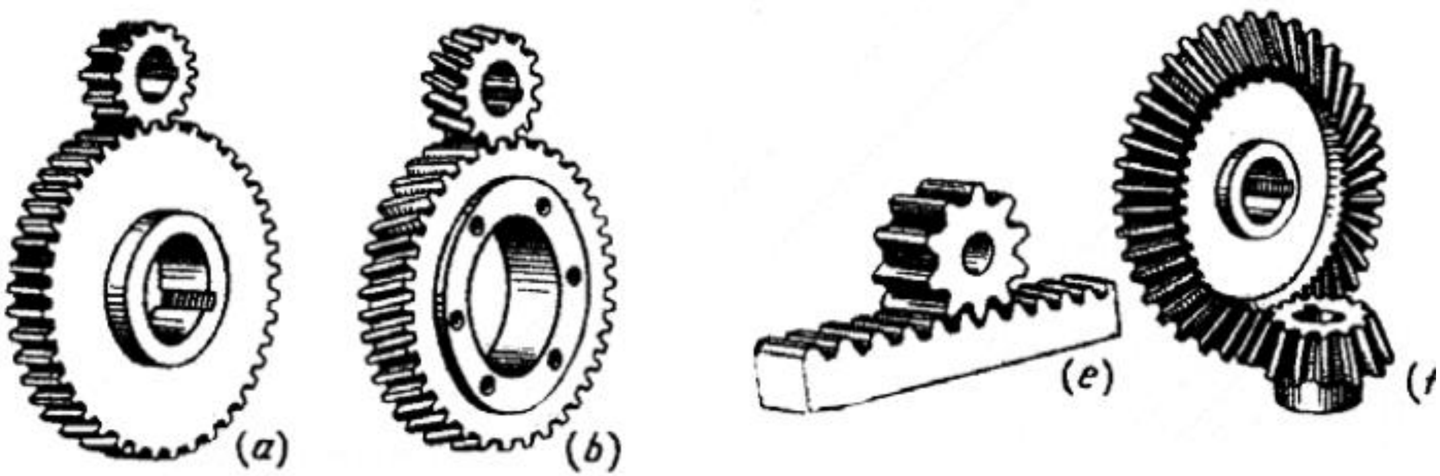
ME 308
MACHINE ELEMENTS II

CHAPTER 5

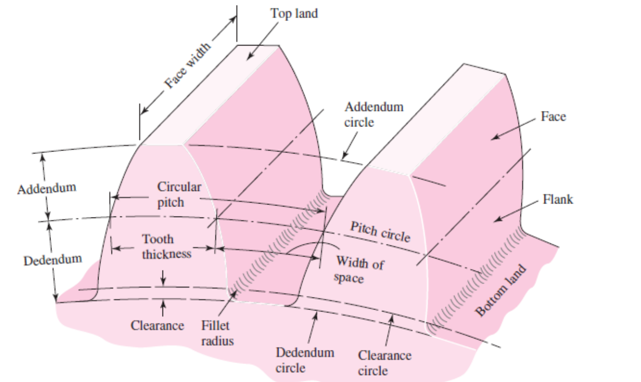
GEARS
PART_1



5.1 GEARS

This chapter addresses

- gear geometry,
- the kinematic relations, and
- the forces transmitted by the gears (spur, helical, bevel, and worm).



The forces transmitted between meshing gears supply torsional moments to shafts for motion and power transmission and create forces and moments that affect the shaft and its bearings.

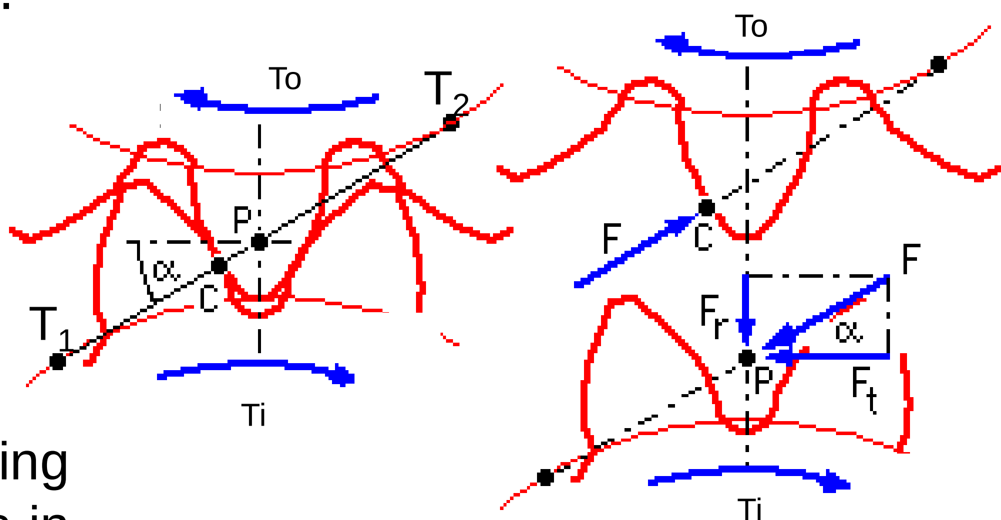
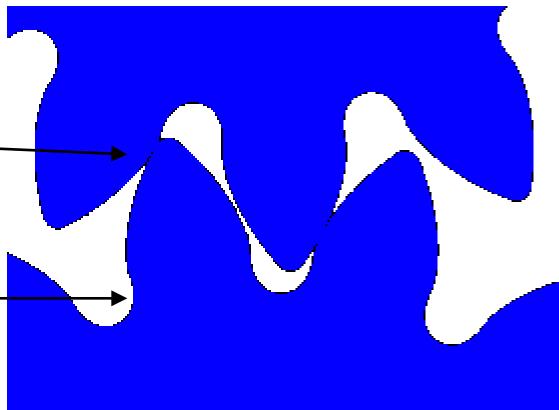


Fig. 3 Tooth forces

The forces transmitted between meshing gears do also effect the teeth of gears in two respects:

- teeth will be in contact under forces creating <u>contact stress</u> at teeth surfaces,
- teeth will be bend by the forces creating <u>bending stress</u> at tooth root.



5.1.1 TYPES OF GEARS

Gears are usually cylindrical sometimes conical disks with some teeth on the circumference to ensure a continuous and positive transmission of rotary motion between the shafts on which they are mounted.

Some examples are:

- Spur gears - → for parallel shaft
- Helical gears
- Bevel gears → can accommodate non-parallel shafts.
- Worm gears

Spur gears especially are used to transmit rotary motion between parallel shafts.

5.1.1 TYPES OF GEARS

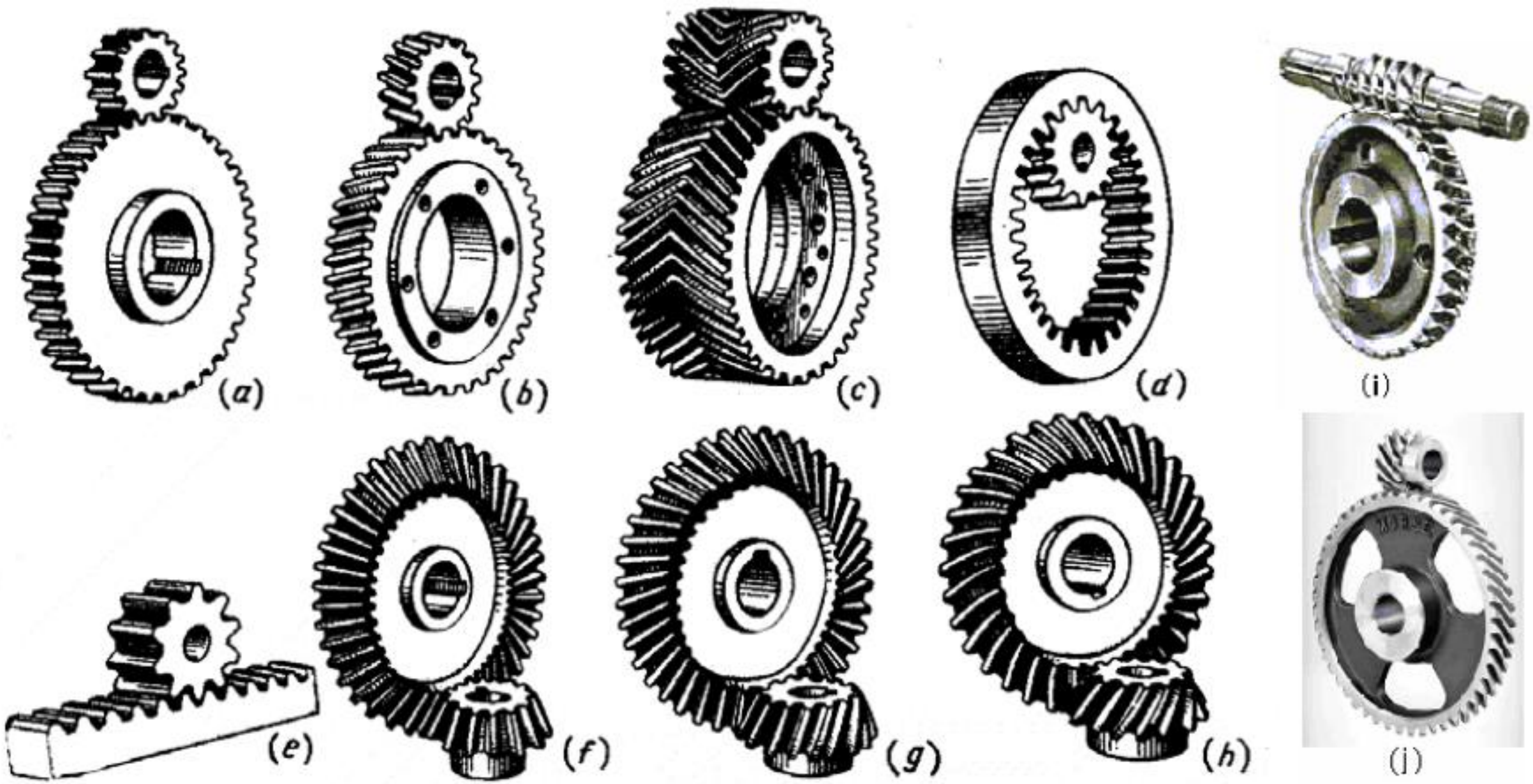
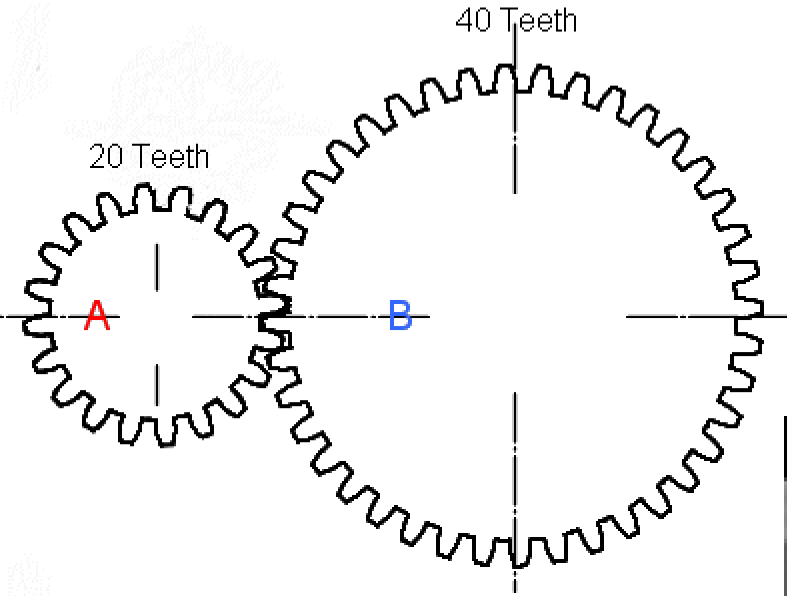
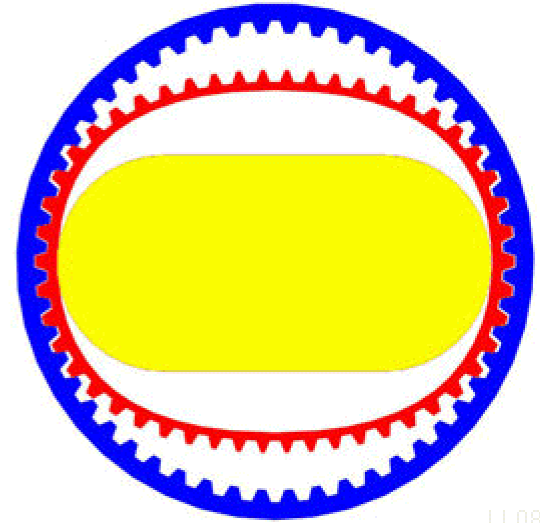
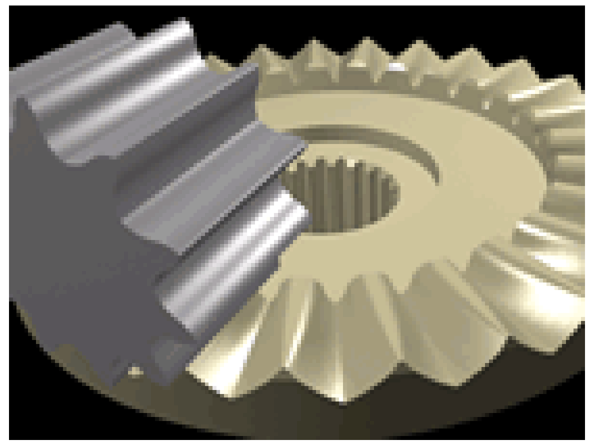
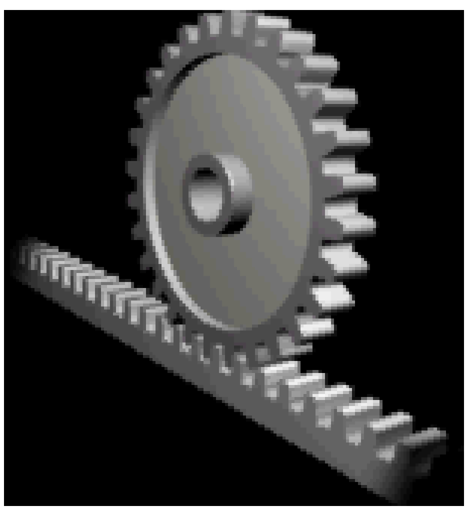


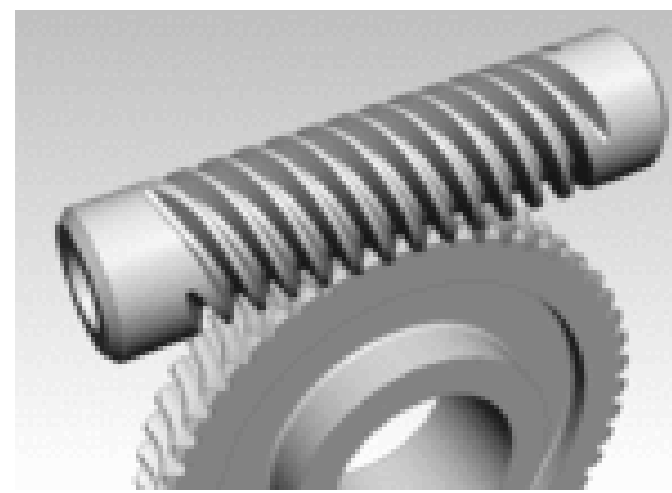
Fig. 5.1 Types of Gears (a) Spur gear, (b) Helical gear, (c) Double helical gear or herringbone gear, (d) Internal gear , (e) Rack and pinion, (f) Straight bevel gear, (g) Spiral bevel gear, (h) Hypoidal bevel gear , (i) worm gear and (j) Crossed helical or spiral gear.

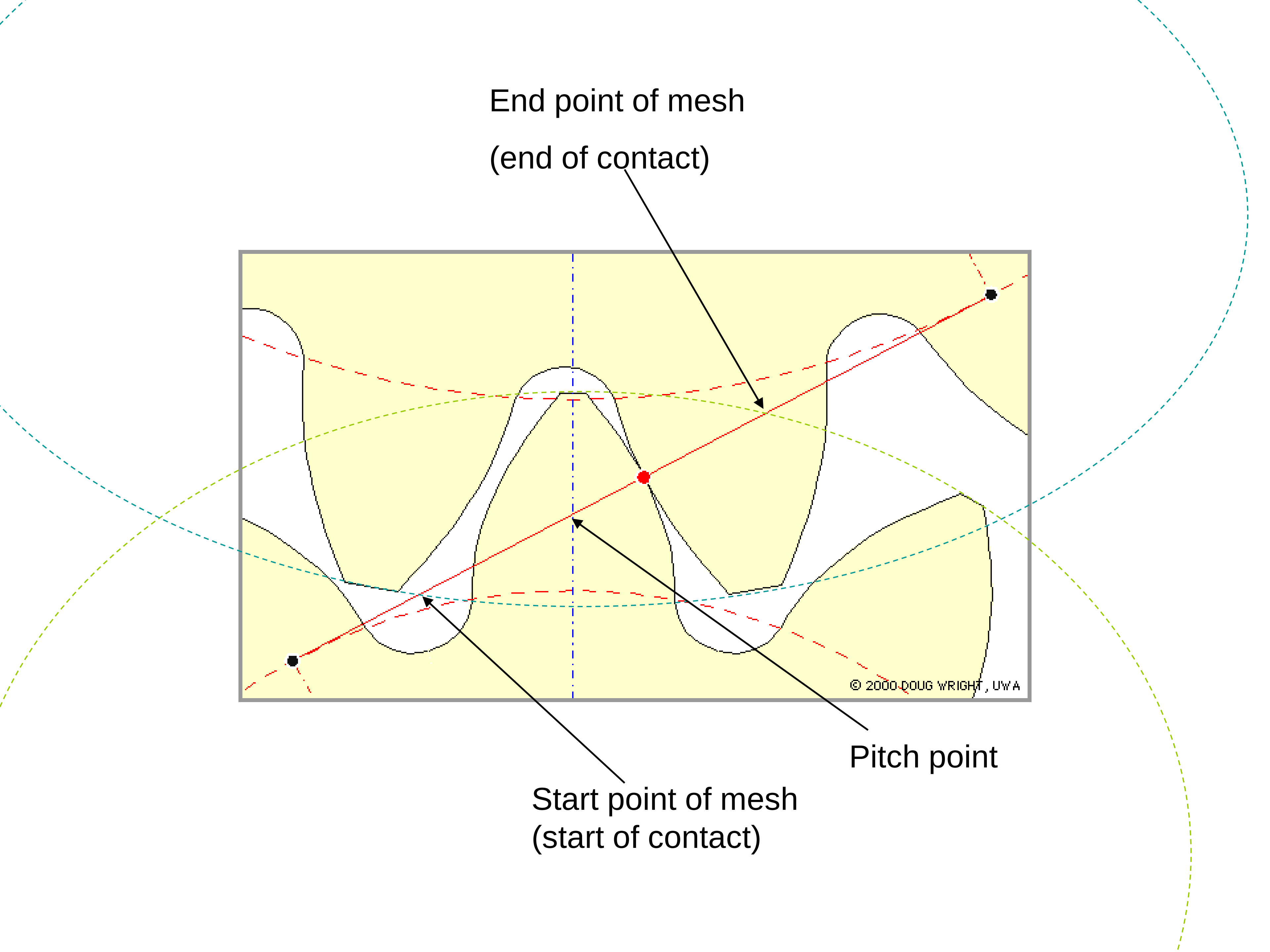

40 Teeth
20 Teeth
A
B





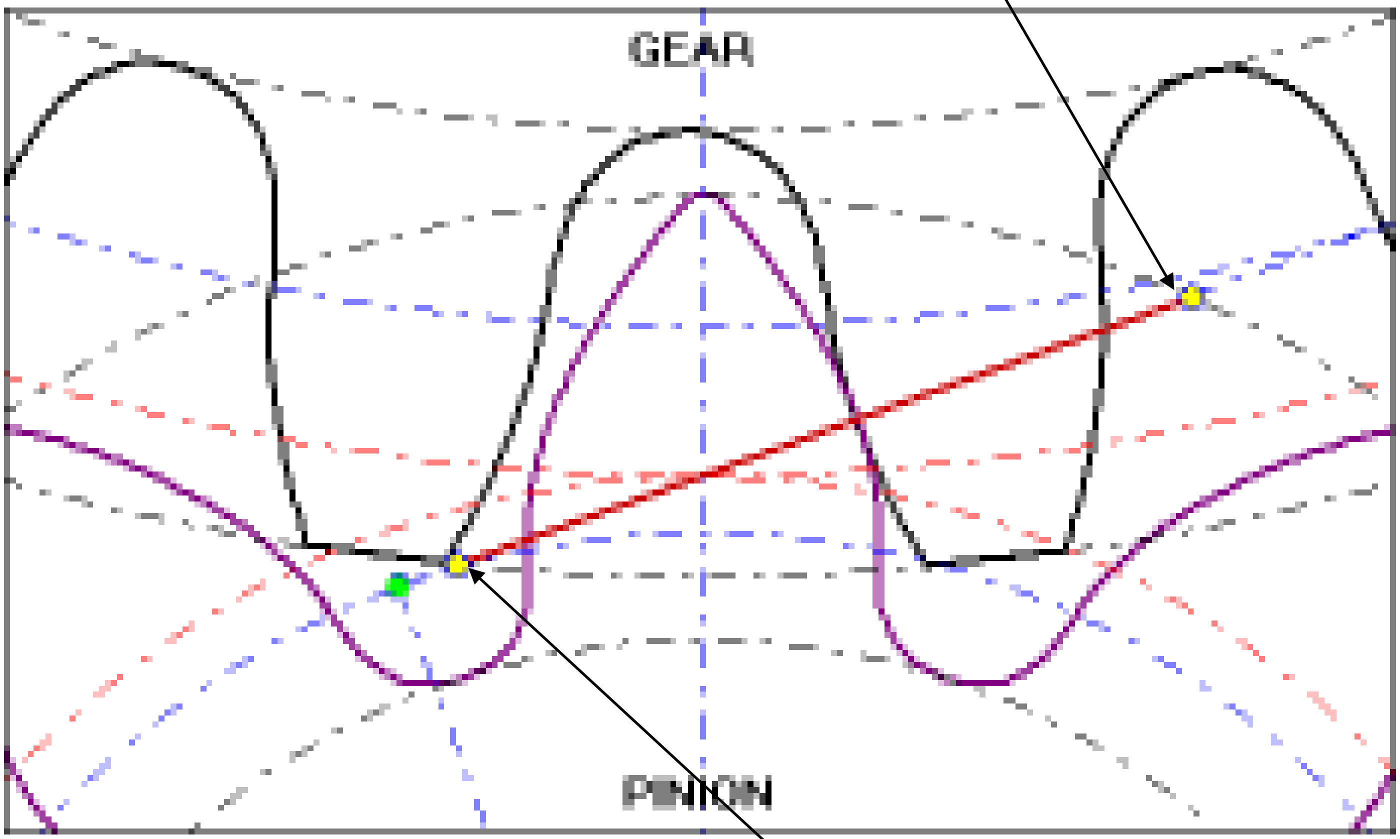





End point of mesh
(end of contact)
© 2000 DOUG WRIGHT, UWA
Pitch point
Start point of mesh
(start of contact)

End point of mesh

(end of contact)



Start point of mesh
(start of contact)

5.1.2 SPUR GEARS

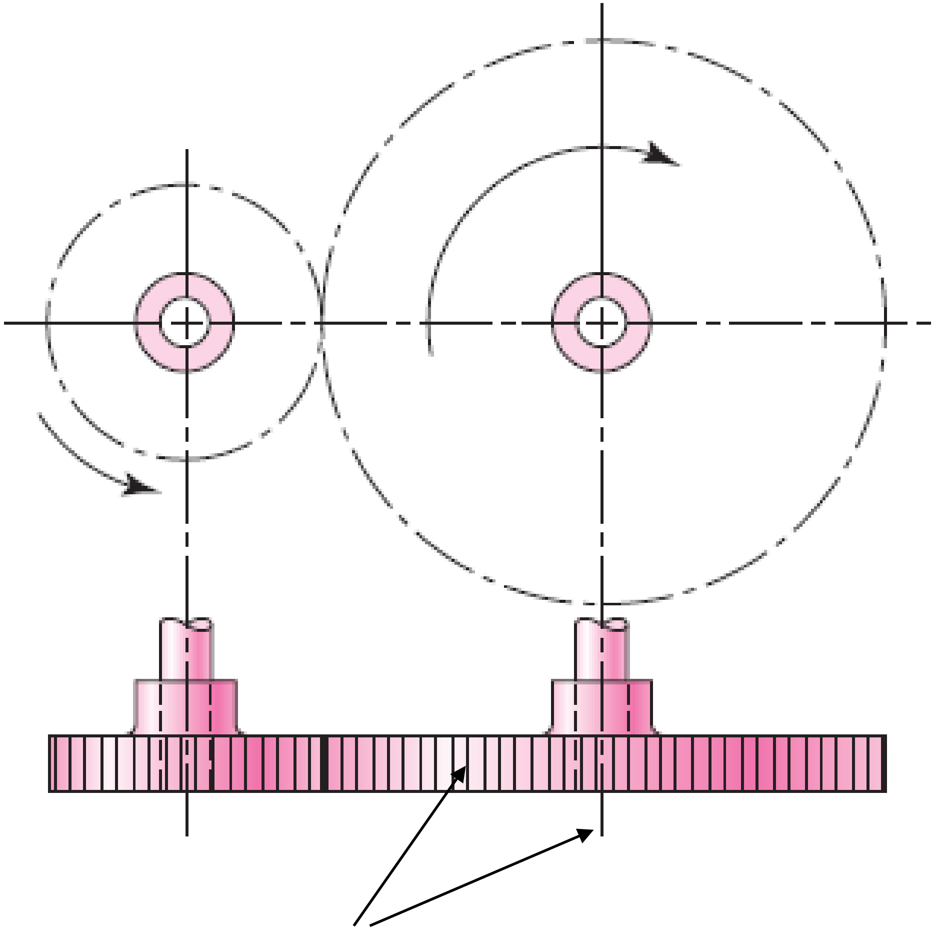


Fig.5.2 Spur gears are used to transmit rotary motion between parallel shafts

Spur gears, illustrated in Fig. 5.2, <u>have teeth parallel to the axis of rotation</u> and are used to transmit motion from one shaft to another, parallel, shaft.

Of all types, the spur gear is the simplest and, for this reason, will be used to develop the primary kinematic relationships of the tooth form.

5.1.3 HELICAL GEARS

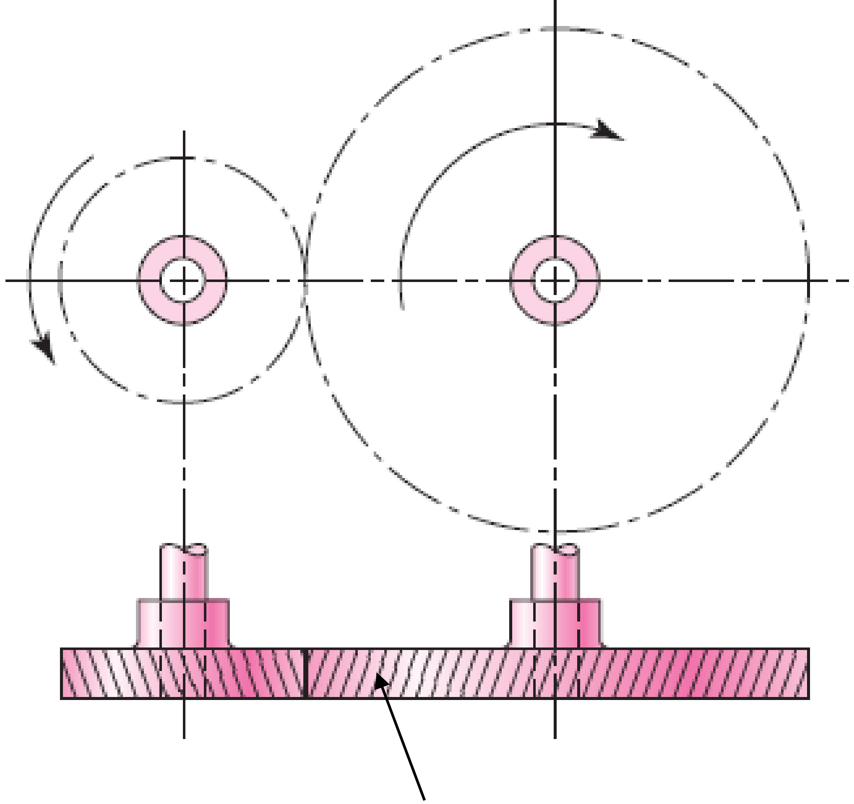


Fig.5.3 Helical gears are used to transmit motion between parallel or nonparallel shafts.

Helical gears, shown in Fig. 5.3, have teeth inclined to the axis of rotation. Helical gears can be used for the same applications as spur gears and, when so used, are not as noisy, because of the more gradual engagement of the teeth during meshing.

The inclined tooth also develops thrust loads and bending couples, which are not present with spur gearing.

5.1.4 BEVEL GEARS

Bevel gears, shown in Fig. 5.4, have teeth formed on conical surfaces and are used mostly for transmitting motion between intersecting shafts. The figure actually illustrates *straight-tooth bevel gears.*

Spiral bevel gears are cut so the tooth is no longer straight, but forms a circular arc.

Hypoid gears are quite similar to spiral bevel gears except that the shafts are offset and nonintersecting.

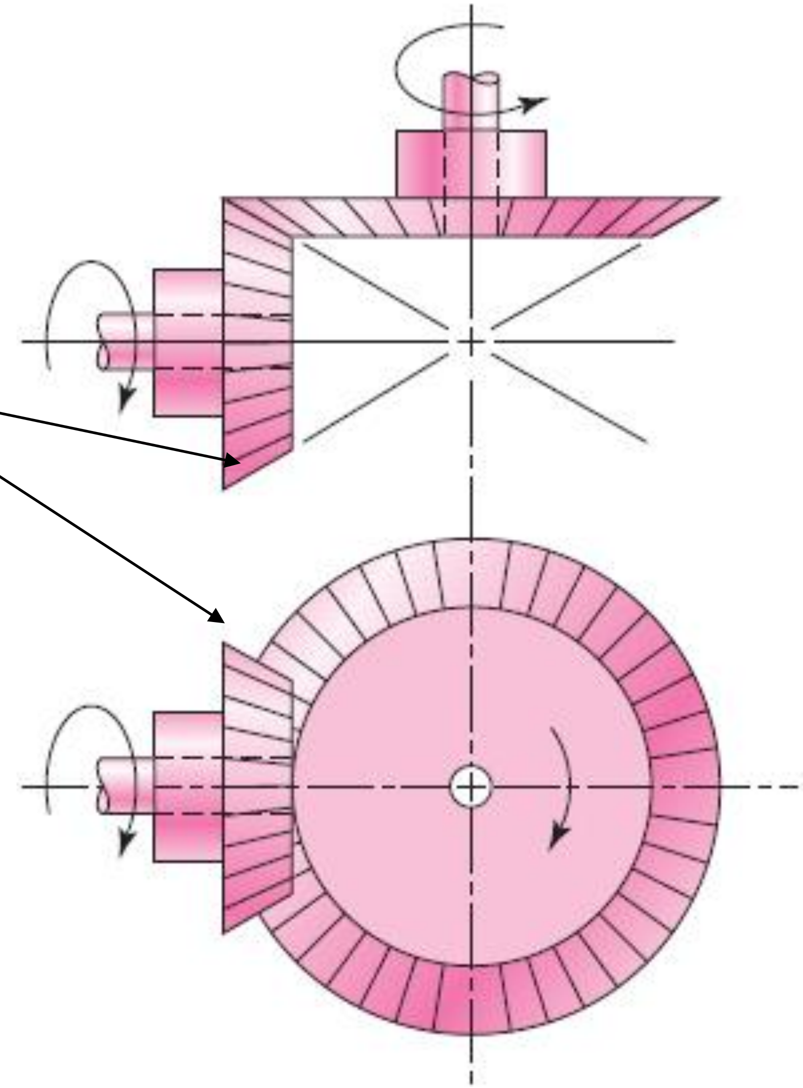
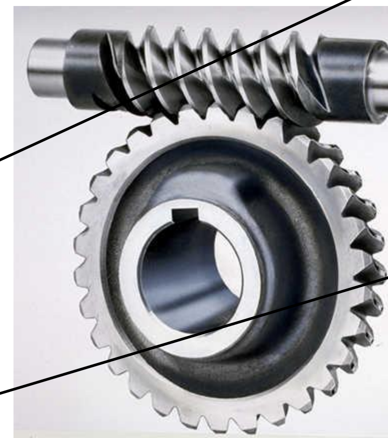
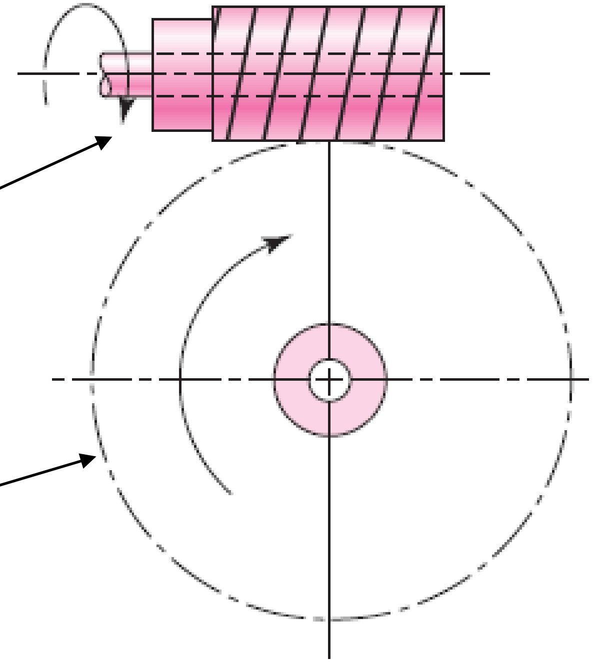


Fig.5.4 Bevel gears are used to transmit rotary motion between intersecting shafts.

5.1.5 WORM GEARS

Fig. 5.5 Worm gear sets are used to transmit rotary motion between nonparallel and nonintersecting shafts.



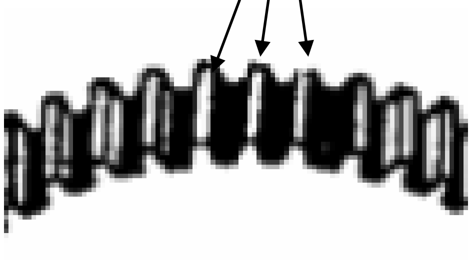


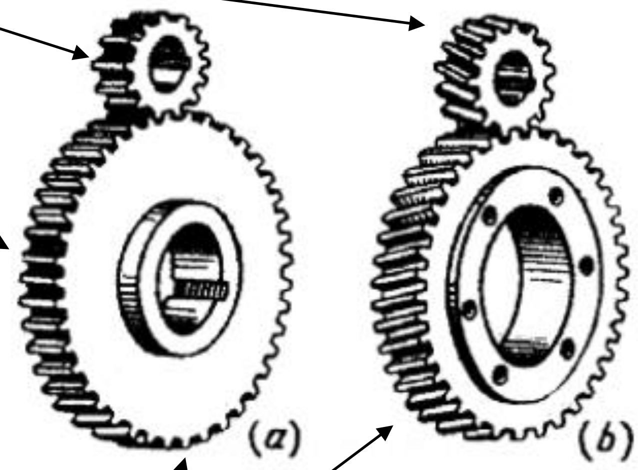
Sonsuz vida dişliler

Worms and worm gears, shown in Fig. 5.5, represent the fourth basic gear type. As shown, the worm resembles a screw. The direction of rotation of the worm gear, also called the worm wheel, depends upon the direction of rotation of the worm and upon whether the worm teeth are cut right-hand or left-hand. Worm-gear sets are also made so that the teeth of one or both wrap partly around the other. Such sets are called *single enveloping* and *double-enveloping worm-gear sets. Worm-gear sets are mostly used* when the speed ratios of the two shafts are quite high, say, 3 or more.

A pinion is the smaller of two mating gears.

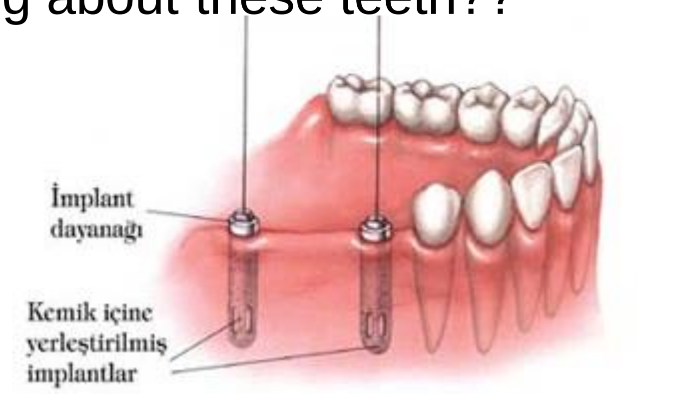
These are the teeth of gear



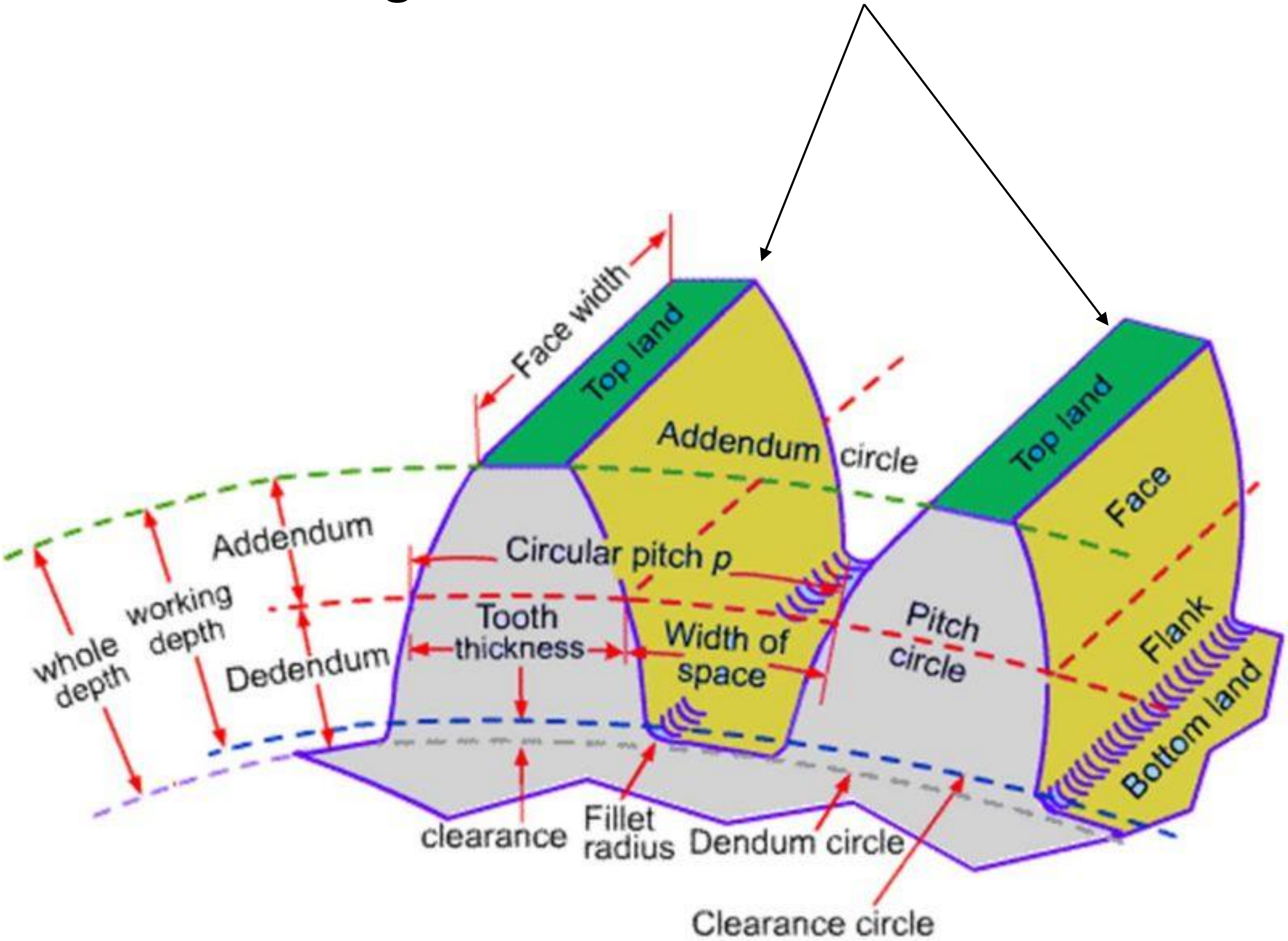


The larger is often called the *gear*

But we are not talking about these teeth??

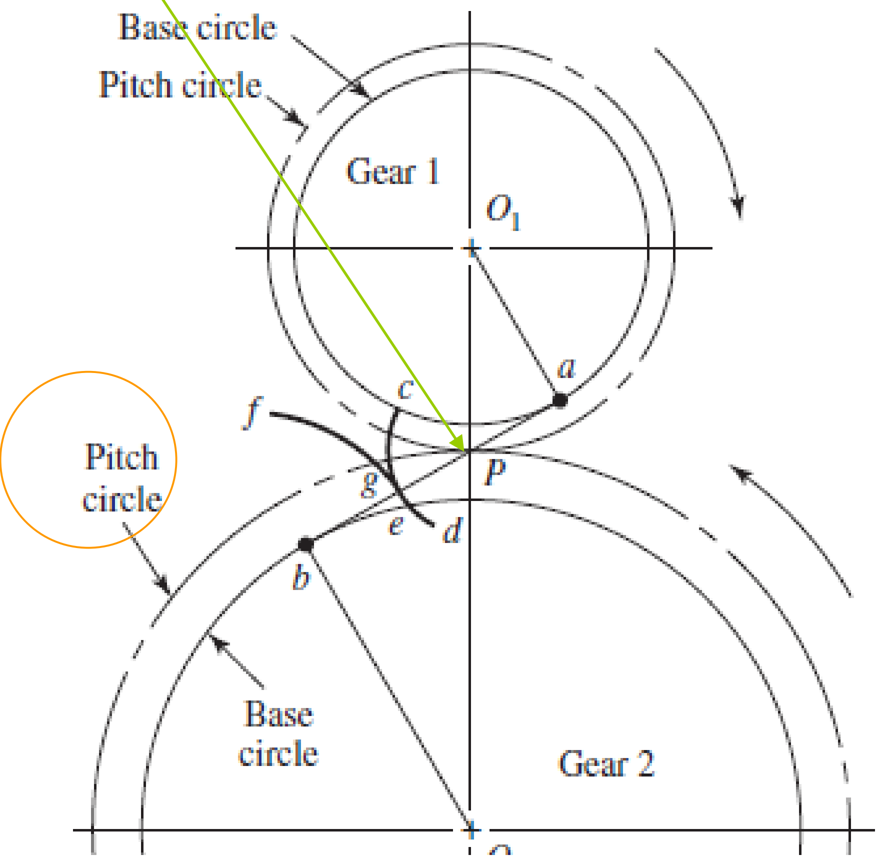


We are talking about those teeth



5.2 NOMENCLATURE OF SPUR GEARS

The terminology of spur-gear teeth is illustrated in Fig. 5.6. The *pitch circle* is a theoretical circle upon which all calculations are usually based; its diameter is the *pitch diameter.* The pitch circles of a pair of mating gears are tangent to each other at pitch point.



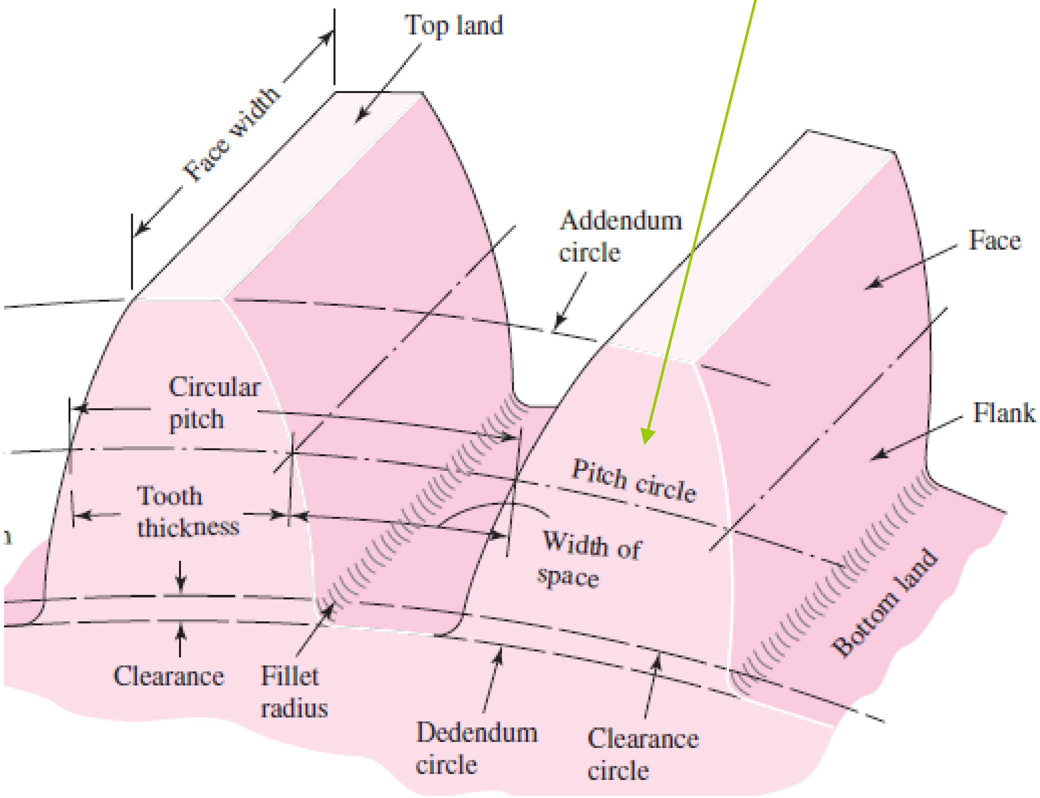
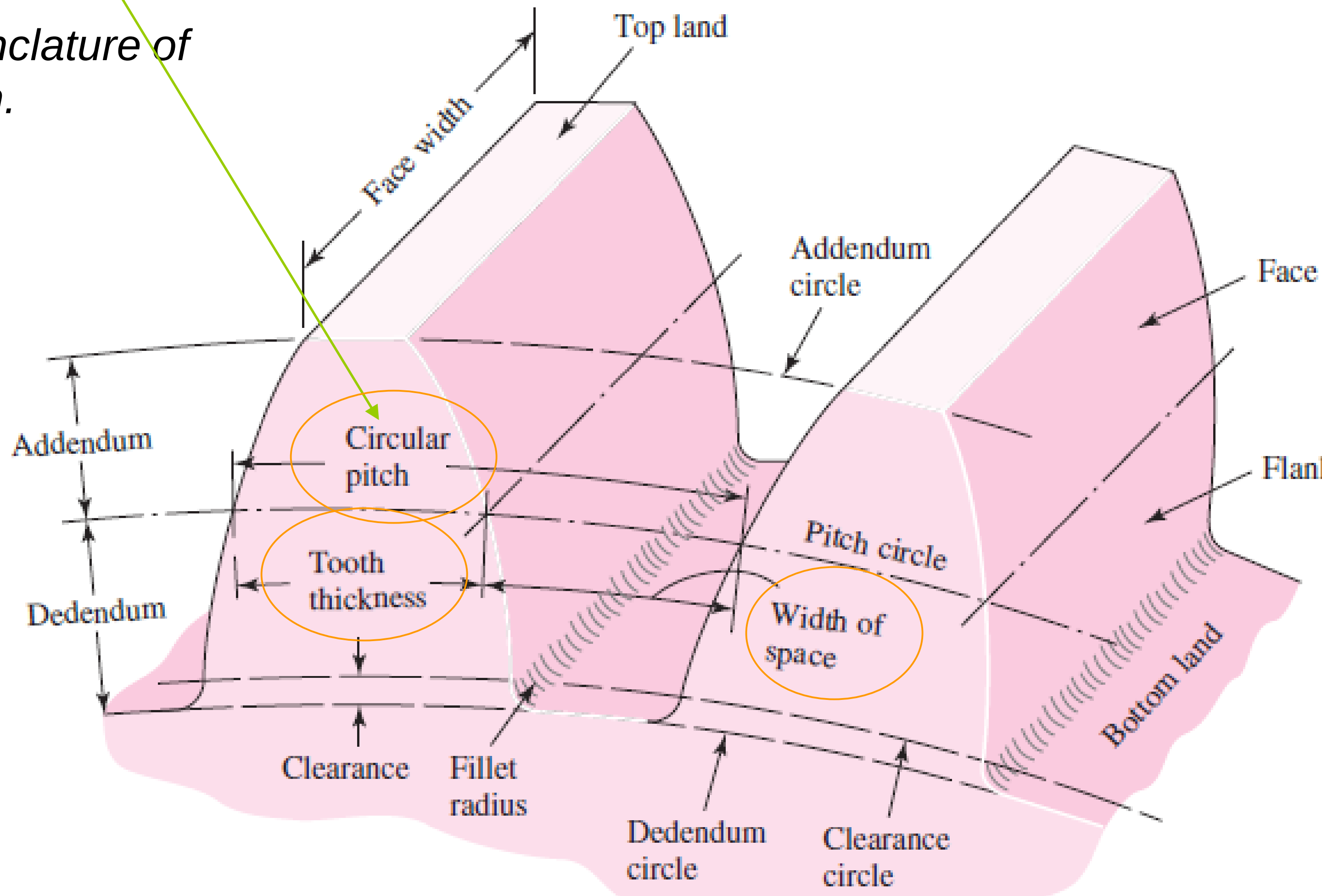


Fig. 5.6 Nomenclature of spur-gear teeth.

5.2 NOMENCLATURE

The *circular pitch p* is the distance, measured on *the pitch circle,* from a point on one tooth to a corresponding point on an adjacent tooth. Thus the circular pitch is equal to the sum of the *tooth thickness and the width of space.*

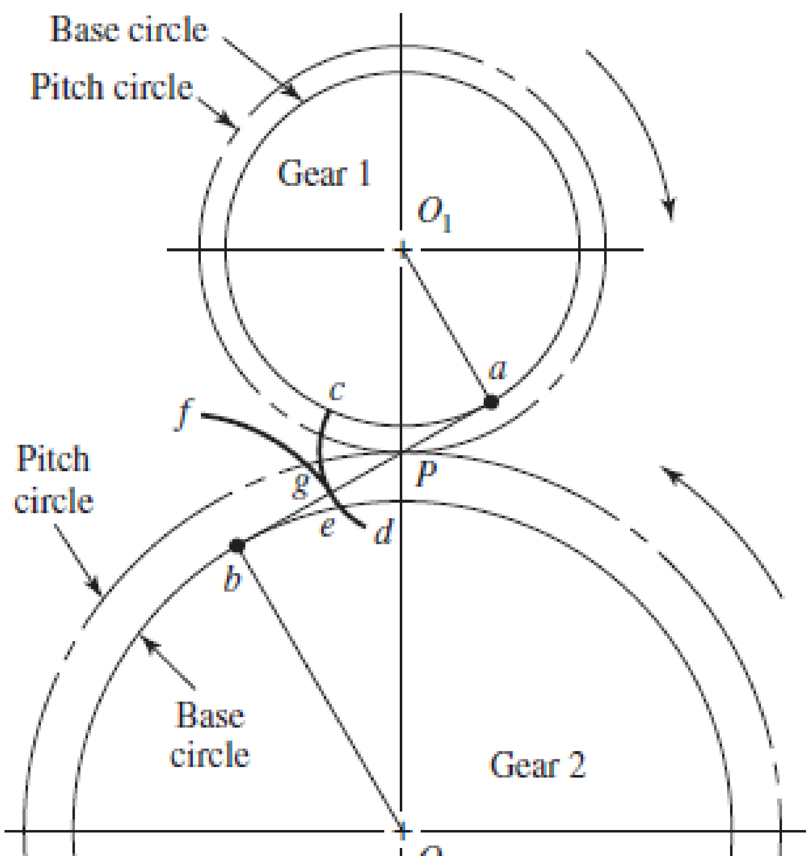
Fig. 5.6 Nomenclature of spur-gear teeth.



5.2 NOMENCLATURE

The *module m* is the ratio of the pitch diameter to the number of teeth. The customary unit of length used is the millimeter. The module is the index of tooth size in SI.

The *diametral pitch P* is the ratio of the number of teeth on the gear to the pitch diameter. Thus, it is the reciprocal of the module. Since *diametral pitch* is used only with U.S. units, it is expressed as teeth per inch..



$m = d / N_T$ or $d = N_T * m$

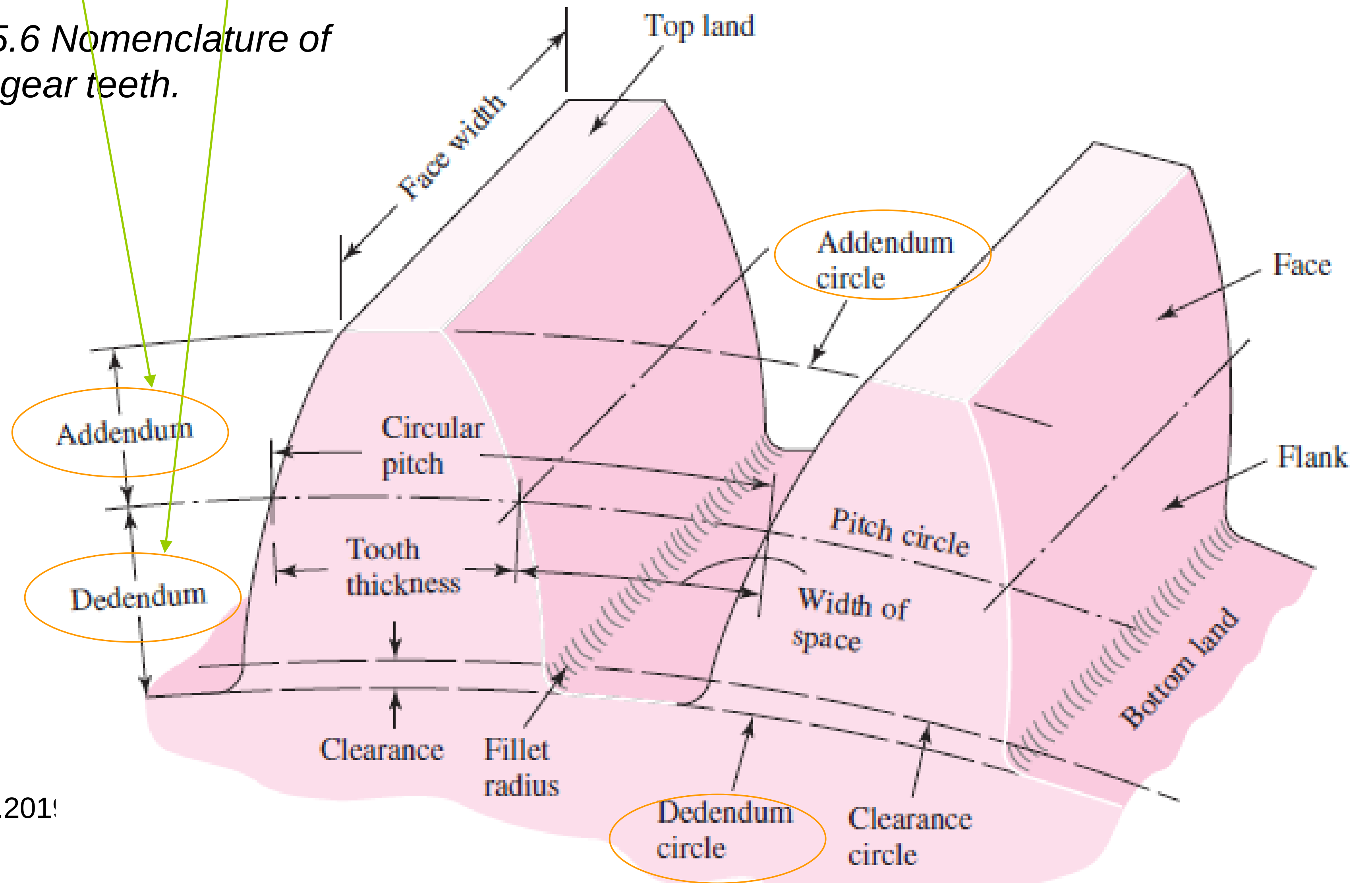
$P = N_T / d$ or $d = N_T / P$

d is the diameter of pitch circle

For two mating gears, module (or P) of both gears should be the same

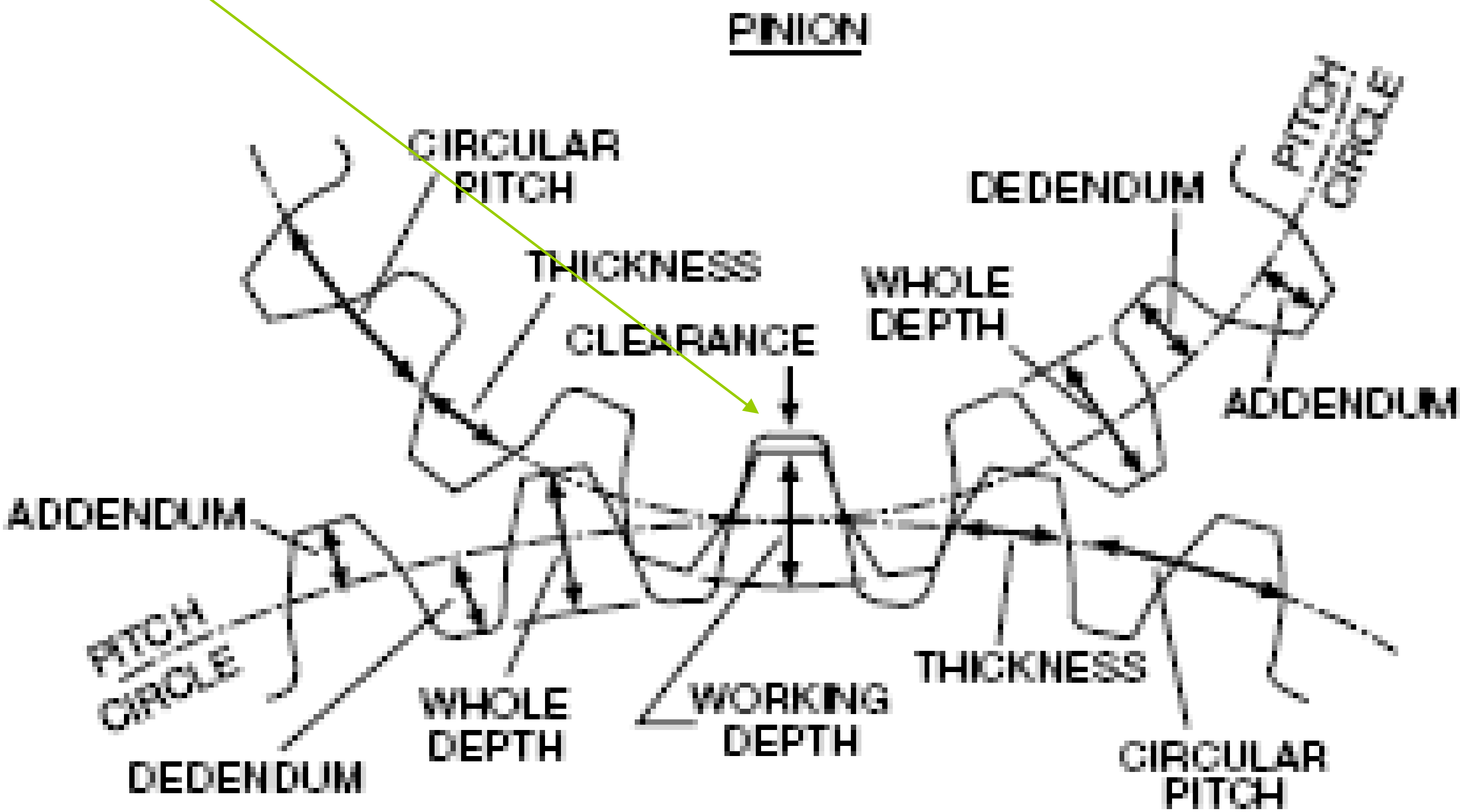
The <u>*addendum* a</u> is the radial distance between the *top land* and the *pitch circle.* The <u>*dedendum b*</u> is the radial distance from *the bottom land* to the pitch circle*. The whole depth h_t* is the sum of the addendum and the dedendum.

Fig. 5.6 Nomenclature of spur-gear teeth.

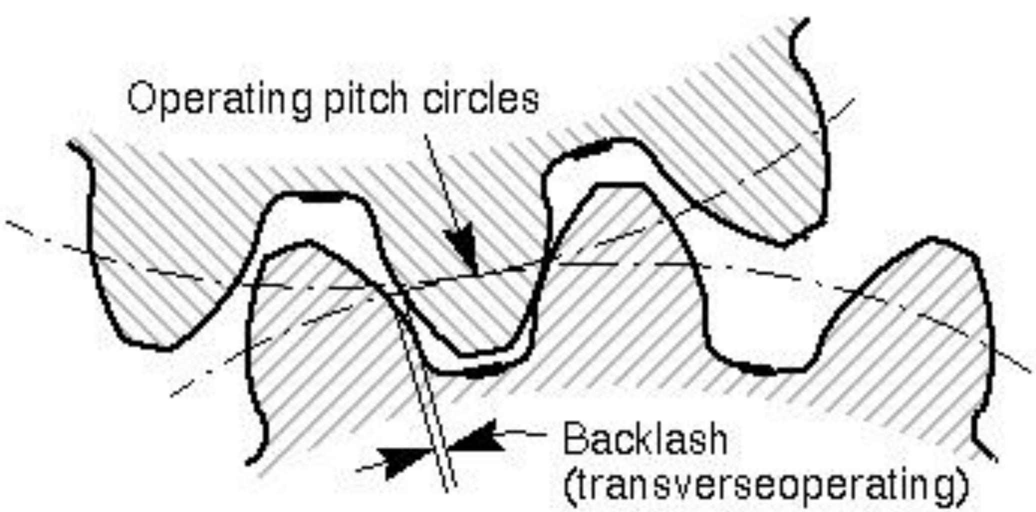


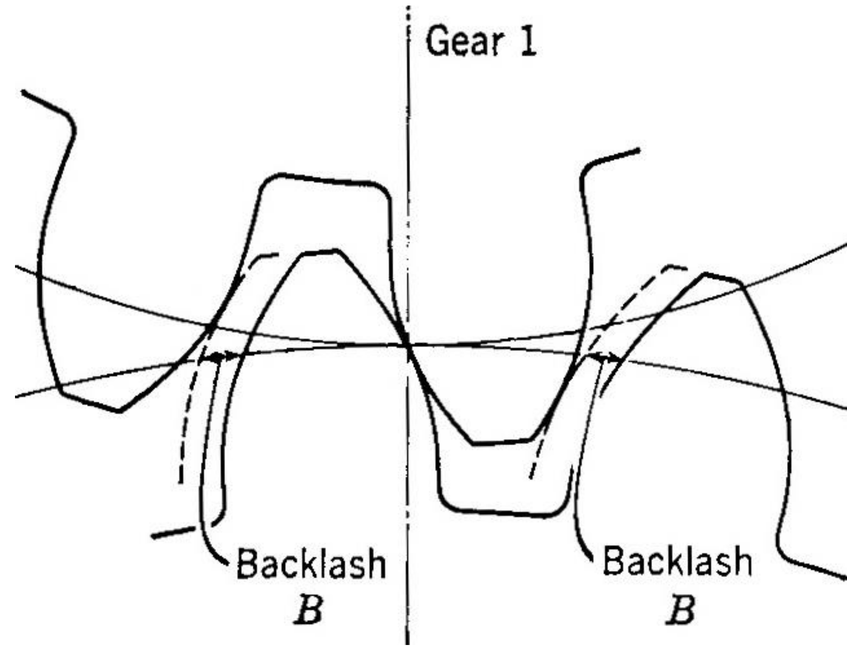
The *clearance circle* is a circle that is tangent to the <u>addendum circle of the mating gear</u>.

The <u>*clearance c*</u> is the amount by which the dedendum in a given gear exceeds the <u>addendum of its mating gear</u>.



The *backlash* is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circles.





Gear teeth size are standardized as the other mechanical elements (bolts, nuts, bearings, etc.) too.

You should prove for yourself the validity of the following useful relations:

In SI Units

Size is in *mm*

N_P # of teeth on pinion

N_G # of teeth on gear

d_P dia. of pinion pitch circle, mm

d_G dia. of gear pitch circle, mm

m: module (index of tooth size, mm)

$$m = \frac{d}{N}; \quad \frac{d_P}{N_P} \; or \quad \frac{d_G}{N_G}$$

p_c : circular pitch (not much used in SI)

$$p_c = \frac{\pi d}{N} = \pi\left(\frac{d}{N}\right) = \pi m; \quad \text{in} \;\; \text{mm.}$$

In American System

Size is in *inches*

N_P # of teeth on pinion

N_G # of teeth on gear

d_P dia. of pinion pitch circle, *in*

d_G dia. of gear pitch circle*, in*

P: diametral pitch (index of tooth size 1/inches)

$$P = \frac{N}{d}; \quad \frac{N_P}{d_P} \; or \quad \frac{N_G}{d_G}$$

p_c : circular pitch used in many calculations

$$p_c = \pi \frac{d}{N} = \frac{\pi}{N/d} = \frac{\pi}{P} \; or$$

$$p_c \times P = \pi \quad \text{in} \;\; \text{inches.}$$

$$p_c = \frac{\pi d}{N} = \pi\left(\frac{d}{N}\right)$$

$$p_c = \pi m; \quad \text{in} \quad \text{mm.}$$

$$p_c = \pi \frac{d}{N} = \frac{\pi}{N/d} = \frac{\pi}{P} \; or$$

$$p_c \times P = \pi \quad \text{in} \quad \text{inches.}$$

$$p_c = \frac{\pi}{P} in \times \frac{25.4 \, mm}{in}$$

$$\pi m = \frac{\pi}{P} 25.4$$

$$m = \frac{25.4}{P}$$

This is the relation between SI module and Imperial(American) diametral pitch

A gear with 8(teeth/inch) diametral pitch will have a corresponding module of

$$m = \frac{25.4}{8} = 3.175 mm$$

5.3 CONJUGATE ACTION

birleşik, çift, aynı kökten türemiş, karşılıklı, eşlenik

Mating gear teeth acting against each other to produce rotary motion are similar to cams. When the tooth profiles (or cams) are designed so as to produce a constant angular velocity ratio during meshing, these are said to have *conjugate action.*

In theory, at least, it is possible arbitrarily to select any profile for one tooth and then to find a profile for the meshing tooth that will give conjugate action.

One of these tooth profile solutions is the *involute profile,* which, with few exceptions, is in universal use for gear teeth and is the only one with which we should be concerned.

Tooth profile shapes: 1)-involute -----> most widely used shape

2)-cyclodial
3)-circular

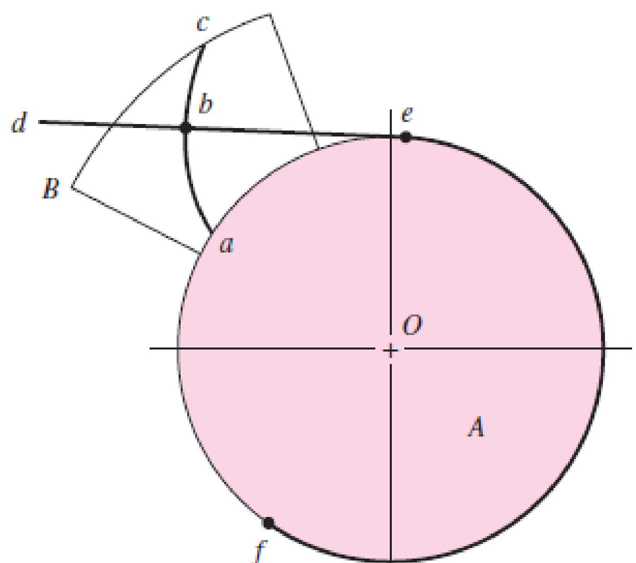
Not much used anymore.

5.4 INVOLUTE PROPERTIES

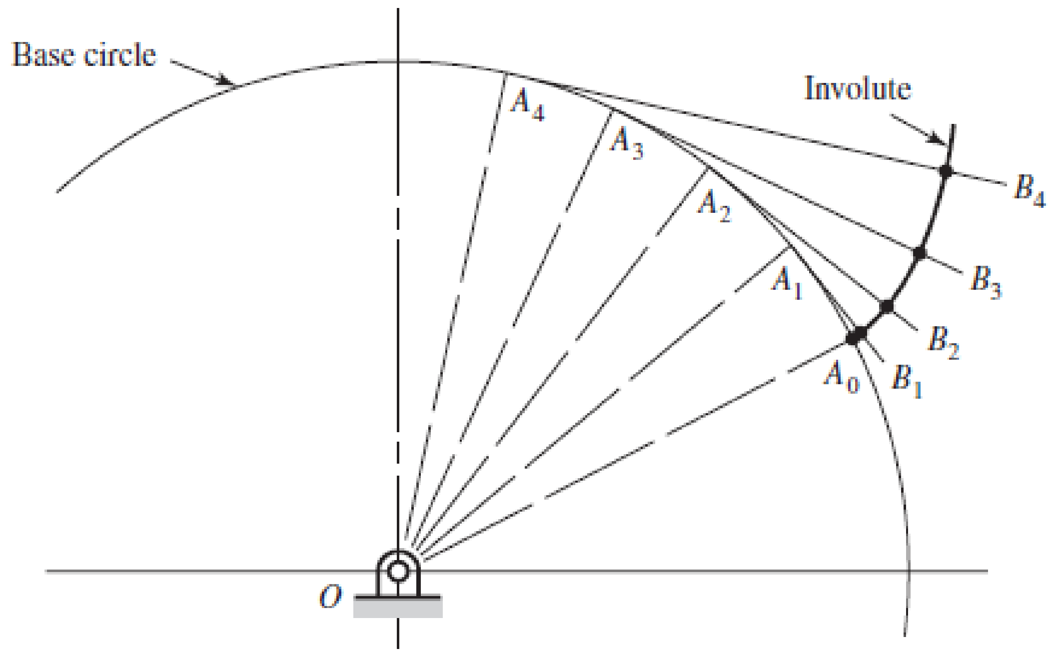
An involute curve may be generated as shown in Fig. 5.8. A partial flange ***B*** is attached to the cylinder ***A***, around which is wrapped a cord ***def***, which is held tight.

Point ***b*** on the cord represents the tracing point, and as the cord is wrapped and unwrapped about the cylinder, point ***b*** will trace out the involute curve ***ac***.

The radius of the curvature of the involute varies continuously, being zero at point ***a*** and a maximum at point ***c***. At point ***b*** the radius is equal to the distance ***be***, since point ***b*** is instantaneously rotating about point ***e***. Thus the generating line ***de*** is normal to the involute at all points of intersection and, at the same time, is always tangent to the cylinder **A**. The circle on which the involute is generated is called the *base circle*

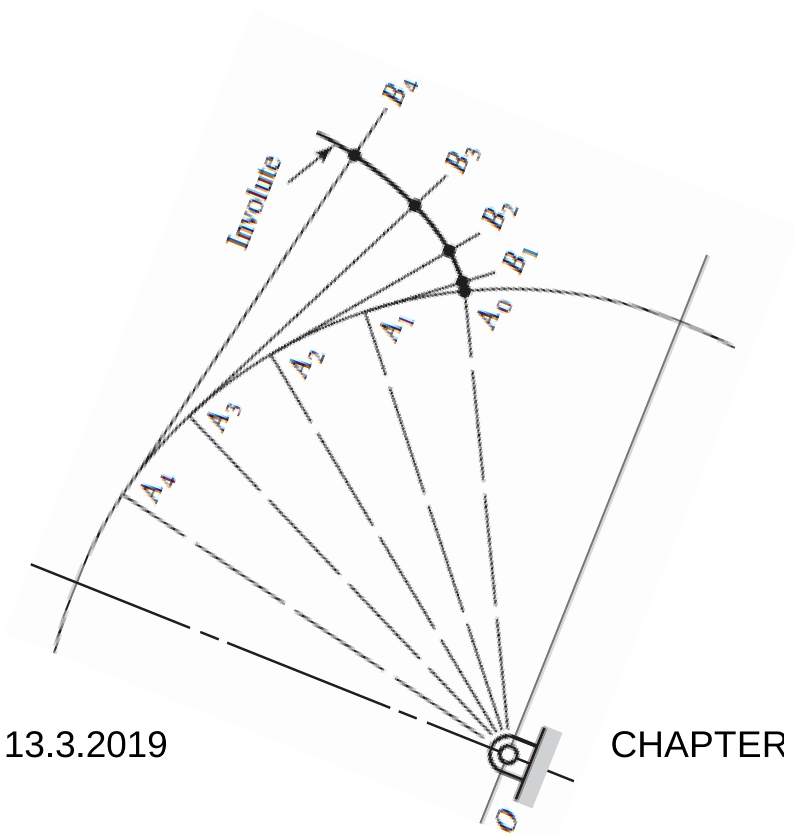


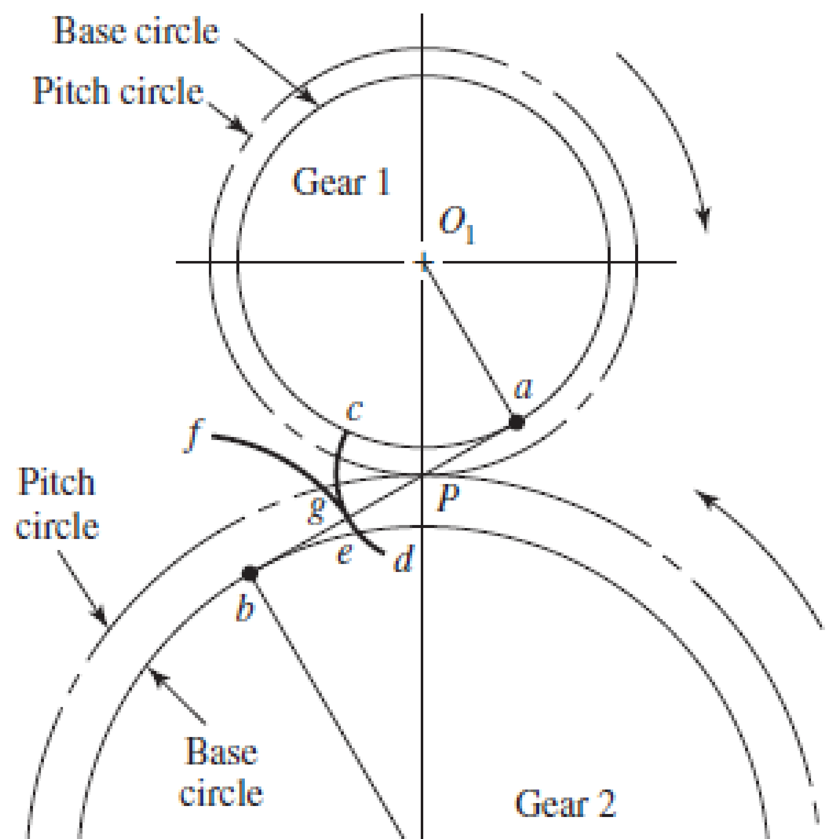
HAPTE



5.4 INVOLUTE PROPERTIES

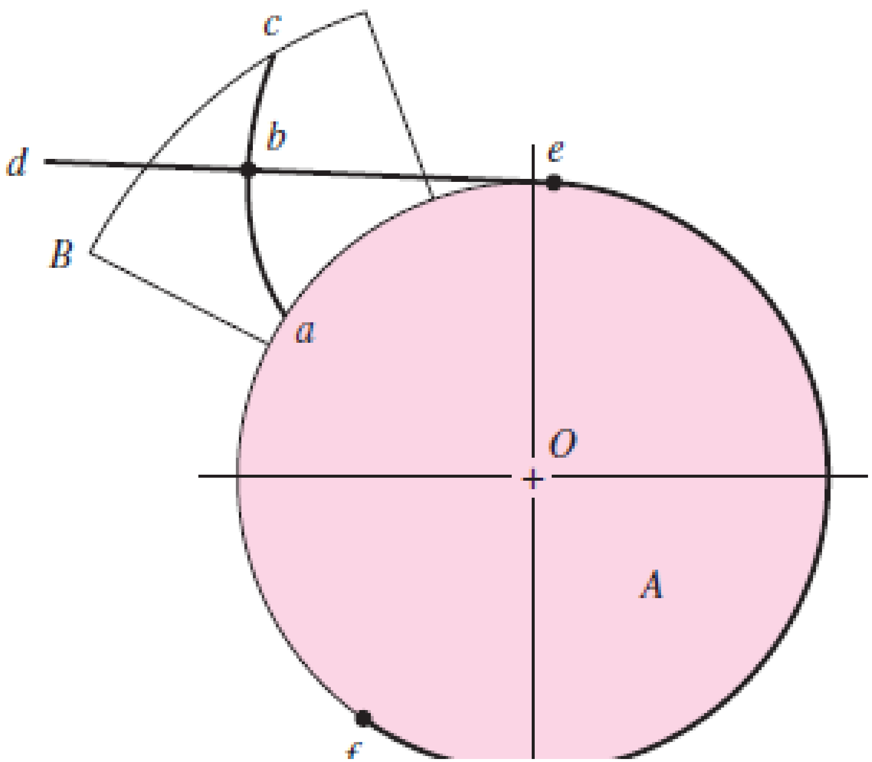
Let us now examine the involute profile to see how it satisfies the requirement for the transmission of uniform motion. In Fig. 5.8*b,* two gear blanks with fixed centers at ***O*$_1$** *and* ***O*$_2$** are shown having base circles whose respective radii are ***O*$_1$*a*** *and* ***O*$_2$*b***. We now imagine that a cord is wound clockwise around the base circle of gear ***1***, pulled tight between points ***a*** and ***b*,** and wound counterclockwise around the base circle of gear ***2***. If, now, the base circles are rotated in different directions so as to keep the cord tight, a point ***g*** on the cord will trace out the involutes ***cd*** on gear 1 and ***ef*** on gear ***2***.

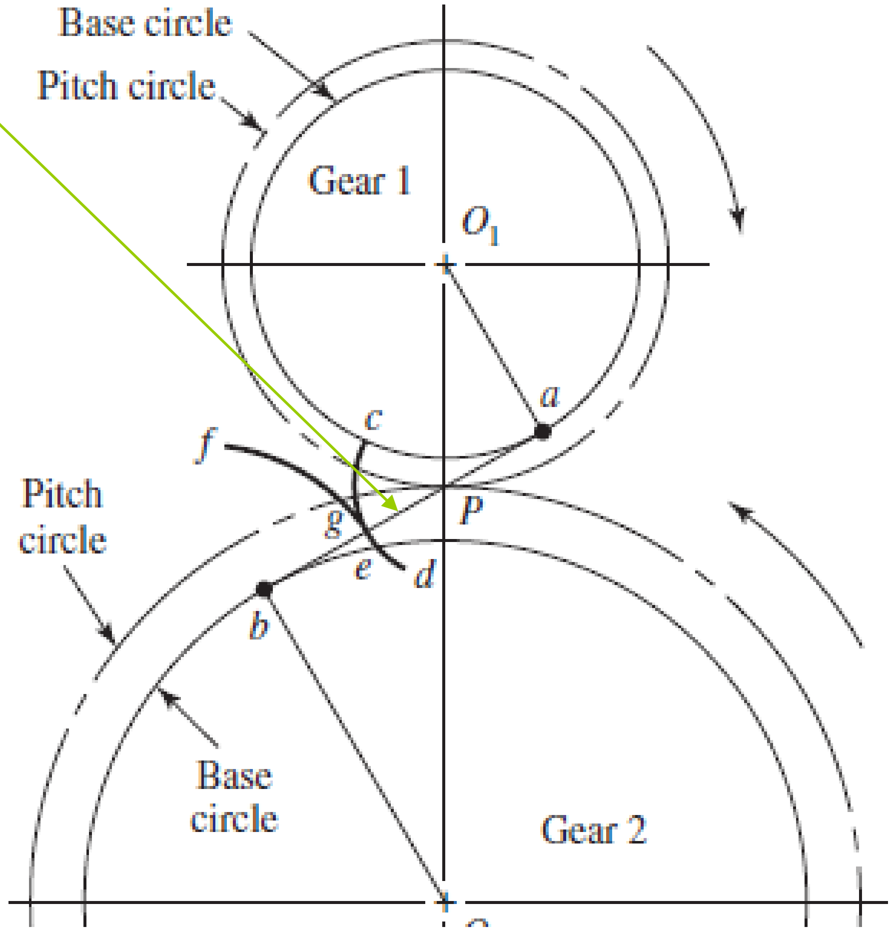


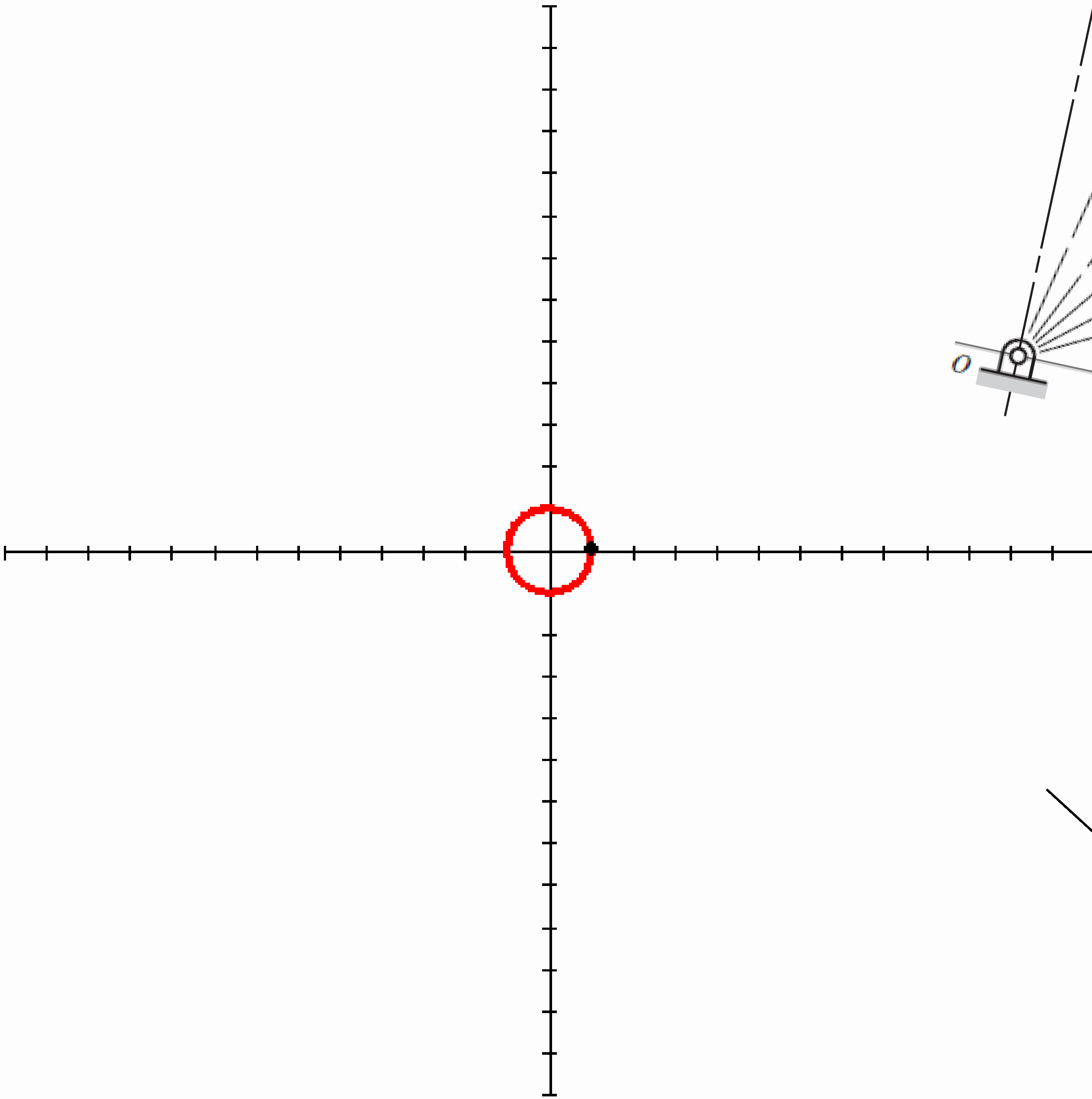


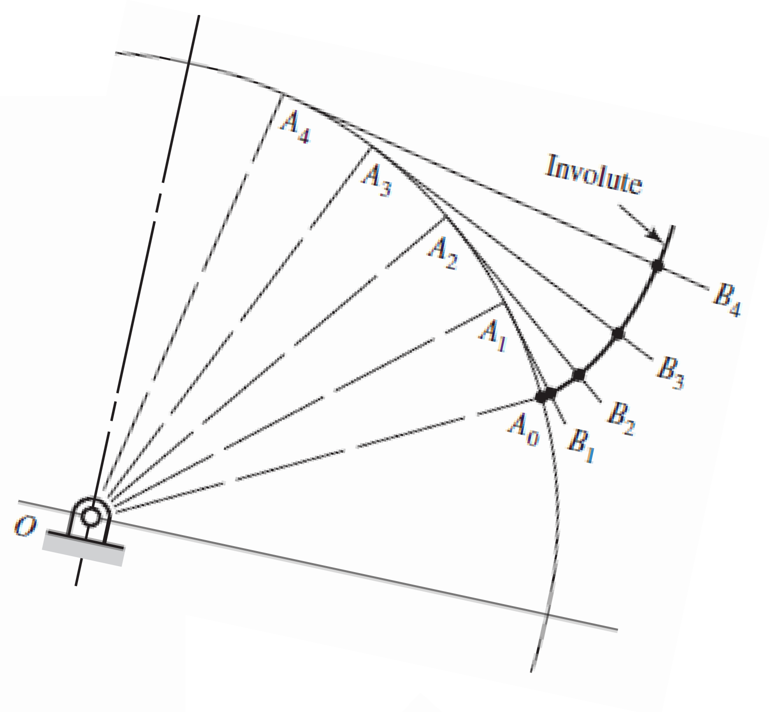
5.4 INVOLUTE PROPERTIES

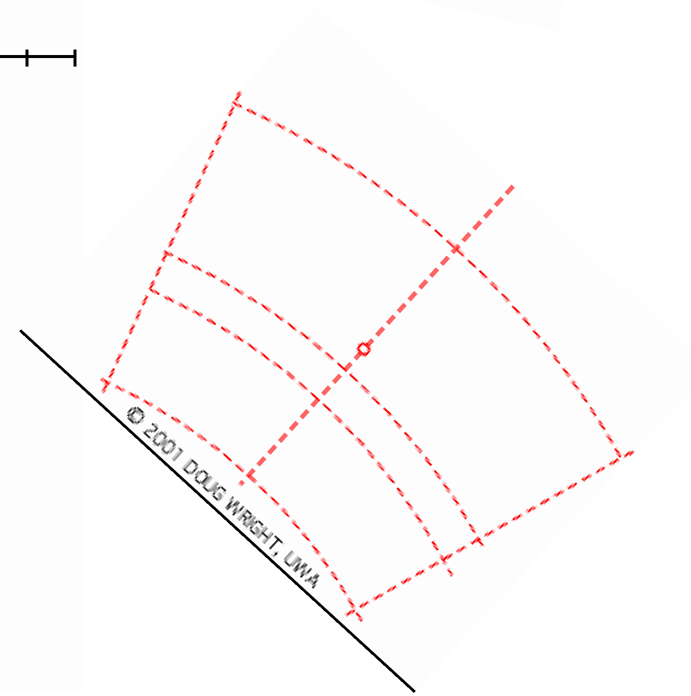
The involutes are thus generated simultaneously by the tracing point. The tracing point, therefore, represents the point of contact, while the portion of the cord ***ab*** is the generating line.

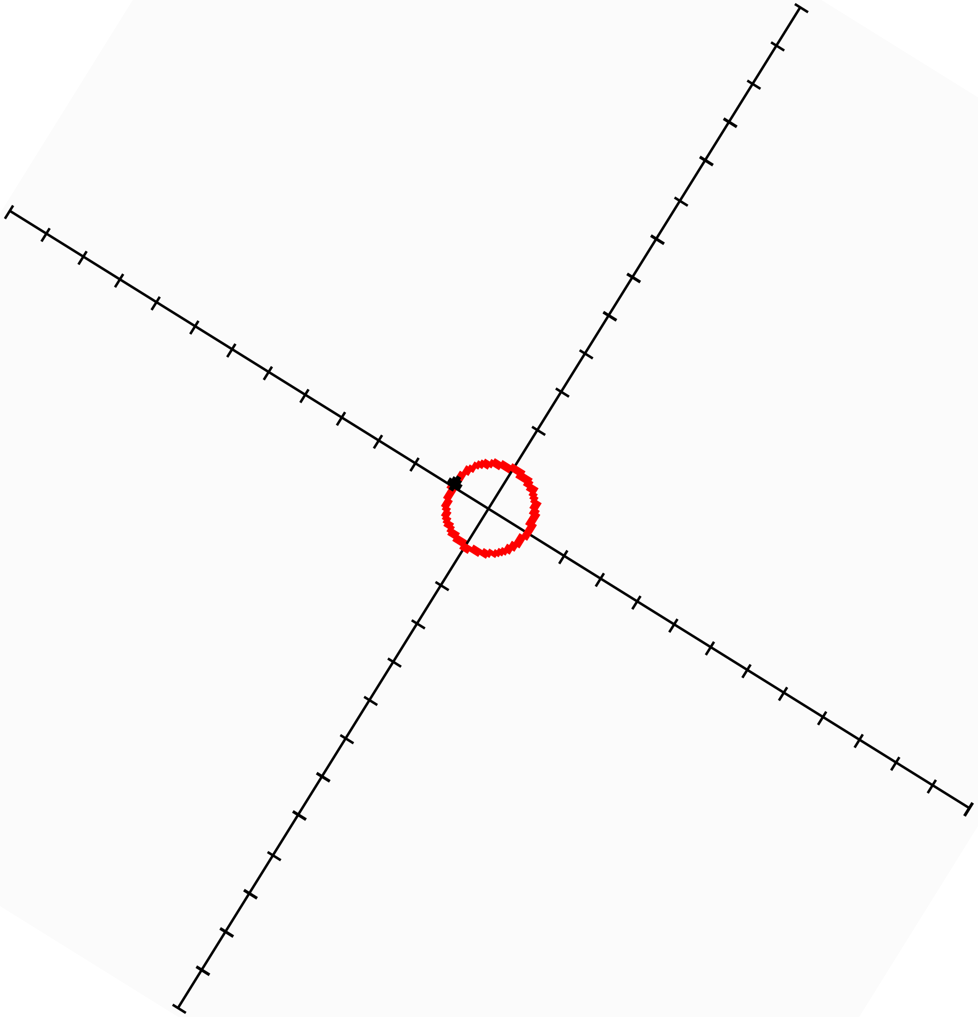


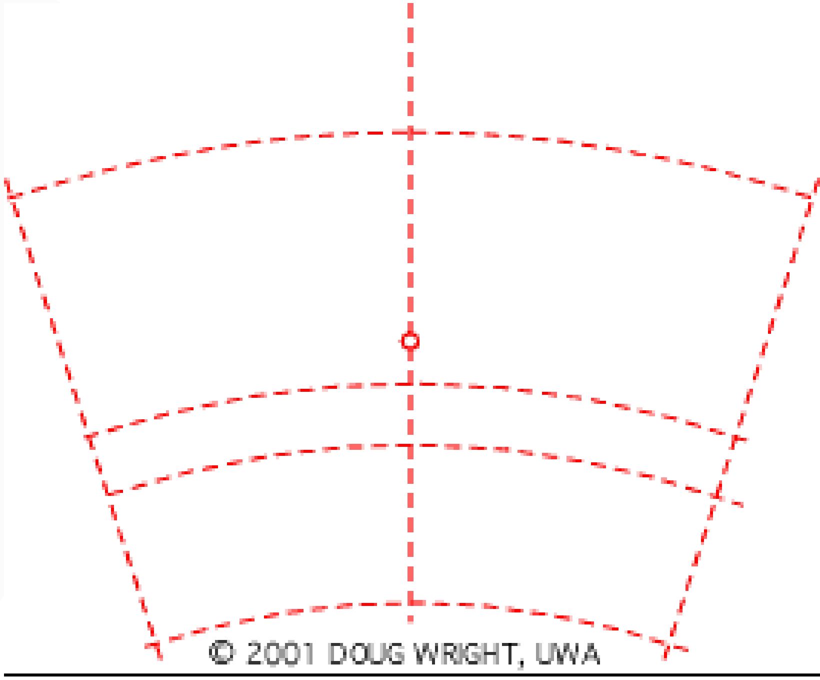


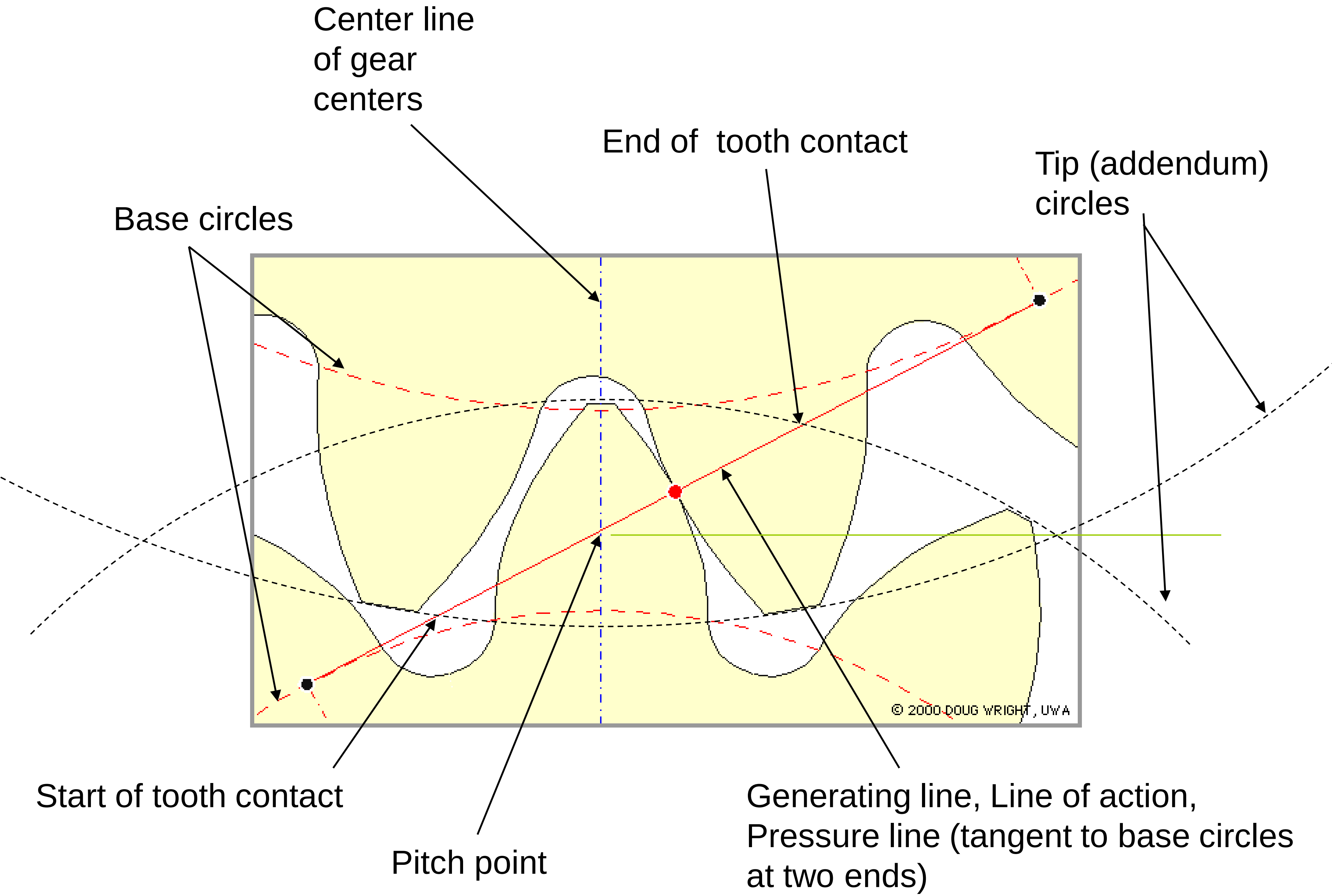



Involute
A_4
A_3
A_2
A_1
A_0
B_4
B_3
B_2
B_1
O


© 2001 DOUG WRIGHT, UWA




© 2001 DOUG WRIGHT, UWA

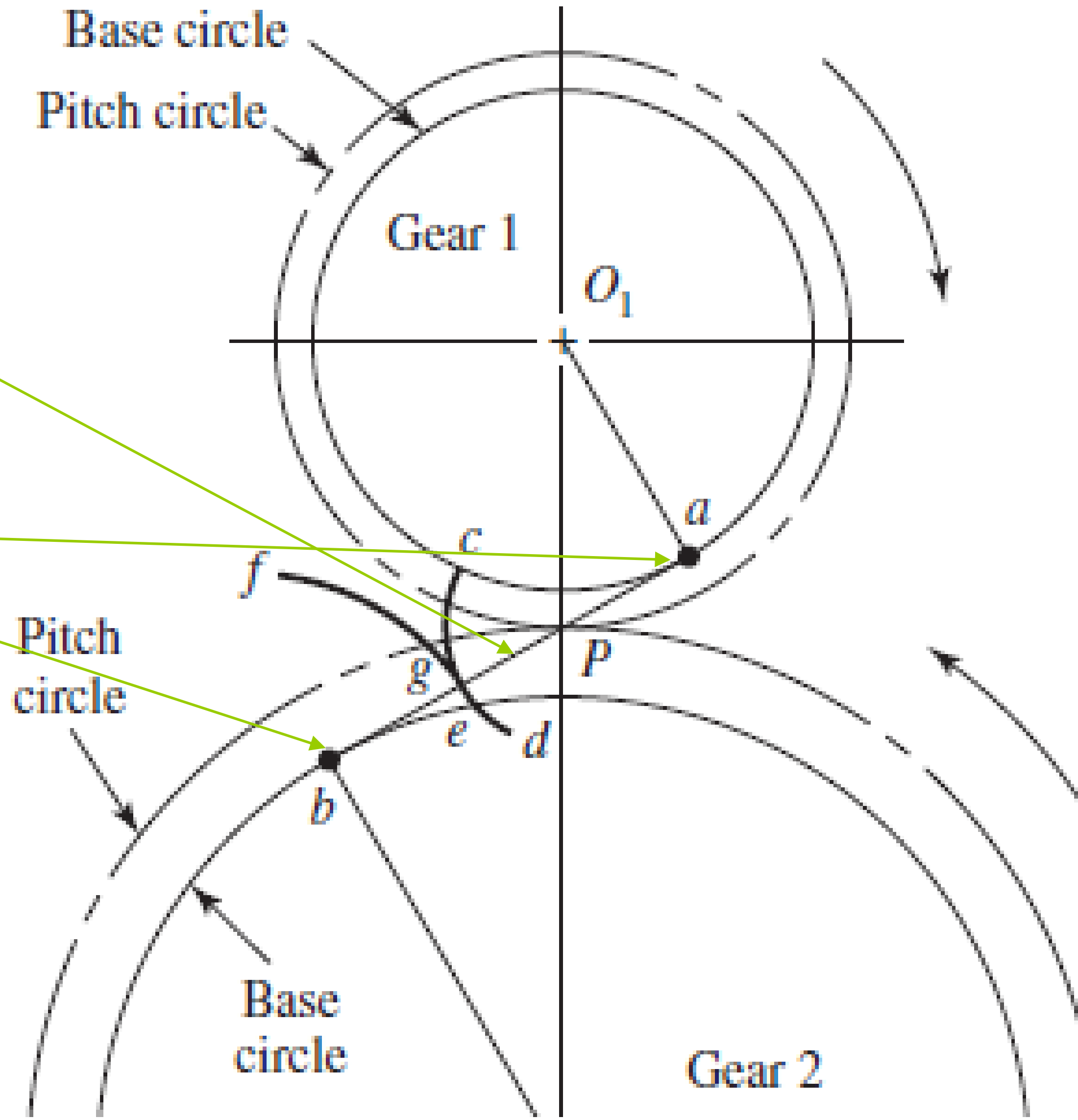

Center line of gear centers
End of tooth contact
Tip (addendum) circles
Base circles
© 2000 DOUG WRIGHT, UWA
Start of tooth contact
Pitch point
Generating line, Line of action, Pressure line (tangent to base circles at two ends)

5.5 FUNDAMENTALS

<u>The point of contact moves along the generating line;</u>

<u>the generating line does not change position, because it is always tangent to the base circles;</u>

<u>and since the generating line is always normal to the involutes at the point of contact, the requirement for uniform motion is satisfied.</u>



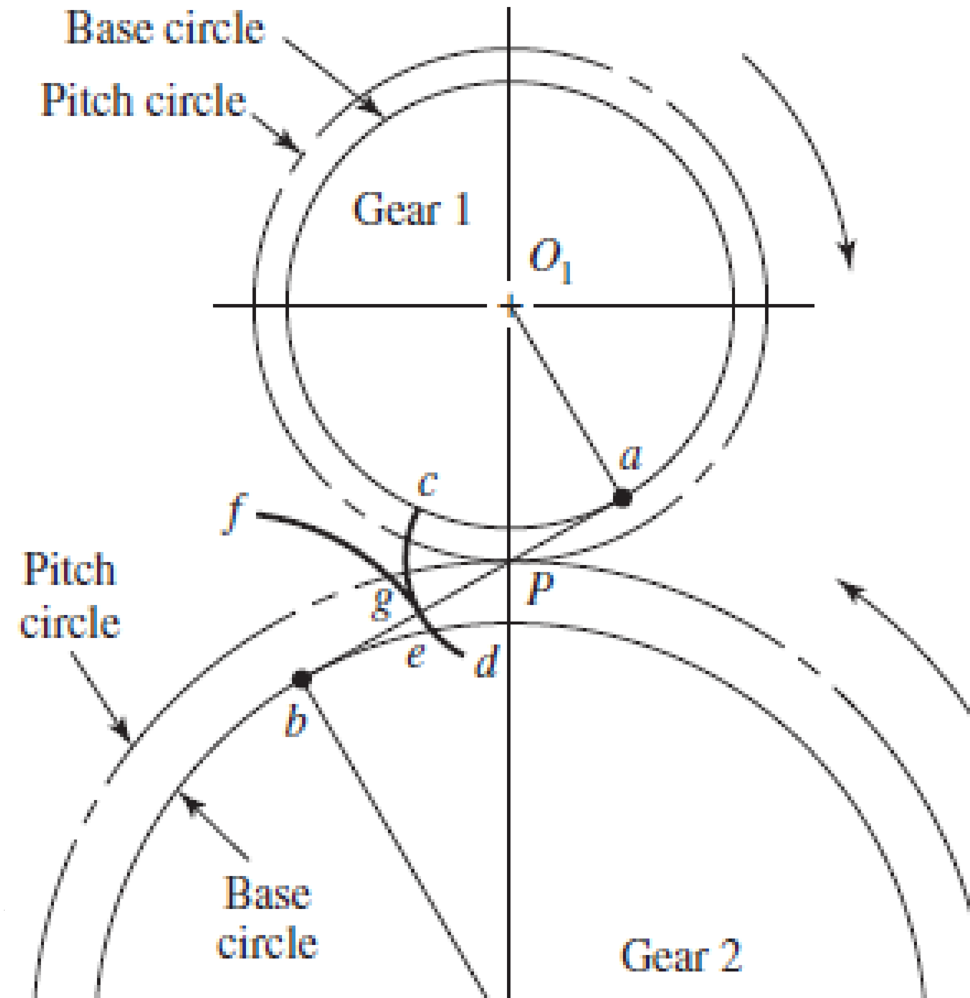
5.5 FUNDAMENTALS

When two gears are in mesh, their pitch circles roll on one another without slipping. Designate the pitch radii as r_1 and r_2 and the angular velocities as ω_1 and ω_2, respectively. Then the pitch-line velocity is

$$V = |r_1\omega_1| = |r_2\omega_2|$$

Thus the relation between the radii on the angular velocities is

$$\left|\frac{\omega_1}{\omega_2}\right| = \frac{r_2}{r_1}$$



5.5 FUNDAMENTALS

Suppose now we wish to design a speed reducer such that the input speed is 1800 rev/min and the output speed is 1200 rev/min.

This is a speed decreasing ratio of 3:2; the pitch diameters would be in the same ratio, for example, a 4-in pinion driving a 6-in gear.

Or we could start with tooth numbers and diametral pitch and then find diameters of gears. The various dimensions found in gearing are always based on the pitch circles.

Suppose we specify that an 18-tooth pinion is to mesh with a 27-tooth gear and that the diametral pitch of the gear set is to be 6 teeth per inch.

Then, from Eq. ($P = \frac{N}{d}$), the pitch diameters of the pinion and gear are, respectively,

$$d_1 = \frac{N_1}{P} = \frac{18}{6} = 3\, in \qquad d_2 = \frac{N_2}{P} = \frac{27}{6} = 4.5\, in$$