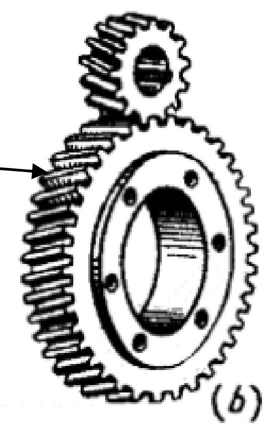
ME 308
MACHINE ELEMENTS II

CHAPTER 6

HELICAL GEARS

This chapter is about helical gears of which teeth are inclined to axis of gear at certain (helix) angles.

Such gears are therefore called helical gears.



Spur gears are actually special helical gears with helix angle of zero degrees

6. HELICAL GEARS

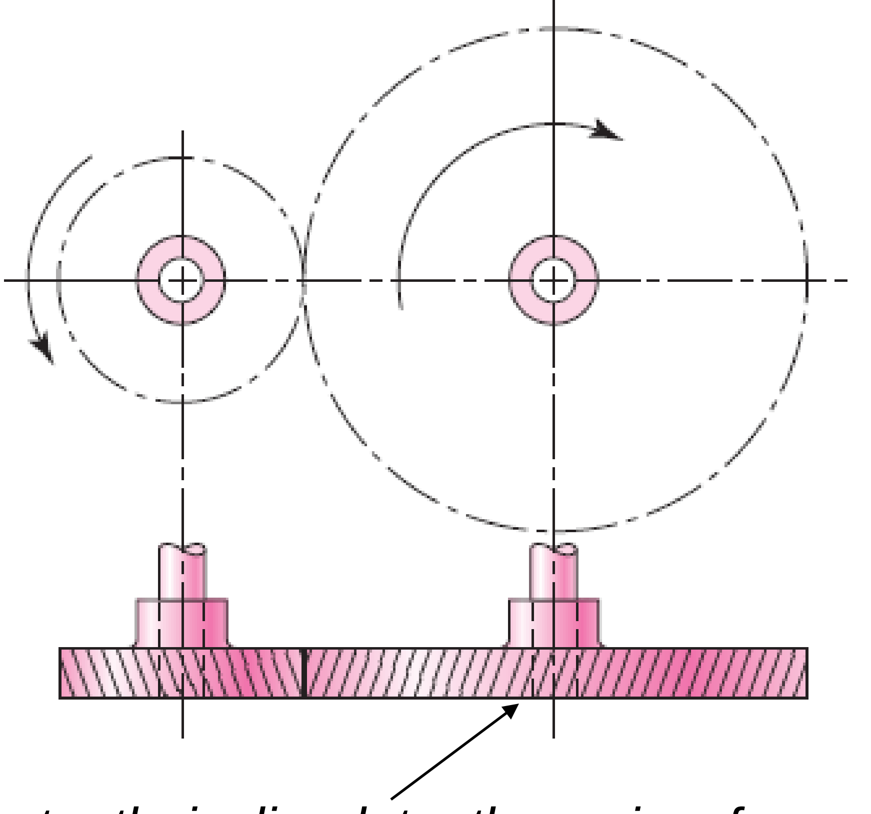


Fig.6.1 Helical gears are used to transmit motion between parallel or nonparallel shafts.

Helical gears, shown in Fig. 6.1, have *<u>teeth inclined to the axis</u>* *of rotation. Helical* gears can be used for the same applications as spur gears and, when so used, are not as noisy, because of the more gradual engagement of the teeth during meshing.

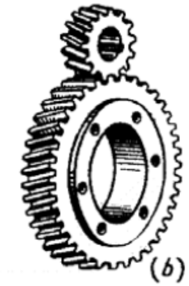
The inclined tooth also develops thrust loads and bending couples, which are not present with spur gearing. Sometimes helical gears are used to transmit motion between nonparallel shafts too.

6.1 HELICAL GEARS

In contrast to spur gears these helical gears can be used to transmit motion between non-parallel shafts too.

- **Helical gears transmit motion between;**

- Parallel shafts

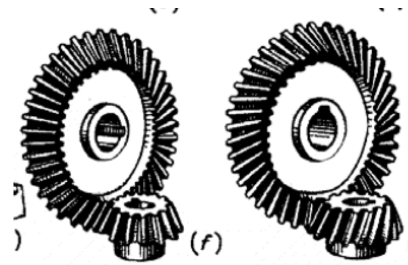


- Non-parallel and non-intersecting shafts.

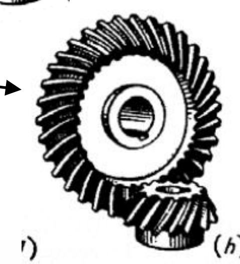
(j)

- **Bevel gears transmit motion between;**

- Non-parallel intersecting shafts



- Non-parallel and non-intersecting shafts.



- **Worm gears transmit motion between;**

- Non-parallel and non-intersecting shafts .

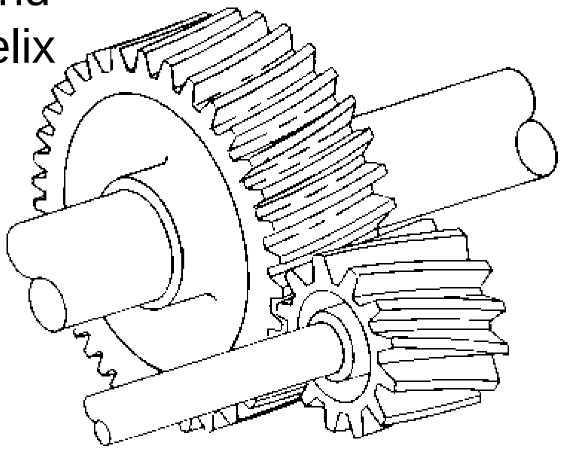
(i)

Parallel shaft helical gears

- The teeth of the helical gears are not parallel to the axis of the gear but inclined at an angle ψ called helix angle.

- ψ is the same for both gears (pinion and gear) but the hand of the helix is opposite, e.g. RH helix for gear and LH helix for pinion or vice versa.

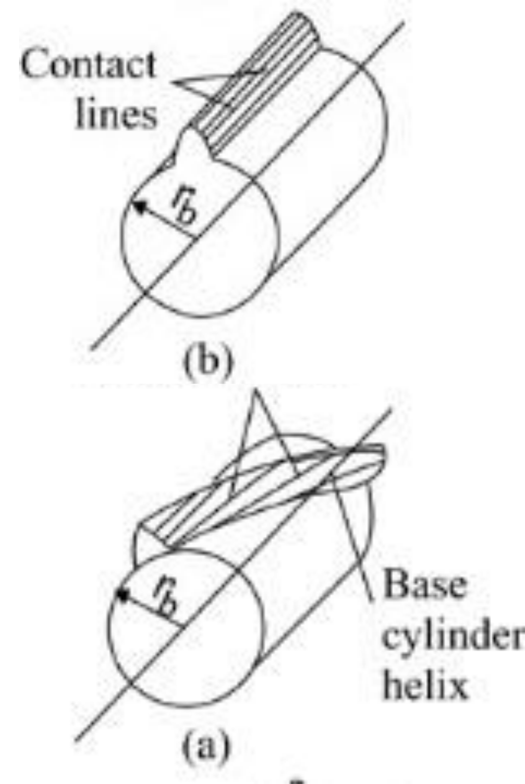
Left hand (LH) helix



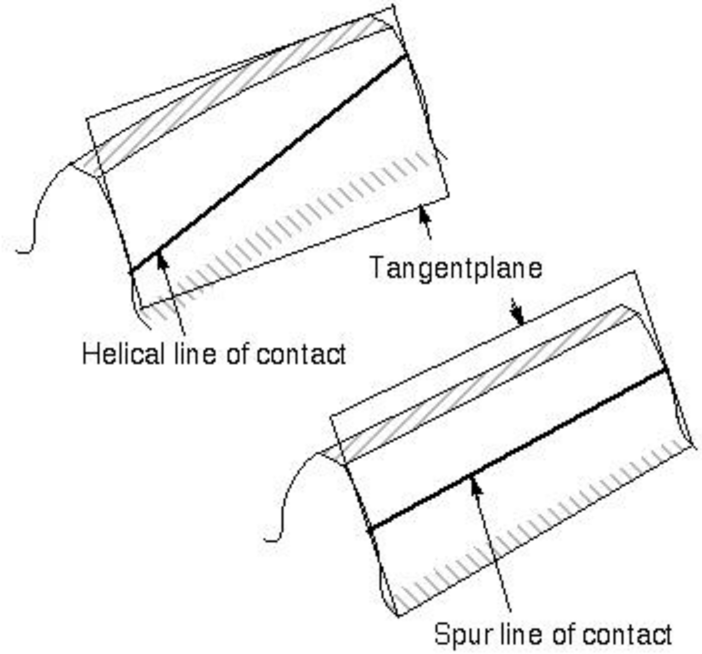
Right hand (RH) helix

- While the contact on spur gears was a line always from start of engagement to the end of engagement with a length equal to face width,

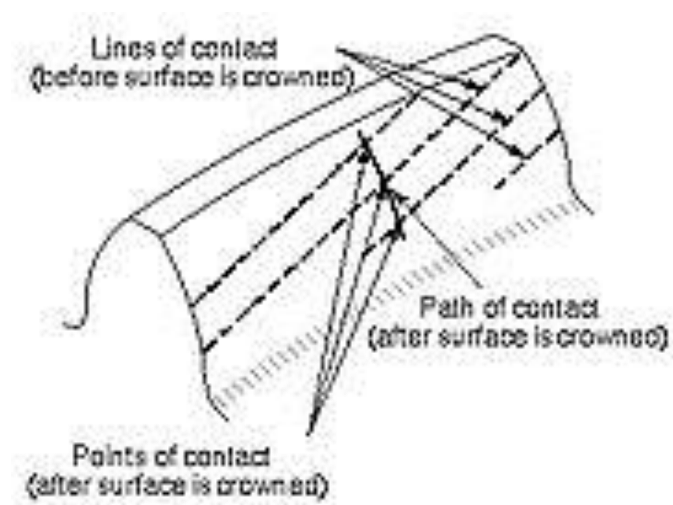
- The contact on the helical gear tooth starts as a point at one corner and changes into an inclined line as the more teeth come into contact. It leaves the contact at other end of the tooth as a point again.



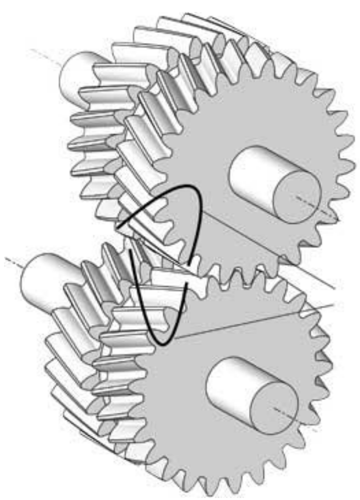
- Due to the helix angle ψ there is a (more) gradual engagement of the teeth and the smooth transfer of the load from one tooth to another which give helical gears the ability to transmit heavy loads at high speeds compared to the spur gears.

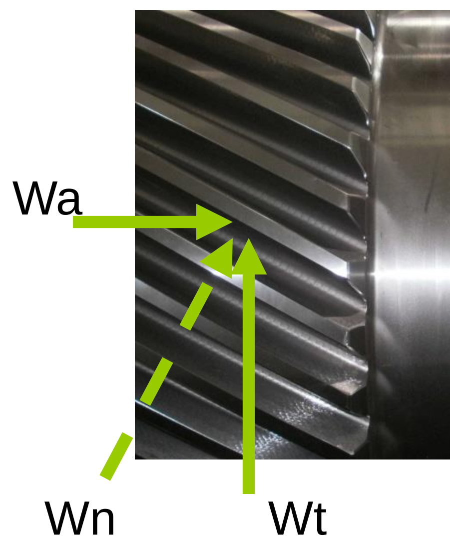


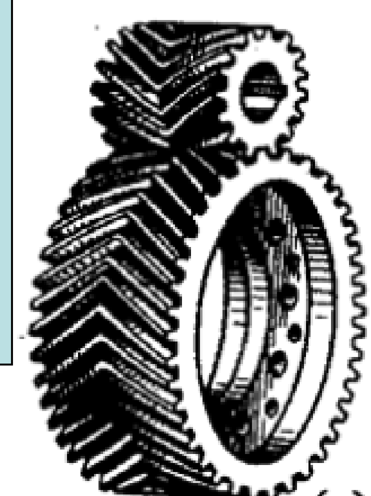
- Due to the helix angle, at one time more teeth on the gears are in contact hence increasing the average number of teeth in contact which means the contact ratio of gears gets larger.



- Due to helix angle *ψ* the tooth load *W* will have an axial component W_a (thrust load) which creates axial bearing loads and also a bending moment on the shaft due to gear radius.
- When W_a becomes large it may create problems in design of the other machine elements such as bearings, shafts etc.
- In such cases two helical gears on the same shafts but with opposite hands can be used to eliminate negative effects of W_a 's .
- Or a herringbone (double helical) gear can be used to cancel effects of W_a ' s.






(c)

Geometry of helical gear tooth

In helical gear geometry there are basically two planes:

1-transverse (front) plane

2- normal plane

Transverse(front) plane is perpendicular to the axis of gear

Whereas normal plane is perpendicular to the gear teeth itself

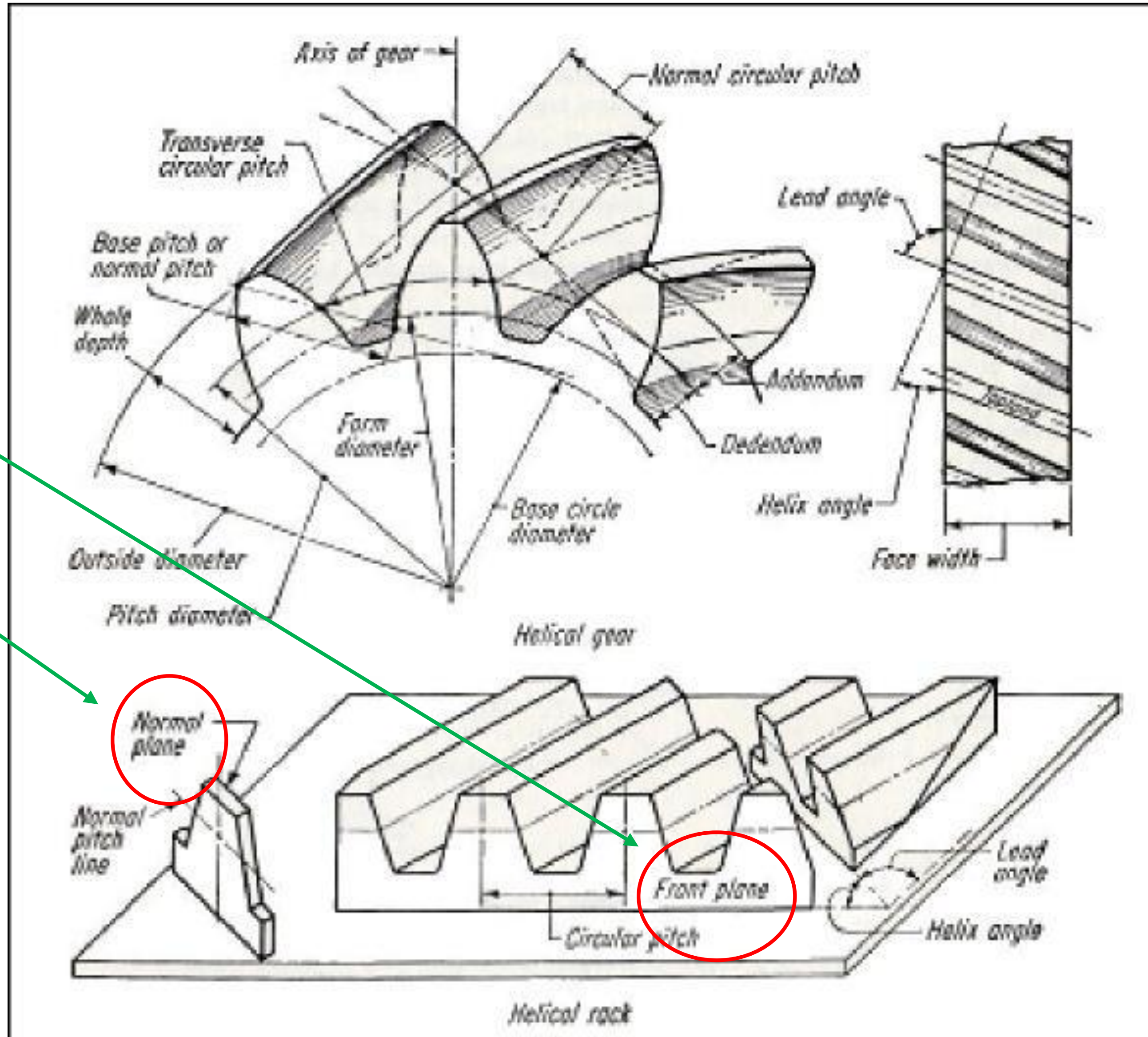


Fig 2: Helical Gear Definitions

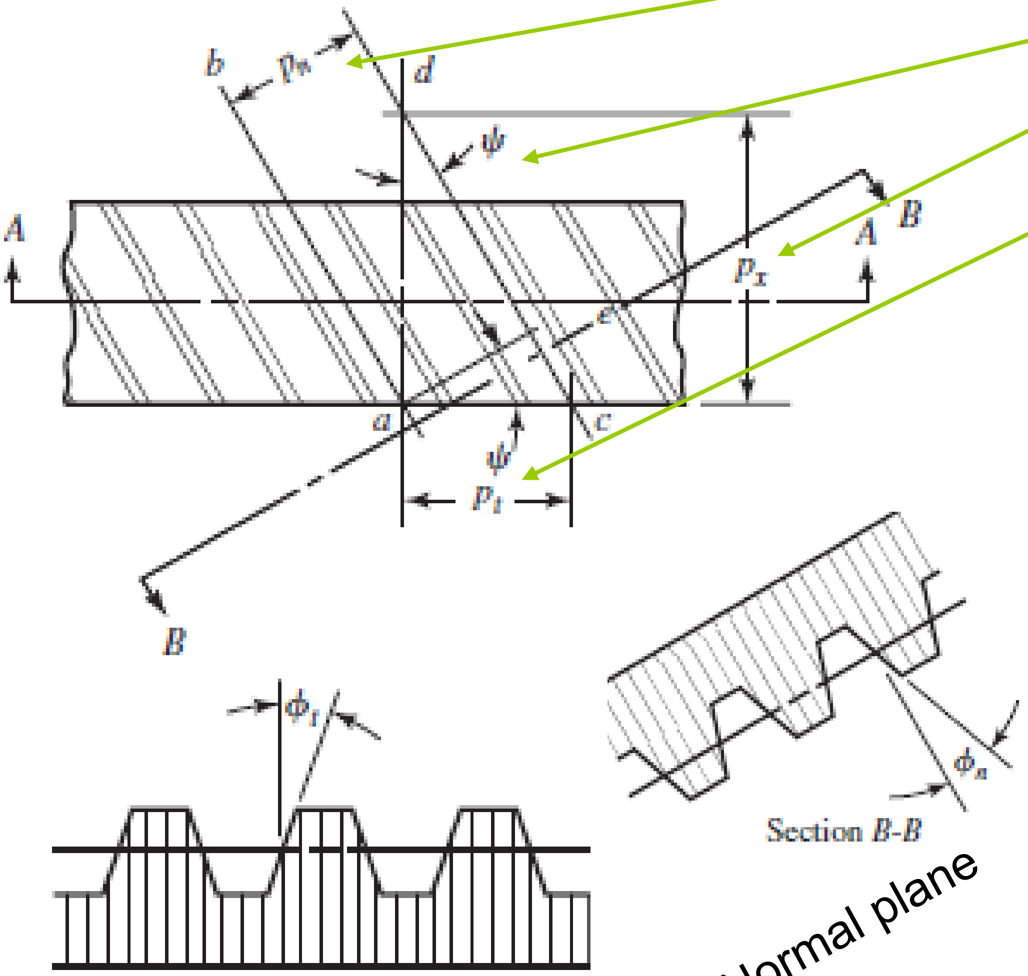
Geometry of helical gear tooth

p_n : normal pitch (a-e)

Ψ : helix angle

p_x: axial pitch (a-d)

p_t : (circular) transverse pitch (a-c)



Section A-A

Transverse plane

Normal plane

For a safe and gradual, smooth power transmission, similar to $3p_c < F < 5p_c$ in spur gears, in helical gears it is recommended that

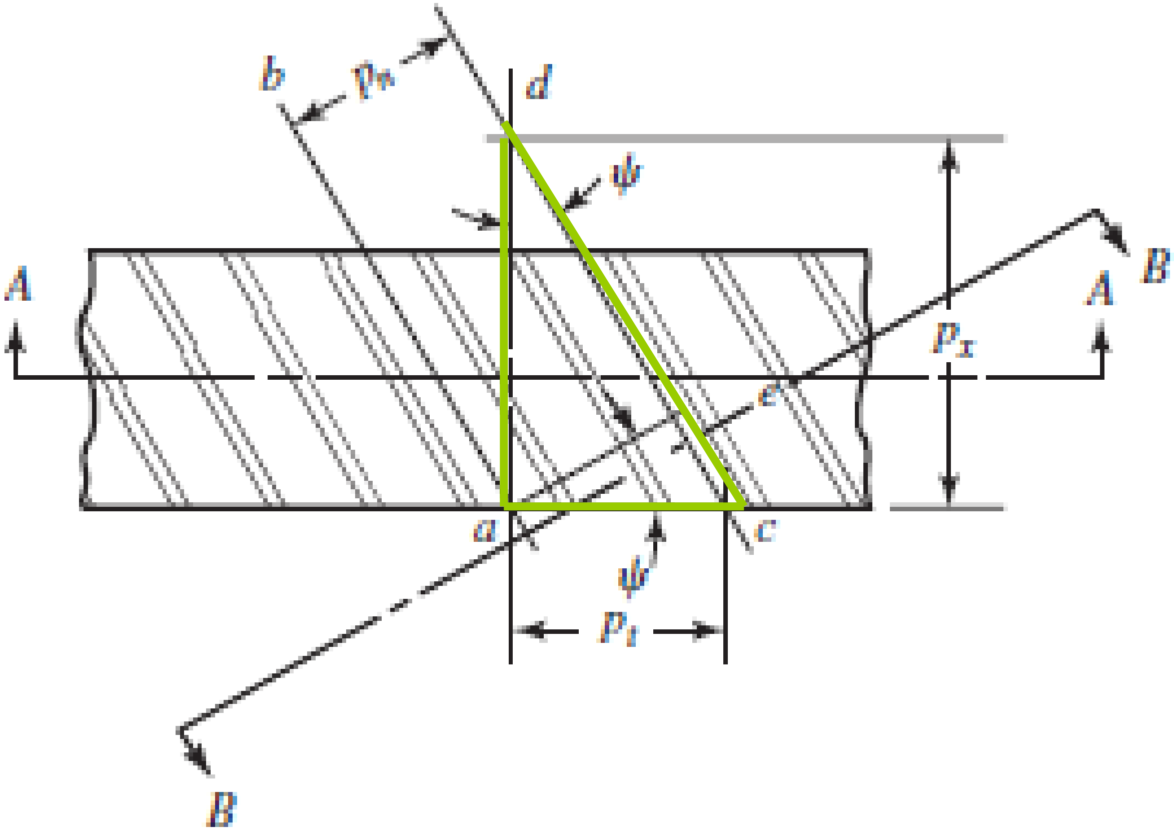
$$F \geq 2p_x$$

but this is not obligatory.

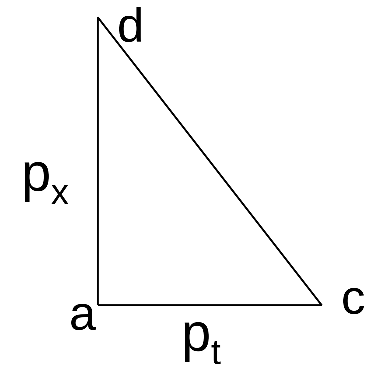
$$axial\_cr = \frac{F}{p_x}$$

$$axial\_cr \geq 2$$

Geometry of helical gear tooth



For triangle acd



$$\tan\psi = \frac{ac}{ad} = \frac{p_t}{p_x}$$

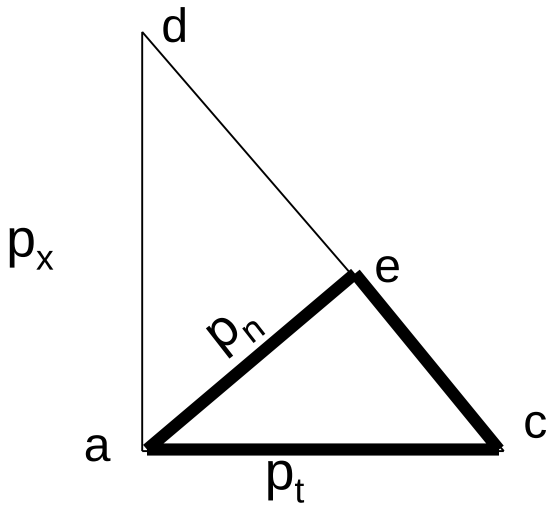
$$\Rightarrow p_x = \frac{p_t}{\tan\psi}$$

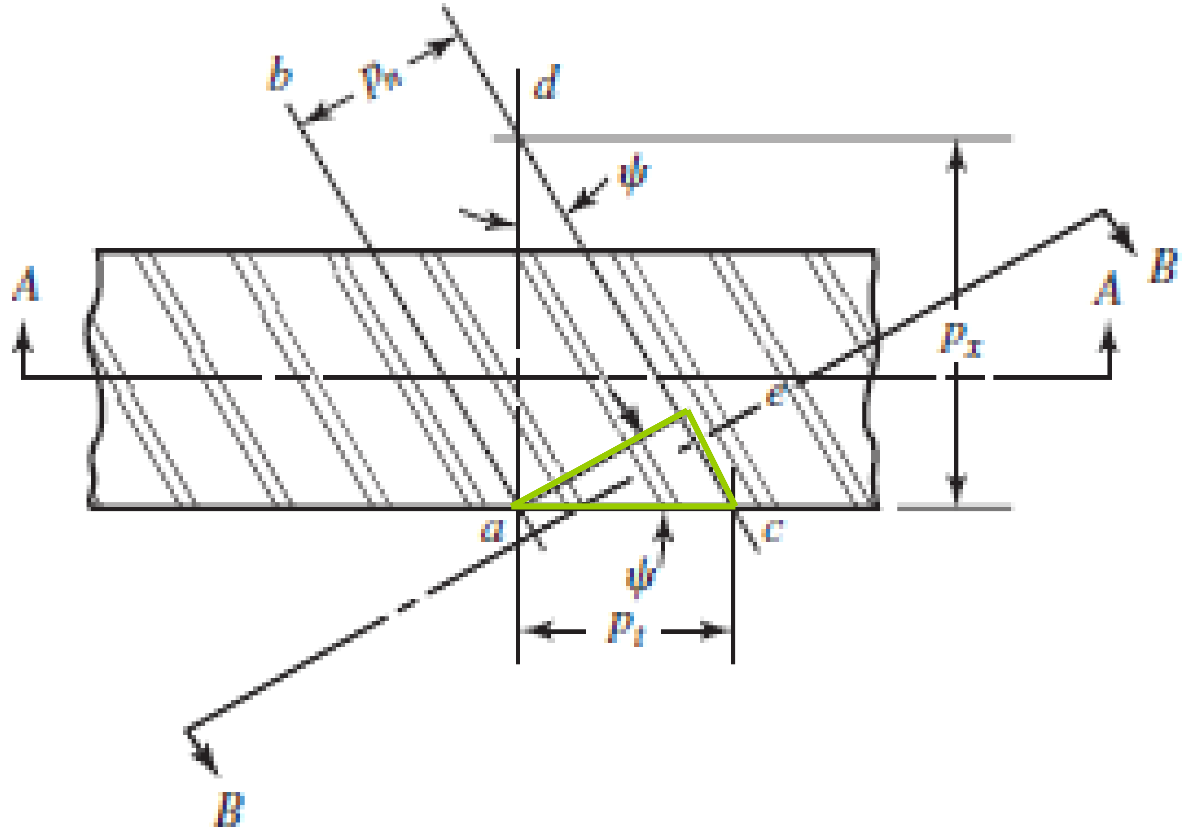
$F \geq 2p_x$ is a recommendation for helical gear action

Also a relation between pressure angles in normal and transverse sections is given as:

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}$$

For triangle ace





$$\cos\psi = \frac{ae}{ac} = \frac{p_n}{p_t}$$

$$\Rightarrow p_n = p_t \cos\psi$$

$$\pi m_n = \pi m_t \times \cos\psi$$

$$m_n = m_t \times \cos\psi$$

$$m_t = \frac{m_n}{Cos\,\psi}$$

$$m = m_t \quad \text{some times used}$$

$$d = m_t \times T_{P,G}$$

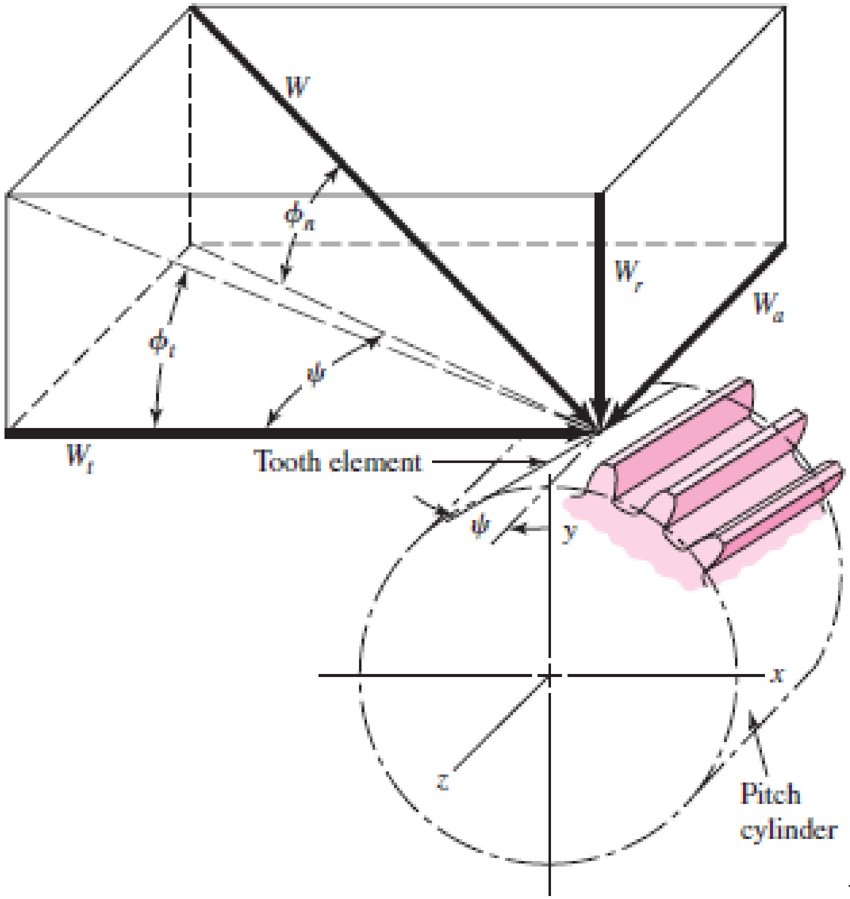
Tooth proportions are generally based on 20^o normal pressure angle ($\phi_n = 20^o$)

Table 13.1 used for helical gears in such case.

Tooth dimensions are calculated by using normal module m_n for helix angle ψ between $0\text{-}30^o$.

$F \geq 2p_x$ is a recommendation for helical gear action

Force analysis in helical gears



$$W: \text{ tooth load}$$

$$W_t: \text{ transverse tooth load (tangential)}$$

$$W_r: \text{ radial tooth load}$$

$$W_a: \text{ axial tooth load (thrust)}$$

$$W_t = (W \times \cos\phi_n) \times \cos\psi$$

$$W_a = (W \times \cos\phi_n) \times \sin\psi$$

$$W_r = W \times \sin\phi_n$$

Since W_t is generally known or given other W_i values are calculated from W_t

$$W_t = \frac{\text{Power}}{\text{pitchline velocity}} = \frac{P(watt)}{V(m/s)}$$

$$W_t = \frac{60 \times P}{\pi \times d \times N} = \frac{60 \times P}{\pi(m \times T_{P,G})N}; \quad W = \frac{W_t}{\cos\psi \times \cos\phi_n}$$

$$W_r = W_t \times \tan\phi_t \quad W_a = W_t \times \tan\psi$$

Since there are 3 components of helical gear force the axial and radial components create axial and radial reaction forces at bearing housings (to be kept in mind)

Helical gear strength analysis

Similar to spur gears, for helical gears too:
- tooth surface durability and
- tooth bending fatigue

are the 2 failure criteria.
- $F \geq 2p_x$ is the third design (not analysis) criteria.

both of these eqn's (bending and contact stresses) of spur gears are valid for helical gears too.

Bending fatigue failure

$$\sigma = \frac{W_t}{K_v \times F \times J \times m}$$

$$n_G = \frac{S_e}{\sigma} = n \times K_o \times K_m$$

surface fatigue

$$\sigma_H = -C_p \sqrt{\frac{W_{tp}}{C_v \times F \times d_p \times I}}$$

$$\text{or } \; S_H = C_p \sqrt{\frac{W_{tp} = n \times C_0 \times C_m \times W_t}{C_v \times F \times d_p \times I}}$$

Some exceptions to the use of spur gear equations are those:

1) Velocity factor $K_v = C_v = \sqrt{\dfrac{78}{78+\sqrt{200V}}}$ is used for helical gears.

2) The geometry factor is taken from Fig.14.8 for $\phi_n = 20^o$ but with a multiplier (M)

Fig.14.8.a assumes that the helical gear (pinion) meshes with another gear with $T=75$ (take J from here for $T_{G(P)} = ?$)

$$\psi = 20^o$$

$$T_P = 30$$

$$T_G = 60$$

$$J_G = ?$$

$$J_P = ?$$

$$J_G = J_{G(75)} \times M_G$$

$$J_P = J_{P(75)} \times M_P$$

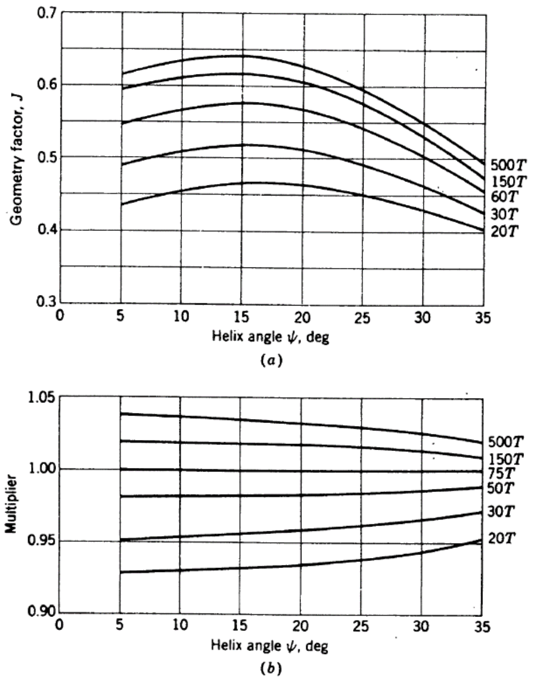
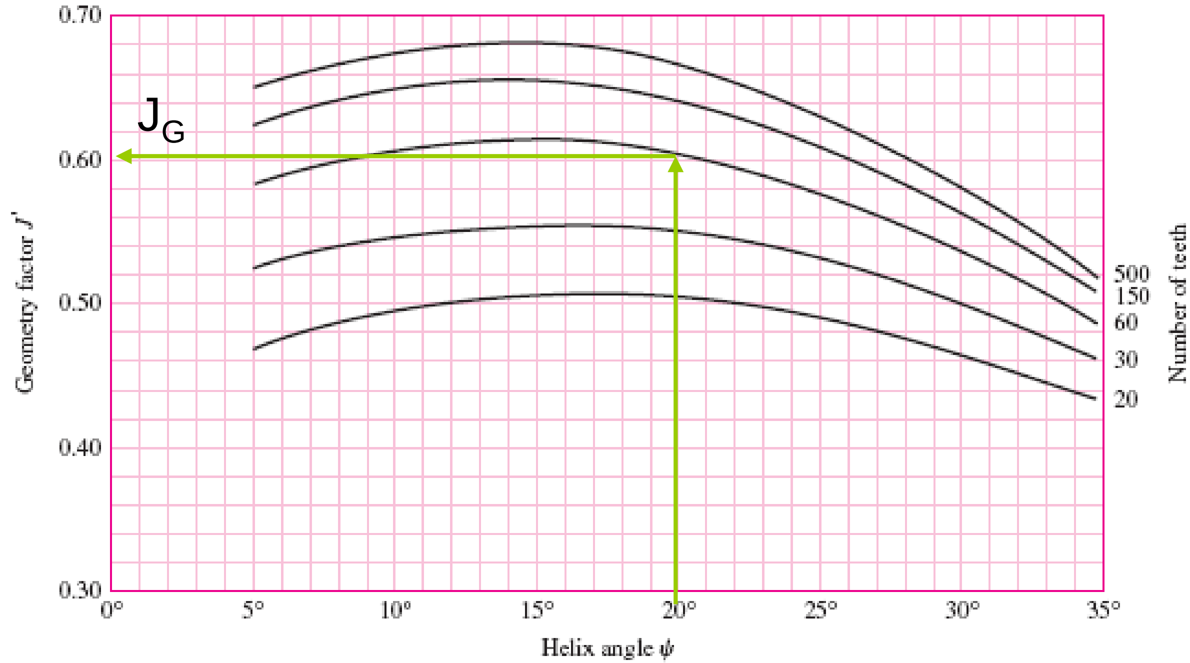
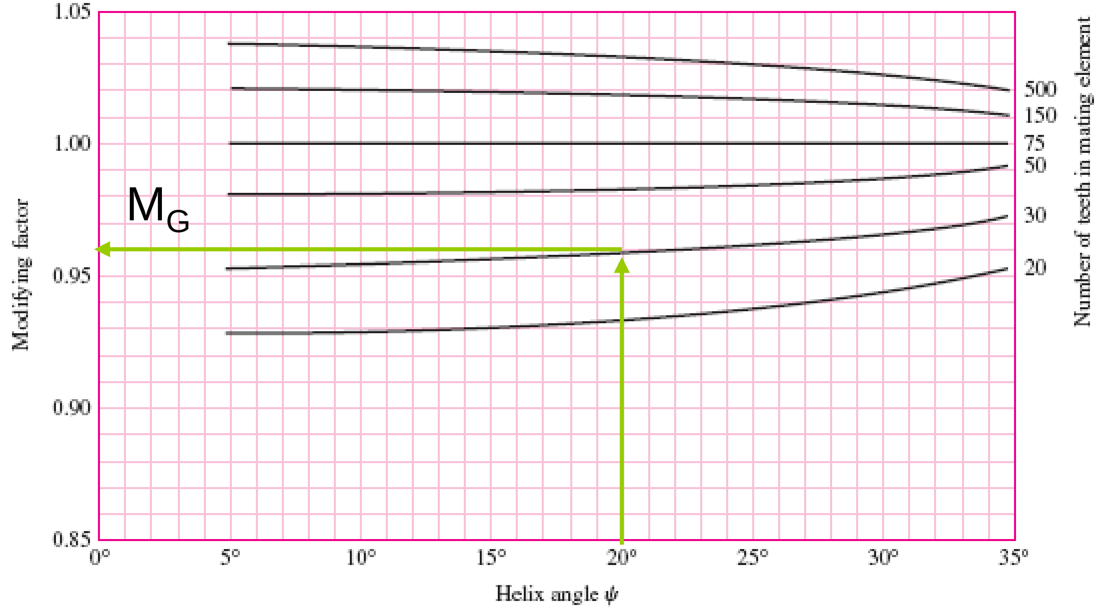


FIGURE 14-8 Geometry factors for helical and herringbone gears having a normal pressure angle of 20°. (*a*) Geometry factors for gears mating with a 75-tooth gear. (*b*) *J*-factor multipliers when tooth numbers other than 75 are used in the mating gear. (*AGMA Information Sheet* 225.01.)



J factor when other than 75 teeth are used in the mating element



$$\psi = 20^o$$

$$T_P = 30$$

$$T_G = 60$$

$$J_G = ?$$

$$J_G = J_{G(75)} \times M_G$$

$$J_G = 0.61 \times 0.96$$

$$J_G = 0.5856$$

$\psi = 20^o$

$T_p = 30$

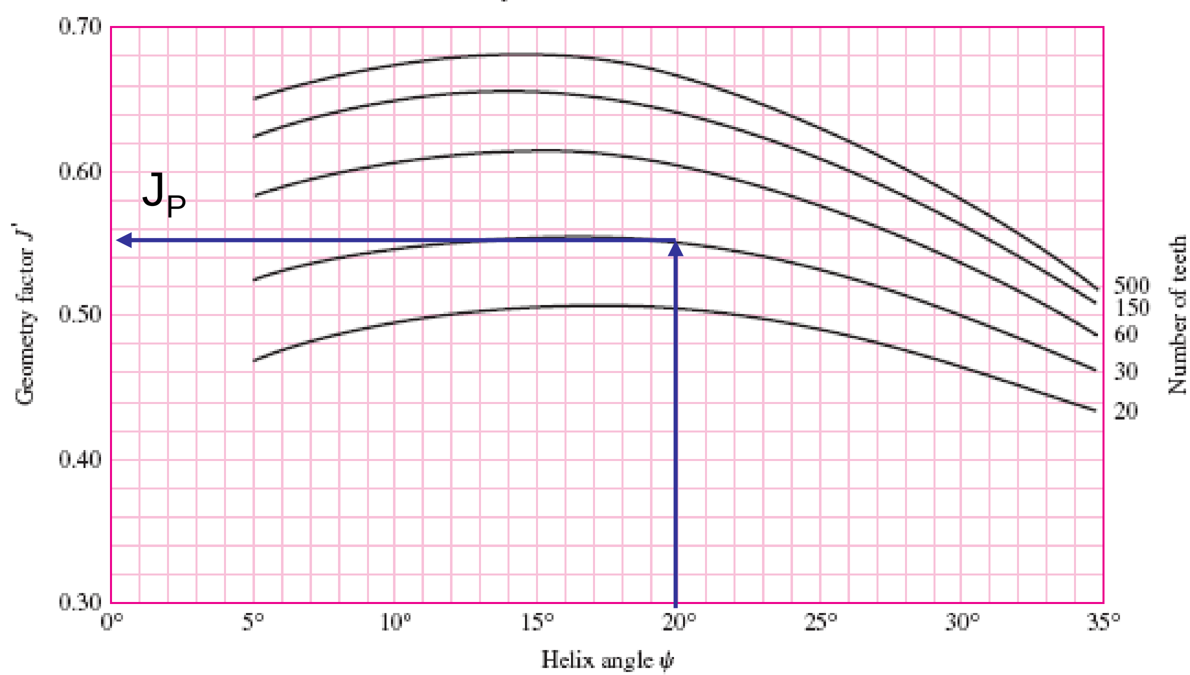
$T_G = 60$

$J_P = ?$

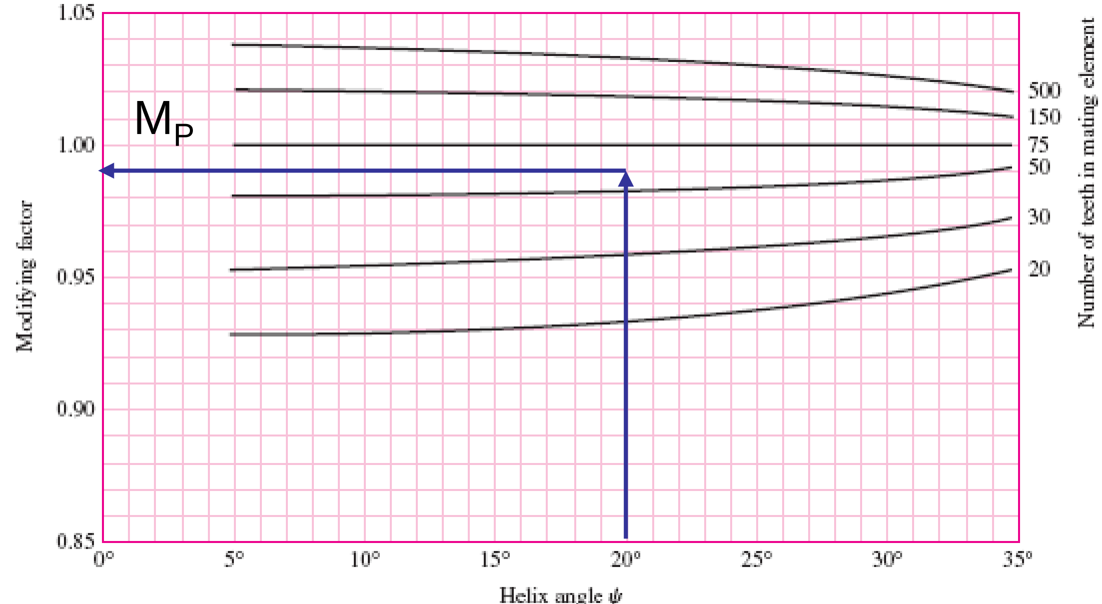
$J_P = J_{P(75)} \times M_P$

$J_P = 0.55 \times 0.99$

$J_P = 0.5445$



J factor when other than 75 teeth are used in the mating element



3) Calculate geometry factor I by using;

$$I = \frac{\sin\phi_t \times \cos\phi_t}{2m_N}\frac{m_G}{m_G+1} \quad and \quad m_N = \frac{p_N}{0.95Z}$$

$p_N =$ normal base pitch

$p_N = p_n \times \cos\phi_n$

$p_N = \pi m_n \times \cos\phi_n$

$Z =$ length of line of action in transverse plane

$$Z = \sqrt{\left(r_p + a\right)^2 - r_{bp}^2} + \sqrt{\left(r_g + a\right)^2 - r_{bg}^2} - \left(r_p + r_g\right)\sin\phi_t$$

1st term 2nd term 3rd term

Regardless of the sign, if either of the 1st or 2nd term is larger than the 3rd term then replace that large term by the 3rd term.

$r_p = \left(m_t \times T_p\right)/2$

$r_g = \left(m_t \times T_g\right)/2$

$r_{bp} = r_p \times \cos\phi_t$

$r_{bg} = r_g \times \cos\phi_t$

Also if effective outside radius (r_{eff}) of either of pinion or gear is less than *(r +a)* use (r_{eff}) instead of *(r +a).*

Contact ratio:

$$pcr = \text{profile (transverse) } cr = \frac{Z}{p_{bt}} = \frac{Z}{p_t \times \cos\phi_t};$$

$$pcr > 1.4 \text{(is usually required)}$$

$$acr = \text{axial cr} = \frac{F}{p_x} = \frac{\text{Face width}}{\text{axial pitch}}$$

Total cr= pcr + acr
e.g. 1.6 + 2= 3.6

acr for helical gears > 0
acr for spur gears = 0

$F \geq 2\,p_x$ $(\text{acr} \geq 2)$ is a recommendation for helical gear action

4) Most modification and correction factors of spur gear are used for helical gears except:

Use $K_m = C_m$ from Table 14.1 for helical gear (not from spur gears) and

C_H from Fig. 14.9 for helical gears

Table 14-1 LOAD-DISTRIBUTION FACTORS C_m AND K_m FOR HELICAL GEARS

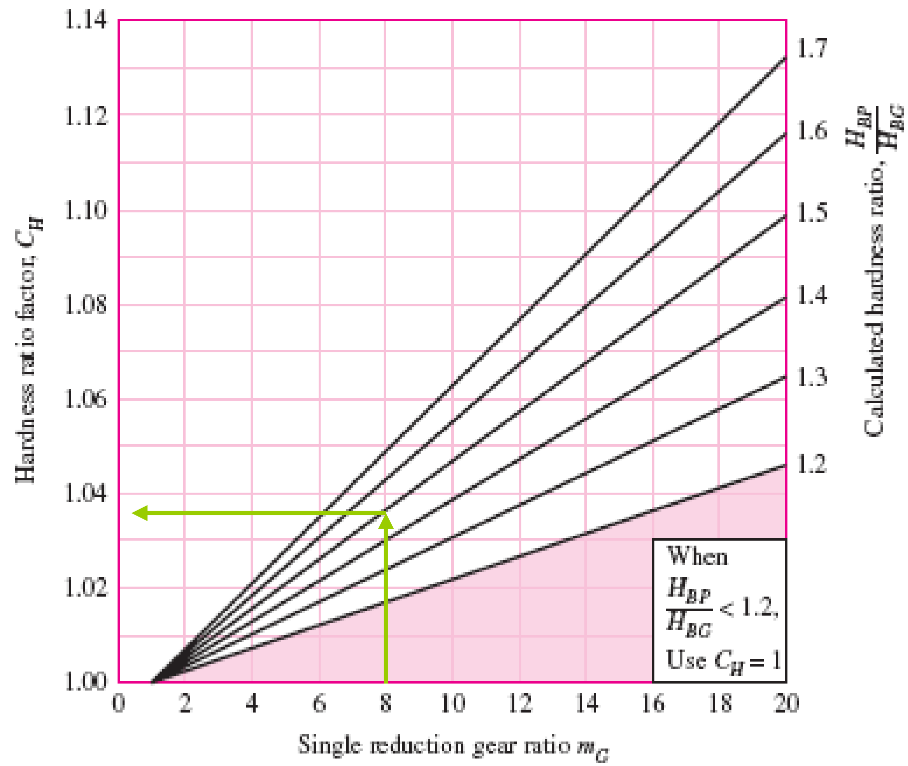
Characteristics of support	Face width, mm 0–50	150	225	400 up
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.2	1.3	1.4	1.7
Less rigid mountings, less accurate gears, contact across full face	1.5	1.6	1.7	2.0
Accuracy and mounting such that less than full-face contact exists	Over 2.0			

$$1.0 \le C_H \le 1.12$$

$$\text{If } K < 1.2; \quad C_H = 1.0$$

otherwise use Fig. 14.9.

$$\left(K = \frac{HB_P}{HB_G} \right)$$



Example 6.1 (14.1)

A parallel helical gear set has information of:
a LH pinion with 18 teeth meshes with a 32 teeth gear. Gear set has a helix angle of *25°* normal pressure angle of (ϕ_n = *) 20°* and a normal module of (m_n =) 3 mm with a facewidth of *(F=) 30 mm.*

Calculate:

Normal pitch $p_n = ?$

Transverse pitch $p_t = ?$

Axial pitch $p_x = ?$

Normal base pitch $p_N = ?$

Tansverse pressure angle $\phi_t = ?$

Pinion pitch diameter $d_P = ?$

Gear pitch diameter $d_G = ?$

$$\cos\psi = \frac{P_n}{P_t} = \frac{m_n}{m_t}; \quad m_t = \frac{m_n}{\cos\psi} = \frac{3}{\cos 25} = 3.31\, mm$$

$$P_n = \pi \times \mathrm{m_n} = \pi \times 3 = 9.424\, mm$$

$$P_t = \pi \times \mathrm{m_t} = \pi \times 3.31 = 10.398\, mm$$

$$\tan\psi = \frac{P_t}{P_x}; \quad P_x = \frac{P_t}{\tan\psi} = \frac{10.398}{\tan 25} = 22.298\, mm$$

$$P_N = P_n \cos\phi_n = 9.424 \times \cos 20^o = 8.855\, mm$$

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}; \quad \tan\phi_t = \frac{\tan\phi_n}{\cos\psi} = \frac{\tan 20}{\cos 25} = 0.4015$$

$$\phi_t = 21.88^o$$

$$d_P = m_t \times T_P = 3.31 \times 18 = 59.58\, mm$$

$$d_G = m_t \times T_G = 3.31 \times 32 = 105.92\, mm$$

Example: 6.2

Check if the helical gear set given below can transmit 1.25 kW power at a pinion speed of 400 rpm safely or not with a safety factor of 4 and reliabilty 99 %. Also find the tooth load components too.

Pinion(steel)

$$HB = 180$$

$$S_{ut} = 550\ MPa$$

$$LH \ \text{helix} \quad \psi = 20^{\circ}$$

$$\phi_n = 20^{\circ} \quad \text{(full depth)}$$

$$N_P = 400\ rpm$$

$$F = 100\ mm$$

$$m_n = 6\ mm$$

$$T_P = 18\ teeth$$

Gear (CI)

$$HB = 196$$

$$S_{ut} = 250\ MPa$$

$$T_G = 27\ teeth$$

$n_{bending}$=? >4 or <4?

$n_{contact}$=? >4 or <4?

Solution:

Check a) surface durability of pinion and gear.

b) bending fatigue of pinion and gear.

c) recommended $F \geq 2P_x$ but not obligatory.

$$m_t = \frac{m_n}{\cos\psi} = \frac{6}{\cos 20}$$

$$m_t = 6.385\, mm$$

Start with (c):

$$P_x = \frac{P_t}{\tan\psi} = \frac{\pi m_t}{\tan\psi} = \frac{\pi (m_n / \cos\psi)}{\tan\psi} = \frac{\pi m_n}{\sin\psi} = \frac{\pi \times 6}{\sin 20} = 55.11\, mm$$

$$2P_x = 2(55.11) = 110.22\ mm$$

Since F =100 mm < $2P_x$ a suitable helix action does not occur but still can work.

a) $HB_P < HB_G$

$$180 < 196$$

Start checking with pinion.

$$(1)\quad S_H = C_p \sqrt{\frac{W_{tp}}{C_v F d_p I}} \qquad n_G = \frac{W_{tp}}{W_t}; \qquad n_G = n \times C_o \times C_m$$

(2) $S_H = \frac{C_L \times C_H}{C_T \times C_R} S_C$

$C_L = 1.0 \quad (Table\, 13.12)$

$C_H = 1.0 \quad K = \frac{HB_P}{HB_G} = \frac{180}{196} = 0.918 < 1.2$

$C_T = 1.0$

$C_R = 1.0 \quad (99\,\%) \quad (Table\, 13.12)$

(3) $S_C = 2.76 \times HB_P - 70\ MPa$

$S_C = 427\ MPa$

$S_H = 427\ MPa$

(4) $\mathrm{C}_P = 174 \quad \text{Steel on CI} \quad (\text{Table } 13.11)$

$F = 100\ mm$

(5) $d_P = m_t \times T_P = \frac{6}{\cos 20} \times 18 = 114.931\, mm$

$d_G = 1.5 \times d_P = 1.5 \times 114.931 = 172.396\, mm$

(6) $$C_V = \sqrt{\frac{78}{78+\sqrt{200V}}} \qquad V = \frac{\pi \times d \times N}{60} = \frac{\pi \times 0.114931 \times 400}{60}$$

$$C_V = 0.883 \qquad V = 2.407\ m/s$$

(7) $$I = \frac{\sin\phi_t \cos\phi_t}{2m_N} \times \frac{m_G}{m_G+1} \qquad \phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = 21.17$$

$$m_G = 1.5$$

$$m_N = \frac{P_N}{0.95 \times Z} \qquad P_N = P_n \times \cos\phi_n = \pi \times m_n \times \cos\phi_n$$

$$P_N = 17.712\ mm$$

$$Z = \text{length of line of action in transverse plane}$$

$$Z = \sqrt{\left(r_p + a\right)^2 - r_{bp}^2} + \sqrt{\left(r_g + a\right)^2 - r_{bg}^2} - \left(r_p + r_g\right)\sin\phi_t$$

1st 2nd 3rd

$$r_p = \frac{1}{2} \times m_t \times T_P = \frac{1}{2} \times \frac{6}{\cos 20} \times 18 = 57.465\ mm$$

$$r_{bp} = r_p \times \cos\phi_t$$

$$r_{bg} = r_g \times \cos\phi_t$$

$$r_g = 1.5 \times r_p = 86.198\ mm$$

$$r_{bp} = 57.465 \times \cos\phi_t = 53.586\ mm$$

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}$$

$$a = 1 \times m_n = 6\ mm$$

$$b = 1.25 \times m_n = 7.5\ mm$$

$$\phi_t = \tan^{-1}\left(\frac{\tan 20}{\cos 20}\right) = 21.17^o$$

$$r_{bg} = 86.198 \times \cos 21.17 = 80.380\ mm$$

1st 2nd 3rd

In $Z = 34.005 + 45.161 - 51.882$

Since neither of 1st and 2nd term is larger than the 3rd term. Then,

$Z = 27.284\ mm$

$$m_N = \frac{P_N}{0.95 \times Z} = \frac{17.712}{0.95 \times 27.284} = 0.6833$$

$$I = \frac{\sin\phi_t \times \cos\phi_t}{2 \times m_N} \times \frac{m_G}{m_G + 1} = \frac{\sin 21.17 \times \cos 21.17}{2 \times 0.6833} \times \frac{1.5}{1.5 + 1}$$

$$I = 0.14785$$

$$\text{profile (transverse) cr} = \frac{Z}{P_{bt}} = \frac{27.284}{\pi m_t \times \cos\phi_t} = \frac{27.284}{\pi\left(\dfrac{m_n}{\cos\psi}\right)\cos\phi_t}$$

$$pcr = \frac{27.284}{\pi\left(\dfrac{6}{\cos 20}\right)\cos 21.17} = 1.458$$

$$\text{axial cr} = \frac{F}{P_x} = \frac{100}{\dfrac{\pi \times m_n}{\sin\psi}} = \frac{100}{\dfrac{\pi \times 6}{\sin 20}} = 1.814$$

Total Contact Ratio

$$(cr) = pcr + acr = 1.458 + 1.814 = 3.272$$

(8) $$S_H = C_p \sqrt{\frac{W_{tp}}{C_v \times F \times d_p \times I}}$$

$C_o = 1.50$ Assume light shock moderate shock

$C_m = 1.60$ Less rigid mounting

$$W_{tp} = \left(\frac{S_H}{C_p}\right)^2 C_v \times F \times d_p \times I$$

$$W_{tp} = \left(\frac{427}{174}\right)^2 \times 0.883 \times 100 \times 114.93 \times 0.14785$$

$$W_{tp} == 9036\ N$$

$$W_t = \frac{60 \times P}{\pi \times d \times N} = \frac{60 \times 1.25 \times 10^3}{\pi \times (0.114931) \times 400}$$

$$W_t = 520\ N$$

$$W_{tp} = W_t \times n_G = W_t \times n \times C_o \times C_m$$

$$n = \frac{W_{tp}}{W_t \times C_o \times C_m} = \frac{9036}{520 \times 1.5 \times 1.6}$$

$n = 7.24 > 4.0$ <u>*SAFE*</u>

Hertzian safety factor for pinion is more than required, that means the power could be safely transmitted.

b) Check for bending fatigue failure $\left(S_{e_{pin}} \,{>?}\!/\!{?<}\, S_{e_{gear}}\right)$

$$S_{e_p} = ?(steel) \quad S_e^{'} = 0.5 S_{ut} = 0.5 \times 550 = 225\ MPa\ (S_{ut} < 1400\ MPa)$$

$$S_{e_g} = ?(CI) \quad S_e^{'} = 0.45 S_{ut} = 0.45 \times 250 = 112.5\ MPa\ (S_{ut} < 600\ MPa)$$

Most likely that even *with* k_a , k_b ,….k_f factors

$S_{e_p} > S_{e_g}$ Thus bending fatigue can be checked for gear

$$n_{Global} = K_o \times K_m \times n_{b_{gear}} = \frac{S_{e_g}}{\sigma_b} \Rightarrow \qquad (n_{b_g} > 4.0)$$

$$S_{e_g} = k_a \times k_b \times k_c \times k_d \times k_e \times k_f \times S_e^{'}$$

$$S_{e_g} = k_a \times k_b \times k_c \times k_d \times k_e \times k_f \times 112.5 = \underline{96\ MPa}$$

0.8 0.894 0.897 1.0 1.0 1.33

$$n_{b_g} = \frac{S_{e_g}}{K_o \times K_m \times \sigma_b}$$

$$W_t = \frac{60 \times P}{\pi d N} = \frac{60 \times 1.25 \times 10^3}{\pi \times (0.114931) \times 400}$$

$$W_t = 520\ Nwtn$$

$$K_0 = 1.50 \ \text{(light shock - moderate sh.)}$$

$$K_m = 1.60 \ \text{(less rigid mount.)}$$

$$K_V = C_V = \sqrt{\frac{78}{78 + \sqrt{200V}}} = 0.883$$

$$F = 100\ mm$$

$$\sigma_b = \frac{W_t}{F \times J \times K_v \times m_t}$$

$$m_t = d / T_P = 114.931/18$$

$$m_t = 6.385\ mm$$

$$\sigma_b = \frac{520}{100 \times 0.5022 \times 0.883 \times 6.385}$$

$$J = 0.54 \times 0.93 = 0.5022$$

$$\sigma_b = 1.83656\ MPa$$

$$\psi = 20^o \qquad \psi = 20^o$$

$$T_G = 27 \qquad T_P = 18$$

$$n_{bg} = \frac{S_{eg}}{K_o \times K_m \times \sigma_b} = \frac{96}{1.5 \times 1.6 \times 1.83656}$$

$n_{bg} = 21.78 > 4.0$ $\underline{SAFE}$! Safety factor of gear for bending fatigue

$$\psi = 20^o$$

$$T_P = 18$$

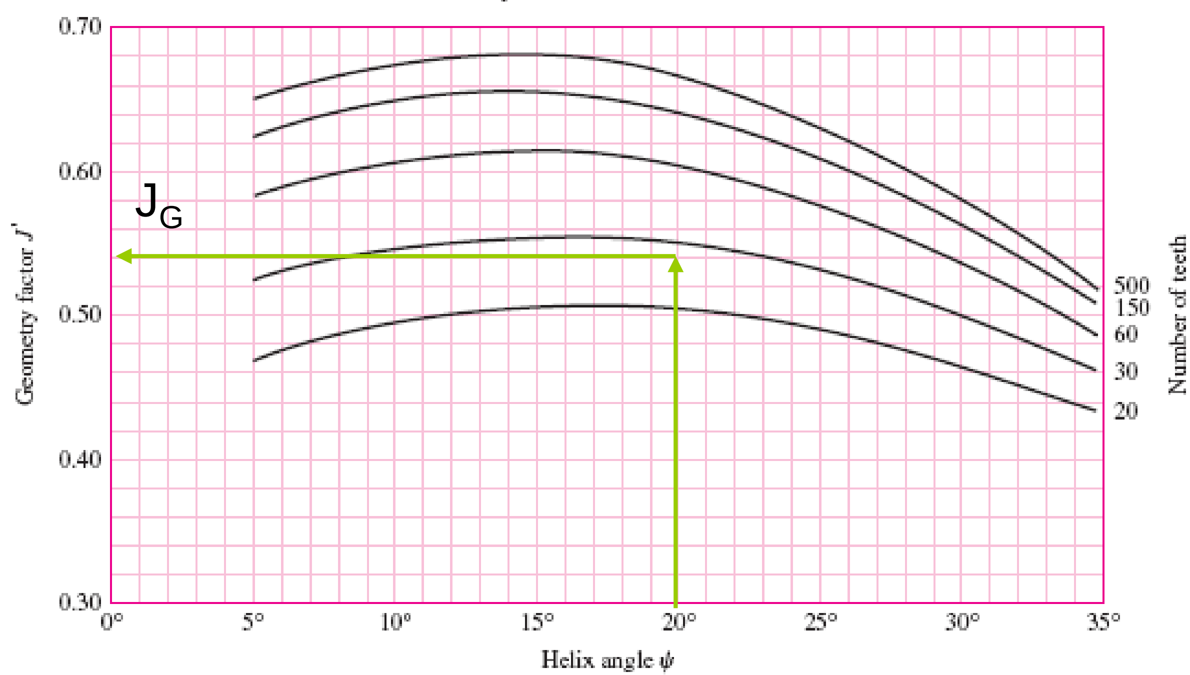
$$T_G = 27$$

$$J_G = ?$$

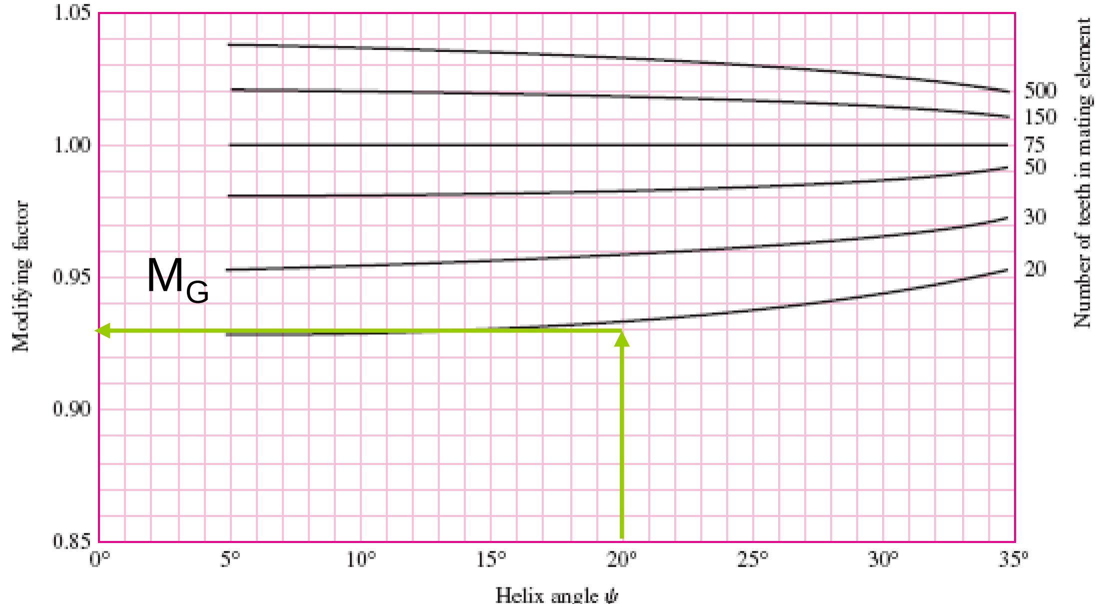
$$J_G = J_{G(75)} \times M_G$$

$$J_G = 0.54 \times 0.93$$

$$J_G = 0.5022$$



J factor when other than 75 teeth are used in the mating element



Tooth load components:

$$W_t = 520\ N \quad (\text{found from } \frac{60 \times P}{\pi \times d \times N})$$

$$W_r = W_t \times \tan\phi_t = 520 \times \tan 21.17 = 201.38\ N$$

$$W_a = W_t \times \tan\psi = 520 \times \tan 20 = 189.26\ N$$

$$W = \sqrt{W_t^2 + W_r^2 + W_a^2} = 588.87\ N$$

$$OR \quad W = \frac{W_t}{\cos\phi_n \times \cos\psi} = \frac{520}{\cos 20 \times \cos 20} = 588.88\ N$$

Example: 6.3

A helical gear set with speed reduction of *1.5:1, n=4* is to be designed.

Determine the required parameters such as T_P , T_G , module, facewidth etc. of the helical gear set.

Pinion(steel)

$$HB_P = 180$$

$$S_{ut} = 550\ MPa$$

$$LH \text{ helix } \psi = 20^{\text{o}}$$

$$\phi_n = 20^{\text{o}} \quad \text{(full depth)}$$

$$N_p = 400\ rpm$$

$$Power = 1.25\ kW$$

Gear (CI)

$$HB_G = 196$$

$$S_{ut} = 250\ MPa$$

Solution:

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t} \rightarrow \phi_t = \tan^{-1}\left(\frac{\tan 20}{\cos 20}\right) = 21.17^o$$

1) Bending fatigue
2) Surface fatigue

→ check first *HB_P =180 < HB_G =196*

Criteria

1) $F \geq 2P_x$

2) $$S_H = C_p\sqrt{\frac{W_{tp}}{F \times d_p \times I \times C_v}}$$

- So start designing pinion based on surface contact stress.
- After finishing design based on surface fatigue of pinion and $F \geq 2P_x$ check other fatigue failure risks.

$$P_x = ? \quad \tan\psi = \frac{P_t}{P_x}$$

$$P_x = \frac{P_t}{\tan\psi} = \frac{\pi \times m_t}{\tan\psi} = \frac{\pi(m_n / \cos\psi)}{\tan\psi} = \frac{\pi \times m_n}{\sin\psi} \qquad F \geq 2\frac{\pi \times m_n}{\sin\psi}$$

$$F=\left(\frac{C_p}{S_H}\right)^2\frac{W_{tp}}{d_p\times I\times C_v} \qquad C_p = 174\ \text{Steel}-\text{Cast Iron} \qquad K=\frac{HB_P}{HB_G}$$

$$S_C = 2.76\times HB_P - 70 = 427\ MPa$$

$$S_H=\frac{C_L\times C_H}{C_T\times C_R}S_C$$

$$K=\frac{180}{196}=0.918<1.2 \qquad \text{Fig. 14.9}$$

$$C_H = 1.0 \qquad C_T = 1.0$$

$$S_H = 427\ MPa$$

$$C_L = 1.0 \qquad C_R = 1.0 \qquad (R = 99\ \%)$$

$$W_{tp}=n\times C_o\times C_m\times W_t$$

$$W_{tp}=4\times 1.25\times 1.6\times W_t \qquad \text{assume } 50 < F < 150\ \text{mm}$$

$$W_t=\frac{60\times P}{\pi\times d\times N}=\frac{60\times P}{\pi(m_t\times T_P)N}=\frac{60\times P\times\cos\psi}{\pi\times T_P\times N\times m_n}$$

Let $N_P = 18$

$$N_G = N_P\times(1.5) = 27$$

$$W_t=\frac{60\times 1250\times\cos 20}{\pi\times 18\times 400\times m_n}=\frac{3.11576}{m_n(meters)}\ Nwtn$$

$$W_{tp}=\frac{24.926}{m_n(meters)}\ (Nwtn)$$

$$d_P = m_t\times T_P=\frac{m_n}{\cos\psi}\times 18=\frac{m_n}{\cos 20}\times 18 = 19.155\times m_n\ \ mm$$

$$C_V = \sqrt{\frac{78}{78 + \sqrt{200V}}} \qquad V = \frac{\pi \times d \times N}{60} = \frac{\pi \frac{m_n}{\cos\psi} N}{60} = 22.288\, m_n \; m/\sec$$

$$I = \frac{\sin\phi_t \times \cos\phi_t}{2m_N} \times \frac{m_G}{m_G + 1} \qquad \phi_t = 21.17^0; \quad m_G = 1.5$$

$$m_N = \frac{P_N}{0.95Z} \qquad P_N = P_n \cos\phi_n = \pi m_n \cos\phi_n$$

$$P_N = 2.952 \times m_n$$

$$Z = \text{length of line of action in transverse plane}$$

$$Z = \sqrt{\left(r_p + a\right)^2 - r_{bp}^2} + \sqrt{\left(r_g + a\right)^2 - r_{bg}^2} - \left(r_p + r_g\right)\sin\phi_t$$

1st 2nd 3rd

$$a = 1.0 \times m_n \; mm$$

$$r_p = \frac{1}{2} m_t \times T_p = \frac{d_P}{2} = \frac{19.155}{2} m_n \; mm$$

$$r_g = \frac{1}{2} m_t \times T_G = \frac{d_G}{2} = m_G \times r_p$$

$$r_g = 1.5 \times \frac{19.155}{2} m_n \; mm$$

$$a = 1.0 \times m_n \; mm$$

$$r_p = \frac{d_P}{2} = \frac{19.155}{2} \times 4 = 38.31 \, mm$$

$$r_g = \frac{1}{2} m_t \times T_G = \frac{d_G}{2} = m_G \times r_p$$

$$r_g = 1.5 \times \frac{19.155}{2} \times 4 = 57.465 \, mm$$

$$\phi_t = \tan^{-1}\left(\frac{\tan 20}{\cos 20}\right) = 21.17^o$$

$$r_{bp} = r_p \times \cos \phi_t$$

$$r_{bg} = r_g \times \cos \phi_t$$

$$r_{bp} = 38.31 \times \cos(21.17) = 35.725 \, mm$$

$$r_{bg} = 57.465 \times \cos(21.17) = 53.587 \, mm$$

Check always 1st, 2nd, 3rd terms;

$$1^{st} \overset{?}{>} 3^{rd}$$

$$2^{nd} \overset{?}{>} 3^{rd}$$

m_n (mm)	V (m/sec)	C_v	W_{tp} (N)	d_p d_g (mm)	1^{st} 2^{nd} 3^{rd} Z	I	F (mm)	$2P_x$ (mm)	Notes
4	0.08915	0.974	6231.50	76.62 114.93	22.669 30.106 34.587 18.187	0.14782	93.8	73.5	$F>2P_x$ OK! Cm=OK! 50<F<150 mm
3	0.06686	0.977	8308.70	57.465 86.1975	17.0018 22.5797 25.9408 13.6380	0.14781	166.2	55.1	F>>2Px not suitable F is very Large > 150 mm Cm=1.6
6	0.13373	0.968	4154.33	114.93 172.395	34.00370 45.15958 51.88175 27.28140	0.147832	41.9	110.2	F<2Px 50<F Not satisfied Cm=1.6

Best solution is $m_n = 4\ mm\ and\ F = 94\ mm$

$$T_P = 18 \quad T_G = 27$$

Then we can (and you have to) check safety factors for other fatigue conditions.

1) Gear surface durability
2) Gear bending fatigue
3) Pinion bending fatigue

to see that actual contact ratio is more than what is required ; n>4.0??

Please do these calculations yourself for this example.

Example: 6.4

A helical gear set with speed reduction of *1.5:1, n=4* is to be designed.

Determine the required parameters such as T_P , T_G , module, facewidth etc. of the helical gear set.

Pinion(steel)

$$HB_P = 180$$

$$S_{ut} = 550\ MPa$$

$$LH \text{ helix } \psi = 20^{\circ}$$

$$\phi_n = 20^{\circ} \quad \text{(full depth)}$$

$$N_p = 600\ rpm$$

$$Power = 2.5\ kW$$

Gear (CI)

$$HB_G = 196$$

$$S_{ut} = 250\ MPa$$

Solution:

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t} \rightarrow \phi_t = \tan^{-1}\left(\frac{\tan 20}{\cos 20}\right) = 21.17^o$$

1) Bending fatigue
2) Surface fatigue

→ check first $HB_P = 180 < HB_G = 196$

Criteria

1) $F \geq 2P_x$

2) $$S_H = C_p\sqrt{\frac{W_{tp}}{F \times d_p \times I \times C_v}}$$

- So start designing pinion based on surface contact stress.
- After finishing design based on surface fatigue of pinion and $F \geq 2P_x$ check other fatigue failure risks.

$$P_x = ? \quad \tan\psi = \frac{P_t}{P_x}$$

$$P_x = \frac{P_t}{\tan\psi} = \frac{\pi \times m_t}{\tan\psi} = \frac{\pi(m_n / \cos\psi)}{\tan\psi} = \frac{\pi \times m_n}{\sin\psi} \qquad F \geq 2\frac{\pi \times m_n}{\sin\psi}$$

$$F = \left(\frac{C_p}{S_H}\right)^2 \frac{W_{tp}}{d_p \times I \times C_v} \qquad C_p = 174 \text{ Steel} - \text{Cast Iron} \qquad K = \frac{HB_P}{HB_G}$$

$$S_C = 2.76 \times HB_P - 70 = 427\ MPa$$

$$S_H = \frac{C_L \times C_H}{C_T \times C_R} S_C \qquad K = \frac{180}{196} = 0.918 < 1.2 \quad \text{Fig.14.9}$$

$$S_H = 427\ MPa \qquad C_H = 1.0 \qquad C_T = 1.0$$

$$C_L = 1.0 \qquad C_R = 1.0 \quad (R = 99\ \%)$$

$$W_{tp} = n \times C_o \times C_m \times W_t$$

$$W_{tp} = 4 \times 1.25 \times 1.6 \times W_t \qquad \text{assume } 50 < F < 150 \text{ mm} \qquad m_t = \frac{m_n}{\cos\psi}$$

$$W_t = \frac{60 \times P}{\pi \times d \times N} = \frac{60 \times P}{\pi (m_t \times T_P) N} = \frac{60 \times P \times \cos\psi}{\pi \times T_P \times N \times m_n} \qquad \text{Let } T_P = 20$$

$$T_G = T_P \times (1.5) = 30$$

$$W_t = \frac{60 \times 2500 \times \cos 20}{\pi \times 20 \times 600 \times m_n} = \frac{3.739}{m_n (meters)}\ Nwtn \qquad W_{tp} = \frac{29.912}{m_n (meters)} (Nwtn)$$

$$d_P = m_t \times T_P = \frac{m_n}{\cos\psi} \times 20 = \frac{m_n}{\cos 20} \times 20 = 21.284 \times m_n \ \ mm$$

$$C_V = \sqrt{\frac{78}{78+\sqrt{200V}}} \quad V = \frac{\pi \times d \times N}{60} = \frac{\pi \frac{m_n}{\cos\psi} N}{60} = 33.432\, m_n \;\; m/\sec$$

$$I = \frac{\sin\phi_t \times \cos\phi_t}{2m_N} \times \frac{m_G}{m_G+1} \quad \phi_t = 21.17^0; \quad m_G = 1.5$$

$$m_N = \frac{P_N}{0.95Z} \quad P_N = P_n \cos\phi_n = \pi m_n \cos\phi_n$$

$$P_N = 2.952 \times m_n$$

$$Z = \text{length of line of action in transverse plane}$$

$$Z = \sqrt{\left(r_p + a\right)^2 - r_{bp}^2} + \sqrt{\left(r_g + a\right)^2 - r_{bg}^2} - \left(r_p + r_g\right)\sin\phi_t$$

1st 2nd 3rd

$$a = 1.0 \times m_n \ mm$$

$$r_p = \frac{1}{2} m_t \times T_p = \frac{d_P}{2} = \frac{21.284}{2} m_n \ mm$$

$$r_g = \frac{1}{2} m_t \times T_G = \frac{d_G}{2} = m_G \times r_p$$

$$r_g = 1.5 \times \frac{21.284}{2} m_n \ mm$$

$$a = 1.0 \times m_n \ mm$$

$$r_p = \frac{d_P}{2} = \frac{21.284}{2} \times 4 = 42.568 \ mm$$

$$r_g = \frac{1}{2} m_t \times T_G = \frac{d_G}{2} = m_G \times r_p$$

$$r_g = 1.5 \times \frac{21.284}{2} \times 4 = 63.852 \ mm$$

$$\phi_t = \tan^{-1}\left(\frac{\tan 20}{\cos 20}\right) = 21.17^o$$

Check always 1st, 2nd, 3rd terms;

$$1^{st} \overset{?}{>} 3^{rd}$$

$$2^{nd} \overset{?}{>} 3^{rd}$$

$$r_{bp} = r_p \times \cos \phi_t$$

$$r_{bg} = r_g \times \cos \phi_t$$

$$r_{bp} = 42.568 \times \cos(21.17) = 39.695 \ mm$$

$$r_{bg} = 63.852 \times \cos(21.17) = 59.543 \ mm$$

m_n (mm)	V (m/sec)	C_v	W_{tp} (N)	d_p d_g (mm)	1st 2nd 3rd Z	I	F (mm)	$2P_x$ (mm)	Notes
4	0.133728	0.9684	7477.84	85.136 127.704	24.431 32.535 38.432 18.534	0.142667	105.57	73.5	$F>2P_x$ OK! Cm=OK! 50<F<150 mm
3	0.06686	0.977	8308.70	57.465 86.1975	17.0018 22.5797 25.9408 13.6380	0.14781	166.2	55.1	F>>2Px not suitable F is very Large > 150 mm Cm=1.6
6	0.13373	0.968	4154.33	114.93 172.395	34.00370 45.15958 51.88175 27.28140	0.147832	41.9	110.2	F<2Px 50<F Not satisfied Cm=1.6

Best solution is $m_n = 4\ mm\ and\ F = 106\ mm$

$$T_P = 20 \quad T_G = 30$$

Then we can (and you have to) check safety factors for other fatigue conditions.

1) Gear surface durability
2) Gear bending fatigue
3) Pinion bending fatigue

to see that actual contact ratio is more than what is required ; n>4.0??

Please do these calculations yourself for this example.

CROSSED HELICAL GEARS (SPIRAL GEARS)

1) Shaft centerlines are neither parallel nor intersecting
2) Teeth have point contact with each other at the beginning and changes to line contact as the gears wear in. For this reason they carry only very small loads and used for instrumental applications mostly.
3) They are definitely not recommended for use in the transmission of power but for the transmission of motion with small loads only.
4) A pair of meshing crossed helical gears usually have the same hand (LH-LH) (RH-RH)
5) Tooth sizes are given in terms of normal pitch (normal module) as in conventional helical gears.

$$a = 1.0 \times m_n \ mm$$

$$b = 1.25 \times m_n \ mm$$

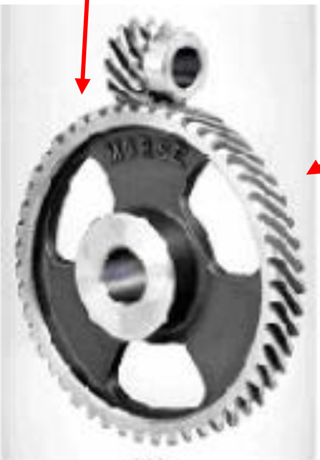
Which type of gears are these??

Right hand?

Left hand?

RH?

LH?



(j)

6) Pitch diameter of gear is given as;

$$d = m_t \times T_{p,g}; \quad m_t = \frac{m_n}{\cos\psi}$$

$$d = \frac{m_n \times T_{p,g}}{\cos\psi} \rightarrow d_P = \frac{m_n \times T_P}{\cos\psi_P}; \quad d_G = \frac{m_n \times T_G}{\cos\psi_G}$$

Might be different

7) Velocity ratio of pinion and gear is obtained from tooth number's of pinion and gear(not from diameters).

$$\frac{W_P}{W_G} = \frac{T_G}{T_P} \neq \frac{d_G}{d_P} \quad \text{since } \psi_P \text{ and } \psi_G \text{ can be different.}$$

8) For minimum sliding velocity in crossed helical gears ψ_P and ψ_G must be equal. If $\psi_P \neq \psi_G$ the gear with larger ψ should be used as driver when both gear have the same hand.

9) Since the teeth are in point contact an effort should be made to obtain a total contact ratio of 2 or more.

When both gears have the same hand, rotations and axial forces are as shown.

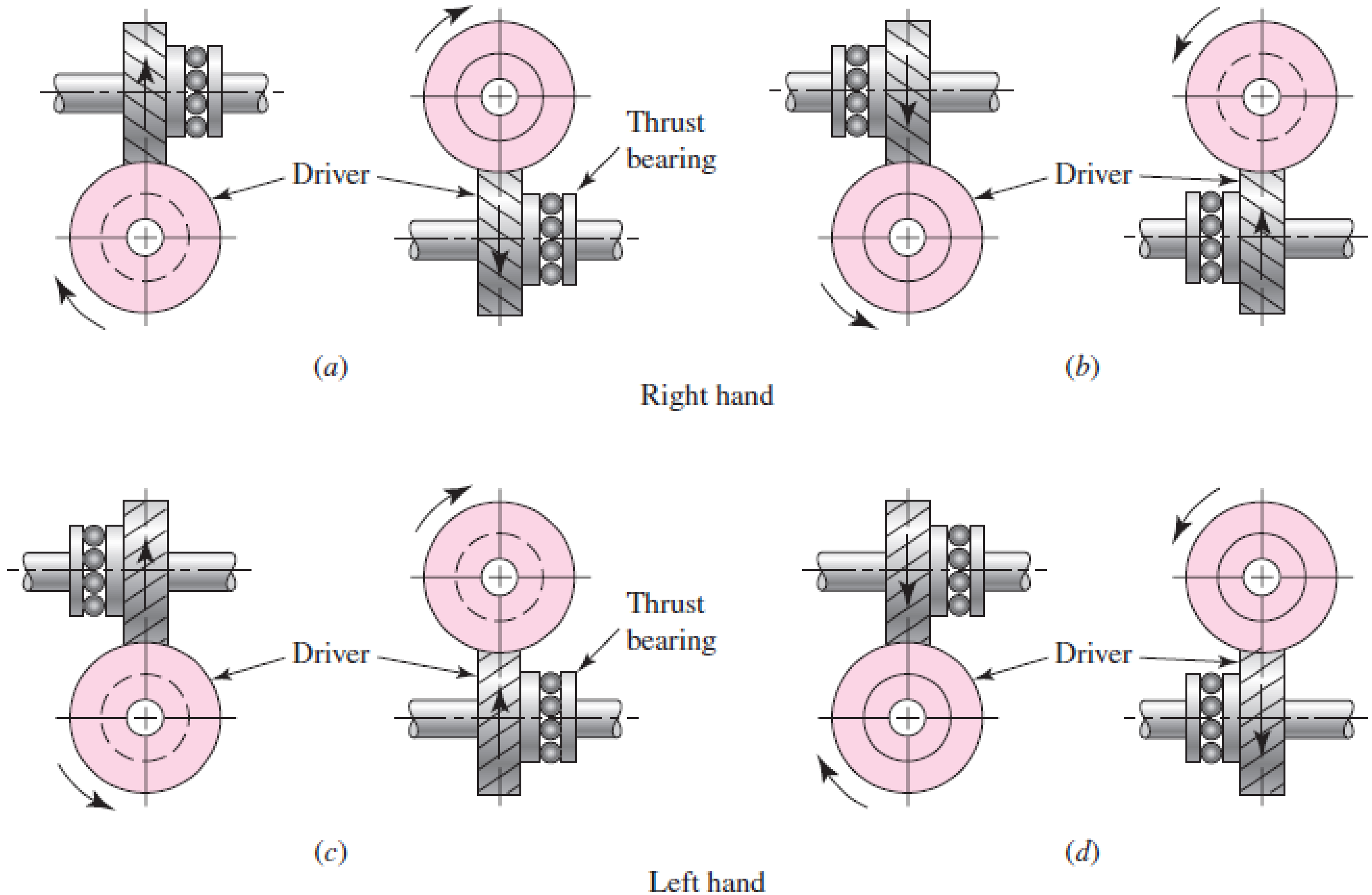


Fig. 6.00 Thrust, rotation, and hand relations for crossed helical gears. Note that each pair of drawings refers to a single gearset. These relations also apply to worm gearsets.