

PROBLEMS

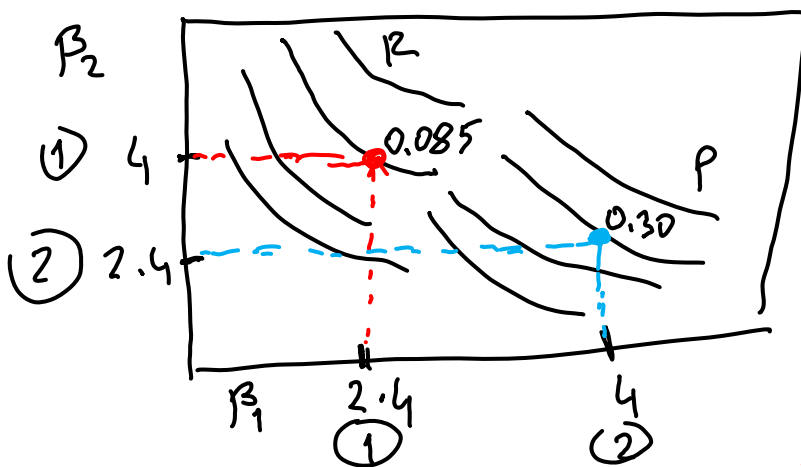
1) Lean beef block (a slab) with dimensions of $1 \times 0.25 \times 0.6 \text{ m}$ is being frozen. The following data are available: $h_c = 30 \text{ W/m}^2 \cdot \text{K}$, $T_{\text{initial}} = 5^\circ\text{C}$, $T_\infty = -30^\circ\text{C}$, density = 1050 kg/m^3 , $L_v = 333.22 \text{ kJ/kg}$ water, moisture content = 74.5% , $k_f = 1.108 \text{ W/m} \cdot \text{K}$, $T_F = -1.75^\circ\text{C}$. Find the time required to freeze to -10°C using Plank's equation.

Solution:

$$t_F = \frac{\rho \times L_v}{(T_F - T_\infty)} \times \left[\frac{\rho \cdot a}{h_c} + \frac{R \times a^2}{k_f} \right]$$

① $\beta_1 = \frac{0.6}{0.25} = 2.4$, $\beta_2 = \frac{1}{0.25} = 4$ and ② $\beta_1 = \frac{1}{0.25} = 4$,

$\beta_2 = \frac{0.6}{0.25} = 2.4$



$R = 0.085$

$P = 0.30$

$a = 0.25$ (thickness for slab)

$$L_v = 333.22 \frac{\text{kJ}}{\text{kg H}_2\text{O}} \times 0.745 \frac{\text{kg H}_2\text{O}}{\text{kg beef}} = 248.25 \frac{\text{kJ}}{\text{kg beef}}$$

$$t_F = \frac{1050 \frac{\text{kg}}{\text{m}^3} \times 248.25 \times 10^3 \frac{\text{J}}{\text{kg}}}{[-1.75 - (-30)]^\circ\text{C}} \left[\frac{0.3 \times (0.25 \text{ m})}{30 \frac{\text{J}}{\text{m}^2 \cdot \text{s} \cdot \text{K}}} + \frac{0.085 \times (0.25 \text{ m})^2}{1.108 \frac{\text{J}}{\text{m} \cdot \text{s} \cdot \text{K}}} \right] \times \frac{1 \text{ hr}}{3600 \text{ s}}$$

$t_F = 16.8 \text{ hr.}$

$\therefore L_v$: Latent heat of fusion per unit mass of food.

2) Lean beef block with dimensions of $1\text{m} \times 0.25\text{m} \times 0.6\text{m}$ is being frozen. The following data are available: $h_c = 30\text{W/m}^2\cdot\text{K}$, $T_{\text{initial}} = 5^\circ\text{C}$, $T_\infty = -30^\circ\text{C}$, $\rho_u = 1050\text{ kg/m}^3$, $\rho_f = 955\text{ kg/m}^3$, $C_{pf} = 2.5\text{ kJ/kg beef}\cdot\text{K}$, $C_{pu} = 3.52\text{ kJ/kg beef}\cdot\text{K}$, $L_v = 333.22\text{ kJ/kgH}_2\text{O}$, moisture content = 74.5%, $k_I = 1.108\text{ W/m}\cdot\text{K}$, $T_F = -1.75^\circ\text{C}$. Find the time required to freeze to -15°C using Pham's equation.

Solution:

$$N_{Bi} = \frac{h_c \cdot d_c}{k_f} = \frac{30 \times \frac{1}{2} \times 0.25}{1.108} = 3.3845$$

$\rightarrow d_c$: half thickness

$$T_{fm} = 1.8 + 0.263 \cdot T_c + 0.105 \cdot T_a \rightarrow T_\infty$$

$$T_{fm} = 1.8 + 0.263 \cdot (-15) + 0.105 \cdot (-30) = -5.295^\circ\text{C}$$

$$\begin{aligned} \Delta H_1 &= C_{pu} \cdot \rho_u (T_i - T_{fm}) \\ &= 3520 \frac{\text{J}}{\text{kg}\cdot^\circ\text{C}} \times 1050 \frac{\text{kg}}{\text{m}^3} \times [5 - (-5.295)]^\circ\text{C} \\ &= 38050320 \text{ J/m}^3 \end{aligned}$$

$$\Delta H_2 = \rho_f [L_f + C_{pf} (T_{fm} - T_c)]:$$

$$L_f = 333.22 \times 0.745 = 248.3 \frac{\text{kJ}}{\text{kg beef}} \equiv 248300 \frac{\text{J}}{\text{kg beef}}$$

$$\begin{aligned} \Delta H_2 &= 955 \frac{\text{kg}}{\text{m}^3} \left[248300 \frac{\text{J}}{\text{kg}} + 2500 \frac{\text{J}}{\text{kg}\cdot^\circ\text{C}} \times [-5.295 - (-15)]^\circ\text{C} \right] \\ &= 260297187 \text{ J/m}^3 \end{aligned}$$

$$\Delta T_1 = \frac{T_i + T_{fm}}{2} - T_a = \frac{5 + (-5.295)}{2} - (-30) = 29.8^\circ\text{C}$$

$$\Delta T_2 = T_{fm} - T_a = -5.295 - (-30) = 24.705^\circ\text{C}$$

$$t_F = \frac{d_c}{E_f \cdot h_c} \left[\frac{\Delta H_1}{\Delta T_1} + \frac{\Delta H_2}{\Delta T_2} \right] \left(1 + \frac{N_{Bi}}{2} \right)$$

d_c : for slab: half thickness

E_f : " " : 1

$$t_F = \frac{\frac{1}{2} \times (0,25)}{1 \times 30} \left[\frac{38050320}{29.8} + \frac{260297187}{24.7} \right] \left[1 + \frac{3.3845}{2} \right]$$

$$t_F = 132503 \text{ s} \equiv 36.7 \text{ hr}$$

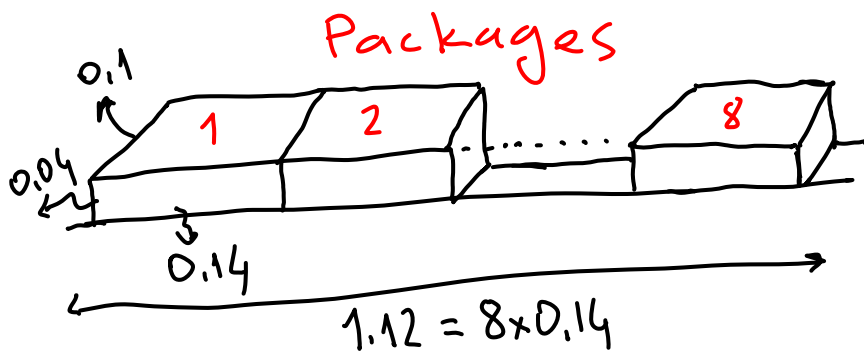
3) A continuous plate freezing system is being designed to freeze 0.5 kg fish packages at a rate of 500 kg fish/h. Package dimensions are 0.04x0.1x0.14 m. Each package enters the freezer at 4.4°C initially. Width of each plate is 1.12 m and can hold 8 packages. Plate temperature is -23 °C, surface heat transfer coefficient 28 W/m².K, thickness of packaging material is 8x10⁻⁴ m, thermal conductivity of packaging material is 0.05 W/m.K, thermal conductivity of frozen fish is 1.125 W/m.K, enthalpy change between initial and plate temperatures is 306 kJ/kg fish, $\rho_u = 880 \text{ kg/m}^3$.

Compute the number of freezing stations (plates) required and the refrigeration requirement.

Solution: Using modified Plank's equation. It accounts for time required to remove sensible heat in pre-cooling and post-freezing period.

$$t_F = \frac{\rho \cdot \Delta H'}{(T_i - T_{\infty})} \left[\frac{P \cdot a}{h_c} + \frac{R \cdot a^2}{k_1} \right], \quad L_v \rightarrow \Delta H'$$

$$\beta_1 = \frac{0.1}{0.04} = 2.5, \quad \beta_2 = \frac{8 \times 0.14}{0.04} = 28 \Rightarrow \beta_1 \text{ and } \beta_2 \text{ are not present in the chart} \Rightarrow$$



It is assumed the plate chain to be an infinite slab.

$$P = 1/2, \quad R = 1/8$$

$$T_i = 4.4^\circ\text{C}$$

$$T_{\infty} = -23^\circ\text{C}$$

$$a = 0.04 \text{ m}$$

$$k = 1.125 \text{ W/m}\cdot\text{K}$$

$$\left. \begin{array}{l} T_i = 4.4^\circ\text{C} \\ T_{\infty} = -23^\circ\text{C} \\ a = 0.04 \text{ m} \\ k = 1.125 \text{ W/m}\cdot\text{K} \end{array} \right\} \text{In one plate} = \frac{1.12 \text{ m}}{0.14 \text{ m}} = 8 \text{ packages}$$

$$\frac{1}{h_c} = \frac{1}{h_{\text{surface}}} + \frac{\Delta x}{k} \quad \rightarrow \text{for packaging material}$$

$$\frac{1}{h_c} = \frac{1}{28} + \frac{8 \times 10^{-4}}{0.05} \Rightarrow h_c = 19.34 \text{ W/m}^2\cdot\text{K}$$

$$t_F = \frac{880 \times 306 \times 1000}{[4.4 - (-23)] \times 3600} \times \left[\frac{1/2 \times (0.04)}{19.34} + \frac{1/8 \times (0.04)^2}{1.125} \right]$$

thermal conduct. of frozen fish

$$t_F = 3.263 \text{ hr.}$$

$$500 \frac{\text{kg fish}}{\text{hr}} \times 3.263 \text{ hr} = 1631 \text{ kg fish capacity}$$

$$\frac{1631 \text{ kg fish}}{8 \frac{\text{package} \times 0.5 \text{ kg fish}}{\text{plate}}} = 408 \text{ stations (plates)}$$

Refrigeration requirement:

$$500 \frac{\text{kg fish}}{\text{hr}} \times 306 \frac{\text{kJ}}{\text{kg fish}} = 153000 \frac{\text{kJ}}{\text{hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}} = 42.5 \text{ kW.}$$

4) A food product at 25°C is being frozen with liquid nitrogen (product final temperature = -35°C). Food product is in brick shape ($0.05 \times 0.1 \times 0.2 \text{ m}$). If the freezing time is 3 minutes, calculate the rate of N_2 utilization for this product ($\text{kg N}_2/\text{kg product}$).

C_p of $\text{N}_2 = 1 \text{ kJ}/(\text{kg N}_2 \cdot ^\circ\text{C})$, λ (latent heat) of $\text{N}_2 = 198 \text{ kJ}/\text{kg N}_2$,

$T_{\text{liq. Nitrogen}} = -196^\circ\text{C}$, density of product = $800 \text{ kg}/\text{m}^3$,

$h_c = 200 \text{ W}/\text{m}^2 \cdot \text{K}$, $k_I = 1.2 \text{ W}/\text{m} \cdot \text{K}$.

Solution:

$$T_i = 25^\circ\text{C}, T_f = -35^\circ\text{C}, a = 0.05 \text{ m}, t_F = 3 \text{ min} \equiv 0.05 \text{ hr.}$$

$$t_F = \frac{\rho \cdot \Delta H'}{(T_i - T_\infty)} \left[\frac{P \cdot a}{h_c} + \frac{R \cdot a^2}{k} \right]$$

$$\beta_1 = \frac{0.1}{0.05} = 2, \quad \beta_2 = \frac{0.2}{0.05} = 4 \Rightarrow$$

$$\beta_1 = \frac{0.2}{0.05} = 4$$

$$\beta_2 = 0.1/0.05 = 2$$

from chart; $P = 0.282, R = 0.083$

$$0.05 = \frac{800 \times \Delta H' \times 1000}{[25 - (-196)]} \left[\frac{0.282 \times 0.05}{200} + \frac{0.083 \times (0.05)^2}{1.2} \right] \times \frac{1 \text{ hr}}{3600 \text{ s}}$$

$$\Rightarrow \Delta H' = 204.09 \text{ kJ/kg product}$$

$$\Delta H_{N_2} = \lambda_{N_2} + c_p(T_f - T_i)_{N_2}$$

$$\Delta H_{N_2} = 198 \text{ kJ/kg } N_2 + 1 \frac{\text{kJ}}{\text{kg } N_2 \cdot ^\circ\text{C}} \times [-35 - (-196)]^\circ\text{C}$$

$$= 359 \text{ kJ/kg } N_2$$

$$\frac{\text{Rate of } N_2 \text{ utilized}}{\text{kg product}} = \frac{204.09 \text{ kJ/kg product}}{359 \text{ kJ/kg } N_2}$$

$$= 0.568 \text{ kg } N_2/\text{kg product}$$

5) Determine the refrigeration requirement for freezing of 50 kg lean beef with 74.5 % moisture content from an initial temperature of 5°C to a final temperature of -15°C. Assume enthalpies of beef at these temperatures are 317 and 58 kJ/kg beef, respectively.

Solution:

$$\Delta H' = (317 - 58) \frac{\text{kJ}}{\text{kg}} = 259 \text{ kJ/kg beef}$$

$$\text{Refrigeration requirement} = 259 \frac{\text{kJ}}{\text{kg beef}} \times 50 \text{ kg beef}$$

$$= 12950 \text{ kJ}$$

6) 100 kg of tomato was frozen by using liquid nitrogen. Initial and final temperatures of tomato were 25°C and -15°C, respectively. Tomato are assumed to be spherical with a diameter 5 cm and density 0.8 g/cm³. Tomato solids makes up 35 % of the total weight of tomato and tomato juice solids makes up 5 % of

total weight of tomato juice. Liquid nitrogen is sprayed on to tomato at a rate of 0.02 kg N₂/tomato.min. Calculate the thermal efficiency of this process. Following data is also given:

$h_c = 100 \text{ W/m}^2\cdot\text{K}$, $k_I = 2.5 \text{ W/m}\cdot\text{K}$, $C_p \text{ of N}_2 = 1 \text{ kJ/kgN}_2\cdot\text{K}$, enthalpy of evaporation of N₂ = 200 kJ/kg N₂, $T_{\text{liq.Nitrogen}} = -196^\circ\text{C}$.

Solution: For fruits and vegetables \Rightarrow

$$\Delta H' = \left[1 - \frac{x}{100} \right] \Delta H'_j + 1.21 \left(\frac{x}{100} \right) \times \Delta T'$$

liquid (juice) \leftarrow $\Delta H'_j$ \rightarrow Solid

x : solids content of fruit.

(Tomato juice): $T_i = 25^\circ\text{C} \Rightarrow H_i = 525 \text{ kJ/kg tomato juice}$
 $T_f = -15^\circ\text{C} \Rightarrow H_f = 80 \text{ kJ/kg tomato juice}$

$\Delta H_{\text{tomato juice}} = 525 - 80 = 445 \text{ kJ/kg tomato juice}$

$$\Delta H' = \left[1 - \frac{35}{100} \right] \times 445 + 1.21 \cdot \underbrace{\left(\frac{35}{100} \right)}_{C_p \text{ solid}} \times [25 - (-15)]$$

$\Rightarrow \Delta H' = 306.19 \frac{\text{kJ}}{\text{kg tomato}}$ heat utilized by the product.

$P = \frac{1}{6}$ $R = \frac{1}{24}$ for sphere

$a = 0.05 \text{ m}$, $T_i = 25^\circ\text{C}$, $T_\infty = -196^\circ\text{C}$

$$t_F = \frac{800 \times 306.19 \times 10^3}{[25 - (-196)]} \times \left[\frac{1/6 \times 0.05}{100} + \frac{1/24 \times (0.05)^2}{2.5} \right] \times \frac{1 \text{ hr}}{3600 \text{ s}}$$

$$t_F = 0.0385 \text{ hr} \equiv 2.309 \text{ min}$$

Enthalpy of liq. $N_2 = h_{N_2} + \text{sensible heat}$

$$= 200 + 1 \times [-15 - (-196)] = 381 \text{ kJ/kg}$$

→ $T_{\text{final } N_2}$

$$V \text{ of 1 tomato} = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \times \left(\frac{0.05}{2}\right)^3 = 6.5 \times 10^{-5} \text{ m}^3$$

$$\text{Wt of 1 tomato} = 6.5 \times 10^{-5} \text{ m}^3 \times 800 \frac{\text{kg}}{\text{m}^3} = 0.052 \text{ kg}$$

$$\frac{0.02 \text{ kg } N_2}{1 \text{ tomato} \cdot \text{min}} \times 2.309 \text{ min} \times \frac{1 \text{ tomato}}{0.052 \text{ kg}} \times 381 \frac{\text{kJ}}{\text{kg } N_2}$$

$$= 338.3 \text{ kJ/kg tomato heat input totally.}$$

$$\text{Thermal efficiency} = \frac{\text{Heat utilized}}{\text{Heat input}} = \eta$$

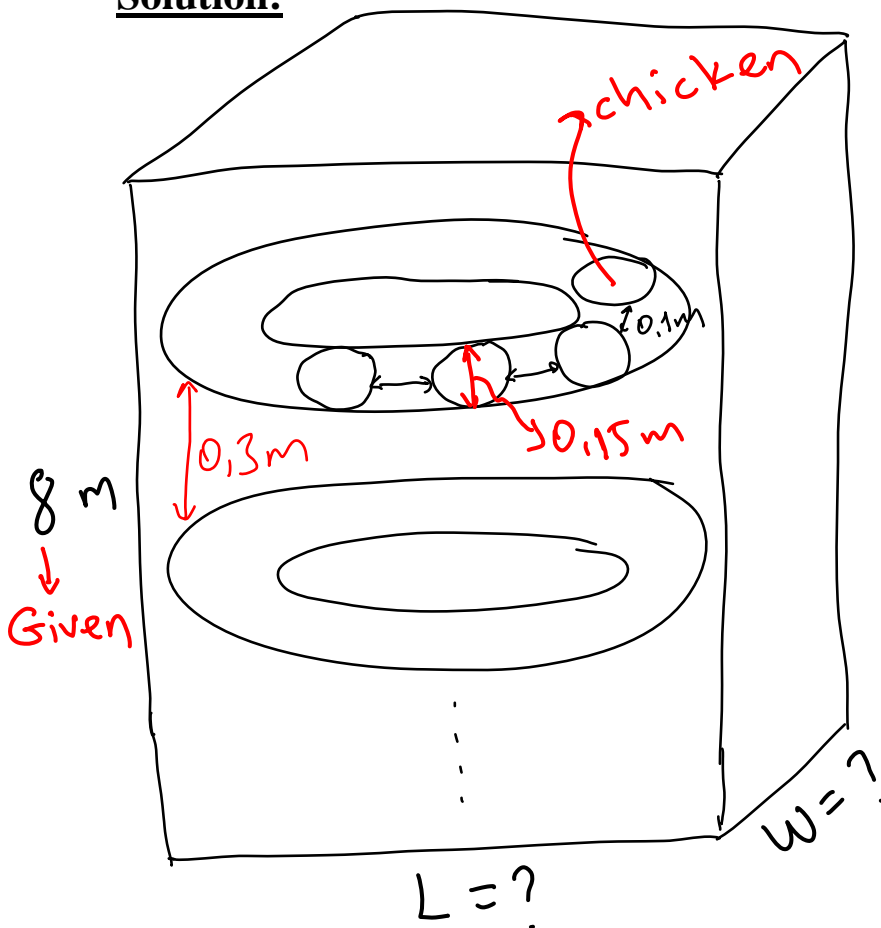
$$\eta = \frac{306.19}{338.19} \times 100 = 90.5\%$$

7) A continuous freezing system for whole chickens in a plastic film is designed using a spiral conveyor and high velocity cold air. The initial product temperature is 5°C , final temperature is -2°C and medium (air) temperature is -30°C . The conveyor that carries the product to freezing chamber is designed to operate at a rate of

3 m/min. Assume the chicken has a spherical shape. Determine the approximate dimensions of freezing room, the capacity of the refrigeration system and total number of chickens on 15 circular sections. The following data were given:

$h_c = 22 \text{ W/m}^2\cdot\text{K}$, height of the freezing chamber = 8 m, clearance between sections of spiral conveyor = 0.3 m, $k_f = 1.298 \text{ W/m}\cdot\text{K}$, density = 855 kg/m^3 , assume the diameter of a chicken is 0.15 m and the distance between each chicken is 0.1 m, in the system there are 15 circular sections, $\Delta H^f = 278.6 \text{ kJ/kg}$ chicken.

Solution:



For a sphere \Rightarrow

$$P = \frac{1}{6}, R = \frac{1}{24}$$

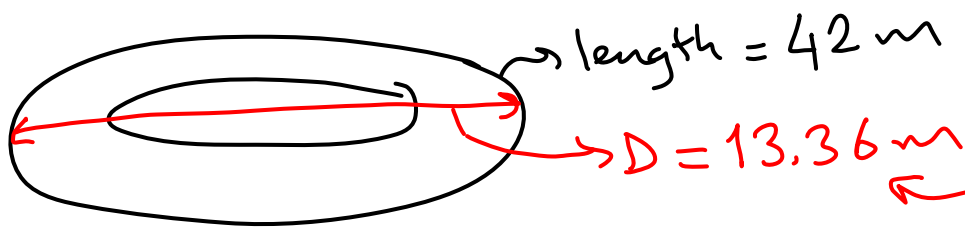
$$t_F = \frac{\Delta H^f \cdot \rho}{T_i - T_\infty} \left[\frac{P \cdot a}{h_c} + \frac{R \cdot a^2}{k} \right]$$

$$= \frac{278.6 \times 10^3 \times 855}{[5 - (-30)]} \times \left[\frac{1/6 \times 0.15}{22} + \frac{1/24 \times (0.15)^2}{1.298} \right] \times \frac{1 \text{ hr}}{3600 \text{ s}}$$

$$t_F = 3.5 \text{ hr}$$

$$\text{Total conveyor length} = \frac{3 \text{ m}}{\text{min}} \times 3.5 \text{ hr} \times \frac{60 \text{ min}}{1 \text{ hr}} = 630 \text{ m}$$

$$\text{Length of each circle} = \frac{630}{15 \text{ circle}} = 42 \text{ m/circle}$$



$$\text{Circumference} = 2\pi r = \pi \cdot D = 42 \Rightarrow D = 13.36 \text{ m}$$

Dimensions: $L = 14 \text{ m}$, width = 14 m at least

Approximate dimensions $\cong 14 \times 14 \times 8 \text{ m}^3$
 $\downarrow \quad \downarrow \quad \downarrow$
 $L \quad W \quad H \text{ (given)}$



$$\frac{42 \text{ m/circle}}{0.25 \text{ m/chicken}} = 168 \text{ chickens on one circle.}$$

Total # of chicken on 15 circles \Rightarrow

$$\frac{168 \text{ chicken}}{1 \text{ circle}} \times 15 \text{ circle} = 2520 \text{ chickens}$$

$$M = \text{Mass of a chicken} = V \times \rho = \frac{4}{3} \pi r^3 \times \rho$$

$$M = \frac{4}{3} \times \pi \times \left(\frac{0.15}{2}\right)^3 \times 855 = 1.51 \text{ kg/chicken}$$

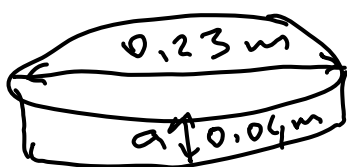
$$\frac{3 \text{ m/min}}{0.25 \text{ m/chicken}} = 12 \frac{\text{chicken frozen}}{\text{min}}$$

Refrigerant requirements (RR) = ?

$$RR = 278.6 \frac{\text{kJ}}{\text{kg chic}} \times \frac{1.51 \text{ kg chic}}{\text{chic}} \times \frac{12 \text{ chic}}{\text{min}} = 5048 \frac{\text{kJ}}{\text{min}}$$

8) A pecon coffee cake (each cake = 0.372 kg) is being frozen in liquid nitrogen in 1.7 minutes. Initial temperature of cake is 22°C, the final temperature is -18°C. The temperature of freezing medim (liquid nitrogen) is -196°C. If 0.665 kg N₂ is required per kg product, estimate the surface heat transfer coefficient of the system. R = 1/8, P = 1/2, k_f = 1.731 W/m.K, latent heat of nitrogen = 197.98 kJ/kg N₂, Cp of N₂ is 1.044 kJ/(kg N₂.°C).

Solution:



=> Assume infinite slab =>

$$R = 1/8, P = 1/2$$

$$V = \pi \cdot r^2 \times a$$

$$\rho = \frac{m}{V} = \frac{0,372 \text{ kg}}{\pi \left(\frac{0,23}{2}\right)^2 \times 0,04 \text{ m}^3} = 224 \text{ kg/m}^3$$

$$\lambda_{N_2} = 197,98 \text{ kJ/kgN}_2$$

$0,665 \frac{\text{kgN}_2}{\text{kg product}}$ is evaporated after it is used.

$$\text{Sensible heat}_{N_2} = C_p \Delta T = 1,044 \times \left[\overset{\text{red } T_{\text{final N}_2}}{-18} - \overset{\text{red } T_{\infty}}{(-196)} \right]$$

$$= 185,83 \text{ kJ/kgN}_2$$

$$\Delta H_{N_2} = \lambda_{N_2} + \text{Sensible heat of N}_2$$

$$= 197,98 + 185,83 = 338,81 \text{ kJ/kgN}_2$$

$$\Delta H'_{\text{product}} = \frac{0,665 \cancel{\text{kgN}_2}}{\text{kg product}} \times 338,81 \frac{\text{kJ}}{\cancel{\text{kgN}_2}}$$

$$= 255,23 \text{ kJ/kg product}$$

$$t_F = 1,7 \text{ min} \equiv 0,028 \text{ hr (given)}$$

Using Plank's modified equation \Rightarrow

$$0.028 \text{ hr} = \frac{224 \times 253.23 \times 10^3}{[22 - (-196)]} \times \left[\frac{\frac{1}{2}(0.04)}{h_c} + \frac{\frac{1}{8}(0.04)^2}{1.731} \right] \times \frac{1 \text{ hr}}{3600 \text{ s}}$$

$$\Rightarrow h_c = 73.2 \text{ W/m}^2 \cdot \text{K}$$

9) Calculate the total refrigeration requirement (in Btu) when 2000 lb of beef slices are frozen from an initial temperature of 40°F to freezing temperature of 23°F and then stored at -4°F (Cp of beef above freezing temperature is 0.82 Btu/(lb.°F), Cp of frozen beef is 0.46 Btu/(lb.°F), the latent heat of fusion of water to ice is 144 Btu/lb water, moisture content of beef is 75 %).

Solution:

1) To bring the T of beef down to freezing T \Rightarrow

$$H_1 = m \cdot C_{p1} (T_{\text{initial}} - T_{\text{freezing}})$$

$$H_1 = 2000 \text{ lb} \times 0.82 \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{F}} (40 - 23)^\circ\text{F} = \boxed{27880 \text{ Btu}}$$

2) To freeze the beef \Rightarrow Heat of fusion:

$$H_2 = L_v \times W \times m = 144 \frac{\text{Btu}}{\text{lb H}_2\text{O}} \times 0.75 \times 2000 \text{ lb P} = \boxed{216000 \text{ Btu}}$$

3) To lower T of frozen beef to storage T \Rightarrow ^{-4°C}
 $H_3 = m \cdot c_{pf} \times (T_{\text{freezing}} - T_{\text{storage}}) \rightarrow T_{\text{final}}$

$$H_3 = 2000 \times 0.46 \times [23 - (-4)] = \boxed{24840 \text{ Btu}}$$

$$\text{Total load} = H_T = H_1 + H_2 + H_3 \Rightarrow$$

$$H_T = \boxed{27880 + 216000 + 24840}$$

$$H_T = \boxed{268720 \text{ Btu}}$$

The equations that we used in this problem set:

- 1. Plank's equation**
- 2. Pham's equation**
- 3. Modified Plank's equation**