PROBLEMS

1) An ice cream mix formulation having a viscosity of 70 centipoises (cp) and a density of 1015 kg/m^3 is being canned aseptically at the rate of 20 L/min. The mix is heated to 285°F, passed through a 100 ft long 0.870 in inside diameter stainless steel pipe and cooled. Calculate the sterilizing value of this process (D_0 = 1.83 min, Z = 24°F, 1000 L = 1 m^3, 1 in = 2.54 cm)

Solution:

$$Q = 20\frac{L}{min} \times \frac{1\,dm^3}{1\,L} \times \frac{1\,m^3}{(10\,dm)^3} = 0{,}02\ m^3/min$$

Diameter = 0,087 in ≡ 0,0221 m

$$r = \frac{0{,}0221}{2} = 0{,}01105\ m$$

$$V_{avg} = \frac{Q}{A} = \frac{0.02\ m^3/min}{\pi \times (0.01105\ m)^2} = 52.138\ m/min$$

$$V_{avg} = 52.138\frac{m}{min} \times \frac{1\,min}{60\,s} = 0.868\ m/s$$

$$Re = \frac{\rho \cdot V_{avg} \cdot D}{\mu} = \frac{(1015)(0{,}868)(0{,}0221)}{70 \times 10^{-3}}$$

(70 cp = 0,07 poise = 0,07 Pa.s)

$Re = 278.15 < 2100 \Rightarrow$ flow is laminar.

$$\Rightarrow V_{max} = 2 \times V_{avg} = 2 \times 52.138 = 104.276\ m/min.$$

$$t = F = \frac{L}{V_{max}} = \frac{100\,ft \times \frac{12\,in}{1\,ft} \times \frac{2.54\,cm}{1\,in} \times \frac{1\,m}{100\,cm}}{104.276\,m/min} \Rightarrow$$

$$F_{285} = 0,292\ min$$

$$\log\frac{D_{285}}{D_0} = \frac{T_0 - T_{285}}{z} = \frac{250-285}{24} = -1.458$$

$$\frac{D_{285}}{D_0} = 10^{-1.458} \Rightarrow D_{285} = D_0 \times 0,0348 \quad (D_0 \rightarrow 1.83)$$

$$D_{285} = 1.83 \times 0.0348 = 0.0636\ min.$$

$$SV = \log\frac{N_0}{N}\ ,\quad F_{285} = D_{285} \times (SV)_{285}$$

$$SV_{285} = \frac{F_{285}}{D_{285}} = \frac{0.292\ min}{0,0636\ min} = \underline{\underline{4.58}}$$

2) Homework: A thermocouple located at the slowest heating point (the center of can) of a picnic can insulated by other cans at the both ends indicated the following temperature-time relationships (the retort temperature is 240°F):

Time (min)	0	10	30	40	50	60	75
T (°F)	140	160	212.5	225	230.5	235	110

Determine the process time by graphical method. The F_0 value for *Cl. botulinum* upon which the process is to be based is 2.55 min and Z = 18°F for 250°F. Determine whether this process would be a safe process. If it is not, what would you recommend ?

3) The Do value of *Cl. botulinum* in a product is 0.3 min. At the pH of the product, the Do value of PA 3679 botulinum is 1.5 min. When conducting an inoculated pack, how many spores of PA 3679 botulinum must be introduced per can such that a spoilage rate of 1/100 cans would satisfy a 20D reduction of PA 3679 *Cl. botulinum* ?

Solution:

$$F_0 = D_0 \cdot (SV) = 0{,}3 \times 20 = 6 \text{ min}$$

(SV → 20D reduction)

$$N_f = \frac{1}{100} = 0{,}01$$

$$\log \frac{N_0}{N_f} = \frac{F_0}{D_0} \Rightarrow \log \frac{N_0}{0{,}01} = \frac{6}{1.5} \Rightarrow N_0 = 100 \text{ spores}$$

must be introduced.

4) Cans of a given food were heated in a retort for sterilization. The Fo for *Cl. botulinum* in this type of food is 2.50 min and Z value is 18ºF. The temperatures in the center of a can were measured and were approximately as follows (the average temperature during each time period was listed below:

t_1 :(0-20 min), T_1:160 ºF; t_2:(20-40 min), T_2:210 ºF; t_3:(40-57 min), T_3:235 ºF

Determine if this sterilization process is adequate using mathematical model.

Solution:

$$F_0 = \sum t \times 10^{\frac{T - T_0}{z}}$$

$$F_0 = 20 \times 10^{\frac{160-250}{18}} + 20 \times 10^{\frac{210-250}{18}} + 17 \times 10^{\frac{235-250}{18}} \Rightarrow$$

$$F_0 = 2.615$$

Since F process (2.615) > F_0 given (2.50) ⇒ the sterilization process is adequate.

5) **Homework: (how to use D and Z values in pasteurization calculations):**

Pooled raw milk at the processing plant has bacterial population of 4×10^5/mL. It is to be pasteurized at 79°C for 21 seconds. The average D value at 65°C for the mixed population is 7 min. The Z value is 7°C. How many organisms will be left after pasteurization? What time would be required at 65°C to accomplish the same degree of lethality?

6) In a given pasteurization process the reduction in the number of viable cells used is 10^{15} and the Fo value used is 9 min. If the reduction is increased by 10^{16} times because of increased contamination, what would be the new Fo value ?

Solution:

$$\circledast \quad \left(\frac{N_o}{N_f}\right)_1 = 10^{15}, \quad (SV)_1 = \log \frac{N_o}{N_f} = \log(10^{15}) = 15$$

$$F_o = (SV) \cdot D_o \Rightarrow 9 = 15 \times D_o \Rightarrow D_o = 0{,}6 \text{ min}$$

$$\circledast \quad \left(\frac{N_o}{N_f}\right)_2 = 10^{16}, \quad (SV)_2 = \log(10^{16}) \Rightarrow (SV)_2 = 16$$

$$\left[F_o = (SV) \times D_o\right]_2 = 16 \times 0{,}6 \text{ min} = 9.6 \text{ min.}$$

7) Apple juice (contains vegetative m.o.) has a viscosity of 5 cp (centi poise) and a density of 1019 kg/m^3. It is to be pasteurized in a continuous system that involves to 95°C holding in a 4 in nominal pipe and cooling. When the sealed tubes (1 mL each) containing equal number of spores of vegetative organisms were heated for 1 and 1.5 min at 95°C, the survivors were 1000 and 100, respectively. (The inside radius of a 4 in pipe is 40 mm).

a) Calculate the length of holding tube for both cases when the flow rate of apple juice is 80 L/min.

b) What would be the length of holding tube for this type of foods containing vegetative microroganisms (assume SV = 5 for vegetative m.o.).

Solution:

a) First, calculate D_0 value at 95°C ⇒

F ~ $t_1 = 1\ \text{min}, \quad N_f = 1000$

F ~ $t_2 = 1.5\ \text{min}, \quad N_f = 100$

$$F = D \times \log \frac{N_0}{N_f} \quad \text{or} \quad F = D \times (SV)$$

1) $$1\ \text{min} = D \times \log \frac{N_0}{1000} \Rightarrow D = \frac{1}{\log N_0 - \log 1000}$$

2) $$1.5\ \text{min} = D \times \log \frac{N_0}{100} \Rightarrow$$

$$1.5 = \frac{1}{\log N_0 - \log 1000} \times [\log N_0 - \log 100]$$

$$1.5 = \frac{\log N_0 - 2}{\log N_0 - 3} \Rightarrow \log N_0 = 5 \Rightarrow N_0 = 100000$$

Substitute into eqn (1) or (2) ⇒

$$1 = D \times \log \frac{100000}{1000} \Rightarrow D = 0.5\ \text{min at } 95°C.$$

$$V_{avg} = \frac{Q}{A} = \frac{0{,}08\ m^3/\text{min}}{\pi \times (0{,}04)^2\ m^2} = 15{,}91\ \frac{m}{\text{min}} \equiv 0{,}265\ m/s$$

($A \to \pi \cdot r^2$)

$$Re = \frac{D \times V_{avg} \times \rho}{\mu} = \frac{0{,}08 \times 0{,}265 \times 1019}{5 \times 10^{-3}} = 4324 > 2100 \Rightarrow$$

(5×10^{-3} ← Pa.s)

Flow is turbulent.

$$V_{max} = \frac{V_{avg}}{0{,}0336 \times \log Re + 0{,}662} \Rightarrow$$

$$V_{max} = \frac{0{,}265}{0{,}0336 \times \log(4324) + 0{,}662} = 0{,}337 \text{ m/s}$$

$$F_1 = \frac{L_1}{V_{max}} \Rightarrow L_1 = 0{,}337 \frac{m}{s} \times 1{,}5 \text{ min} \times \frac{60 s}{1 \text{ min}} \Rightarrow$$

$L_1 = 30{,}33$ m (Length of holding tube for 1.5 min heating)

$$F_2 = \frac{L_2}{V_{max}} \Rightarrow L_2 = 0{,}337 \times 1 \times 60 = 20{,}22 \text{ m}$$

(For heating 1 min)

b) For vegetative m.o., spoilage should be one can in 1×10^5 cans, i.e., SV = 5.

$$\log \frac{N_0}{N_f} = 5, \quad F = D \times (SV) = 0{,}5 \times 5 = 2{,}5 \text{ min.}$$

↳ $F_{95} = D_{95}^{*} (SV)$

$F_3 = \frac{L_3}{V_{max}} \Rightarrow L_3 = 0{,}337 \times 2.5 \times 60 = 50.55\ m$

(This is the length of holding tube for vegetative microorganisms)

8) The following temperature history resulted from the slowest heating point of a heat penetration run on a certain low acid canned food product:

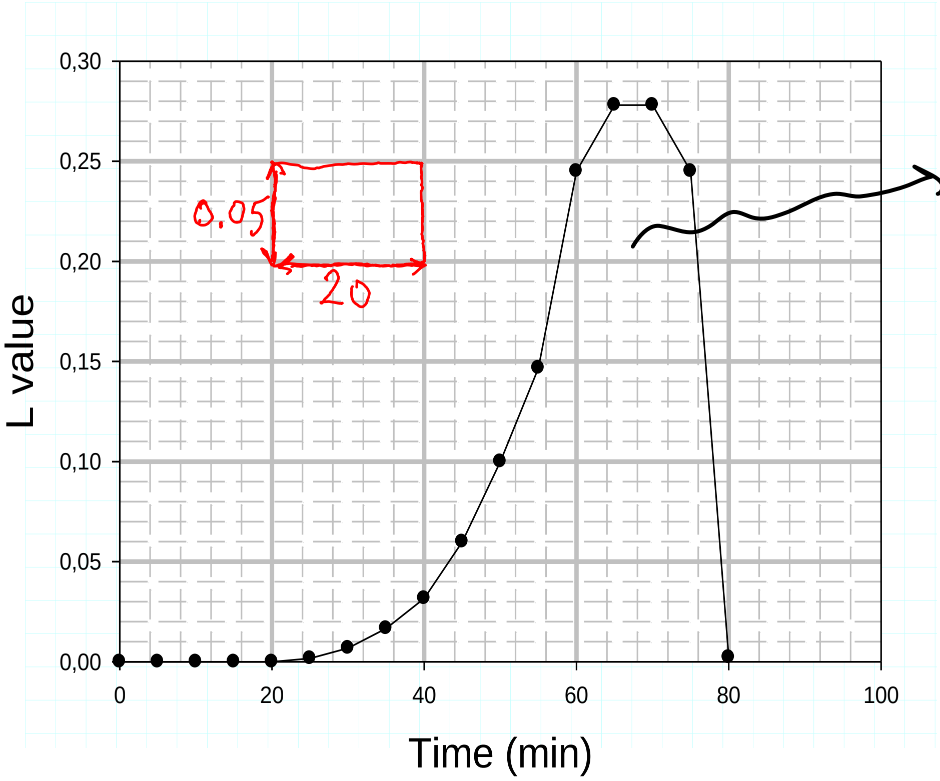
Calculate L values → $L = 10^{(T-250)/18}$

Time (min)	T (ºF)	$L = 10^{(T-To)/z}$	
0	141	0,000	→ 8.8×10^{-7}
5	149	0,000	→ 2.45×10^{-6}
10	162	0,000	→ 1.3×10^{-5}
15	172	0,000	→ 4.64×10^{-5}
2	189	0,000	→ 4.1×10^{-4}
25	200	0,002	→ 1.66×10^{-3}
30	211	0,007	
35	218	0,017	
40	223	0,032	
45	228	0,060	
50	232	0,100	
55	235	0,147	
60	239	0,245	
65	240	0,278	
70	240	0,278	

75	**239**	**0,245**
80	**202**	**0,002**

Does this process impart commercial sterility with respect to *Cl. botulinum* (i.e., if the sterilization process is adequate or not) using graphical method ? The Fo value for *Cl. botulinum* is 2.40 min, Z = 18°F, To = 250°F.

Solution:



Area = L×dt

1 square

0,05

20

Area of 1 square = 0,05×20 = 1 unit²

Find the total # of squares and multiply by 1 unit² = Area under curve.

\# of squares ≅ 7.25

Area = L×dt = 7.25 min = Fo calculated

Since Fo calculated (7.25) > Fo of Cl. botulinum (2.4 min)

=> The product is commercially sterile.

OR The sterilization process is adequate.

9) A process is based on an Fo value of 2.88 min. If a can contained 10 spores of organisms having a Do value of 1.5 min, then, calculate the probability of spoilage from the later organism.

Solution:

$$F_o = D_o \times SV, \quad SV = \log \frac{N_o}{N_f} = \frac{F_o}{D_o} = \frac{2.88}{1.5} = 1.92$$

$$\log \frac{N_o}{N_f} = 1.92 \Rightarrow \log N_o - \log N_f = 1.92$$

$$\log N_f = \log N_o - 1.92 = \log 10 - 1.92 = -0.92$$

$$\log N_f = -0.92 \Rightarrow N_f = 0.12 \Rightarrow$$

Probability of spoilage is 12 cans in 100 cans.

10) The Fo for *Cl. botulinum* type B is 1.1 min for 99.999 % inactivation. If the Z = 18ºF, what would F be at 275ºF ? Also, calculate F at 275ºF for 99.9999 % inactivation.

Solution:

Assume 100 spores initially $\Rightarrow$

$$F_o = 1.1 \text{ mins}, \quad SV = \log \frac{N_o}{N_f} = \log \frac{100}{(100 - 99.999)} \Rightarrow$$

$$SV = 5$$

$$\log \frac{F_{275}}{F_o} = \frac{T_o - 275}{18} \Rightarrow \log \frac{F_{275}}{1.1} = \frac{250 - 275}{18} \Rightarrow$$

$F_{275} = 0{,}0449$ min.

$F_{275} = D_{275} \times SV_{275} \Rightarrow 0{,}0449 = D_{275} \times \log \frac{100}{100 - 99.999}$ → 5

$D_{275} = \frac{0{,}0449}{5} = 0{,}00898$ min

For 99.9999 % inactivation at 275 °F ⇒

$F_{275} = D_{275} \times SV_{275} \Rightarrow$

$F_{275} = 0{,}00898 \times \log \frac{100}{100 - 99.9999}$ → 6

$F_{275} = 0{,}00898 \times 6 = 0{,}054$ min heating is required at 275°F for 99,9999 % inactivation of microorganism.

11) Browning reaction in the milk has been shown to have a Q_{10} of 1.585. This product is processed at 285°F and 260ºF to an equivalent heating time of Fo = 15 min ($T_0 = T_{ref} = 250$°F) separately in the holding tube of an aseptic canning system. Compare the extent of formation of the brown pigment between the product processed at these temperatures, i.e., C_{260}/C_{285} = ? Z = 22 °F. Assume a zero order kinetics for formation of brown pigment and initial concentration of pigment is zero.

Solution:

$\frac{dc}{dt} = +k \cdot c^n$, zero order $\Rightarrow n = 0 \Rightarrow \frac{dc}{dt} = k$

Integrate $\Rightarrow \int_{c_0}^{c} dc = k \cdot \int_{t_0}^{t} dt \Rightarrow c - \cancel{c_0}^{\,0} = k(t - \cancel{t_0}^{\,0})$

$\Rightarrow \boxed{c = k \cdot t}$ $t \rightsquigarrow F$

$F_0^{22} = 15$ min, $F_{260} = ?$, $F_{285} = ?$

$c_{260} = k_{260} \times F_{260}$, $c_{285} = k_{285} \times F_{285}$

$$\frac{F}{F_0} = 10^{\frac{250-T}{z}}$$

$F_0 \rightsquigarrow$ at 250°F

$$\frac{F}{F_0} = \frac{F_{260}}{F_0} = 10^{\frac{250-260}{22}} \Rightarrow \frac{F_{260}}{15} = 10^{-0,454} = 0,351$$

$\Rightarrow F_{260} = 15 \times 0,351 = 5,266$ min.

$$\frac{F_{285}}{F_0} = \frac{F_{285}}{15} = 10^{\frac{250-285}{22}} \Rightarrow F_{285} = 0,384 \text{ min}$$

$$Q_{10} = \frac{k_{285}}{k_{260}} = 1.585$$

$$k_{285} = 1.585 \times k_{260}$$

$$C_{260} = k_{260} \times F_{260}$$

$$C_{285} = k_{285} \times F_{285}$$

$$C_{285} = 1.585 \times k_{260} \times 0.384$$

$$\frac{C_{260} = k_{260} \times 5.266}{C_{285} = 1.585 \times k_{260} \times 0.384}$$

$$\frac{C_{260}}{C_{285}} = \frac{5.266}{0.608} = 8.66 \Rightarrow \frac{C_{260}}{C_{285}} = 8.66$$

∴ There would be 8.66 times more intense browning in the 260°F processed product compared to the one processed at 285°F.

i.e., LTLT = more quality damage.